Further discussion on Bayes' law

2012/2013 Semester I | MA4264 | Game Theory

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Here I would like to give some further discussion on Bayes' law.

What is Bayes' law Given an information set I, where this information set contains n decision nodes: $k_1, k_2, \ldots, k_n$, if the decision node k_i will be reached with probability p_i for each $i = 1, 2, \ldots, n$, then the belief on this information set should be as follows:

1. If $p_1 + p_2 + \cdots + p_n \neq 0$, then the player with the move should believe that the decision node k_i has been reached with probability

$$\frac{p_i}{p_1 + p_2 + \cdots + p_n}.$$

2. If $p_1 + p_2 + \cdots + p_n = 0$, the belief should be arbitrary.

When do we need to apply Bayes' law Given an information set I, we need to find the smallest subgame containing this information set I, then apply Bayes' law in this subgame.

It should be noted this criterion and the criterion I discuss in the tutorial are equivalent.

Examples 1 Assume that $((A, L, L'), (p, 1-p))$ is a perfect Bayesian equilibrium.

The subgame starting from the decision node of player 2 is the smallest subgame containing player 3's information set. In this subgame, we know the left node will be reached with probability 1, and right node will be reached with probability 0. Then by Bayes' law, we know that player 3's belief on this information set should be: player 3 believes the left node and right node have been reached with probabilities

$$\frac{1}{1+0} = 1 \text{ and } \frac{0}{1+0} = 0 \text{ respectively.}$$

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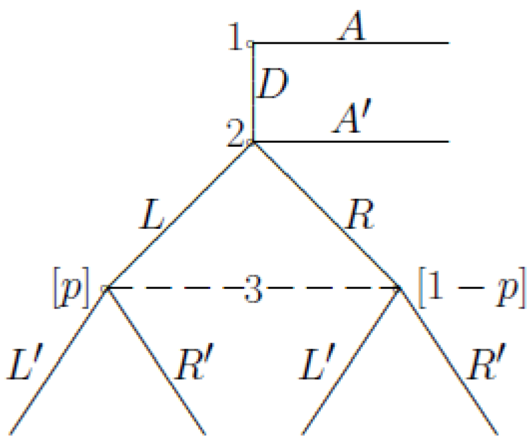

Figure 1

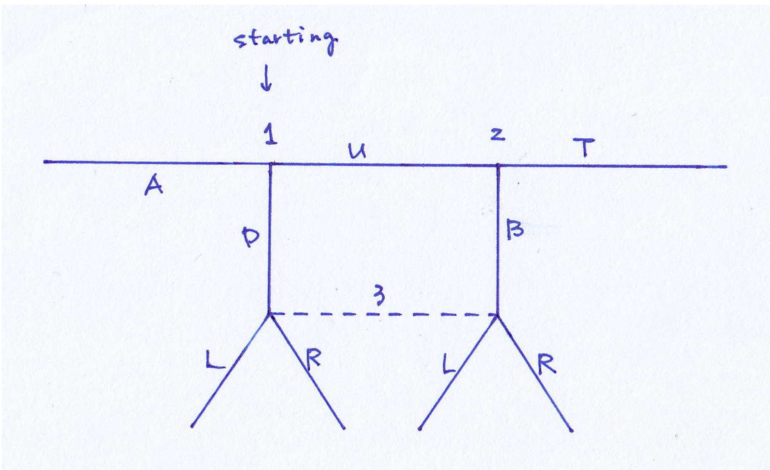

Figure 2

Examples 2 Assume $((A, B, L), (p, 1 - p))$ is a perfect Bayesian equilibrium, where p is the player 3's belief for the left decision node.

The only game containing player 3's information set is the original game, and in this game, the nodes in player 3's information set will be reached with probability 0 since player 1 chooses A and game ends. Then Bayes' law implies player 3's belief on it should be arbitrary.