Solution to Assignment 1

2012/2013 Semester I MA4264 Game Theory

Tutor: Xiang Sun*

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1. Question 4 part 2, in Tutorial set 1;
2. Question 3 part 2, in Tutorial set 2;
3. Question 8, in Tutorial set 2.

Solution for Question 1 Under the new payoff rules, the best response becomes:

$$R_i(s_j) = \begin{cases} \emptyset, & \text{if } s_j < \frac{1}{\sqrt{2}}; \\ [0,1], & \text{if } s_j \geq \frac{1}{\sqrt{2}}, \end{cases}$$

where $(i,j) = (1,2)$ or $(2,1)$. Note that when $s_j < \frac{1}{\sqrt{2}}$, Player i does not have the best response, because he will try to choose s_i as close as possible to $\sqrt{1/2 - s_j^2}$, but can not achieve $\sqrt{1/2 - s_j^2}$. The detailed discussion is as follows:

- For any $1 \geq s_i \geq \sqrt{1/2 - s_j^2}$, Player i's payoff is 0, which is less than the payoff when Player i chooses $\frac{1}{2}\sqrt{1/2 - s_j^2}$; Hence such a s_i can not be a best response.
- For any $0 \leq s_i < \sqrt{1/2 - s_j^2}$, Player i's payoff is s_i, which is less than the payoff when Player i chooses $\frac{s_i + \sqrt{1/2 - s_j^2}}{2}$; Hence such a s_i can not be a best response.

Therefore, the pure-strategy Nash equilibria are

$$\left[\frac{1}{\sqrt{2}}, 1\right] \times \left[\frac{1}{\sqrt{2}}, 1\right].$$

Solution for Question 2 If $0 < c_1 < c_2 < a$ and $2c_2 > a + c_1$, then $a - c_1 > a - c_2 > \frac{a-c_2}{2} > 0$ and $\frac{a-c_1}{2} > a - c_2 > 0$. Hence we have the Figure (1), and from it we will obtain the Nash equilibrium: $(\frac{a-c_1}{2}, 0)$.

*Email: xiangsun@nus.edu.sg. Suggestion and comments are always welcome.

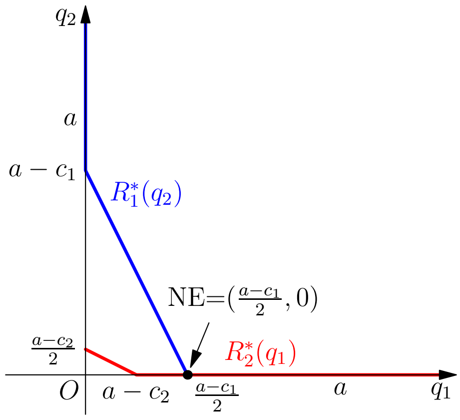

Figure 1: Intersection of best-response correspondences

Solution for Question 3

(i) (1) C is strictly dominated by A and will be eliminated;

(2) L is strictly dominated by M and will be eliminated;

(3) B is strictly dominated by D and will be eliminated.

Hence we will obtain the reduced game G_1.

G_1:

		Player 2: M	Player 2: R
Player 1	A	$2, 5$	$2, 0$
	D	$1, 1$	$3, 3$

G_2:

		Player 2: M	Player 2: R
Player 1	A	$2, 5$	$2, 0$
	D	$1, 1$	$3, 3$

(ii) From the bi-matrix G_2, we obtain the pure-strategy Nash equilibria: (A, M) and (D, R) (red pairs) with payoffs $(2, 5)$ and $(3, 3)$, respectively.

(iii) Let $p_1 = (r, 1-r)$ be a mixed strategy in which Player 1 plays A with probability r. Let $p_2 = (q, 1-q)$ be a mixed strategy in which Player 2 plays M with probability q. Then Player 1's expected payoff is:

$$U_1(A, p_2) = 2q + 2(1-q) = 2,$$
$$U_1(D, p_2) = q + 3(1-q) = 3 - 2q.$$

Hence

$$r^*(q) \equiv \arg\max_{0 \le r \le 1} U_1(p_1, p_2) = \begin{cases} \{1\}, & \text{if } q > \frac{1}{2}; \\ \{0\}, & \text{if } q < \frac{1}{2}; \\ [0, 1], & \text{if } q = \frac{1}{2}. \end{cases}$$

Similarly, Player 2's expected payoff is:

$$U_2(p_1, M) = 5r + (1-r) = 1 + 4r,$$
$$U_2(p_1, R) = 3(1-r).$$

Hence

$$q^*(r) \equiv \arg\max_{0\le q\le 1} U_2(p_1,p_2) = \begin{cases} \{1\}, & \text{if } r > \frac{2}{7}; \\ \{0\}, & \text{if } r < \frac{2}{7}; \\ [0,1], & \text{if } r = \frac{2}{7}. \end{cases}$$

We draw the graphs of $r^*(q)$ and $q^*(r)$ together:

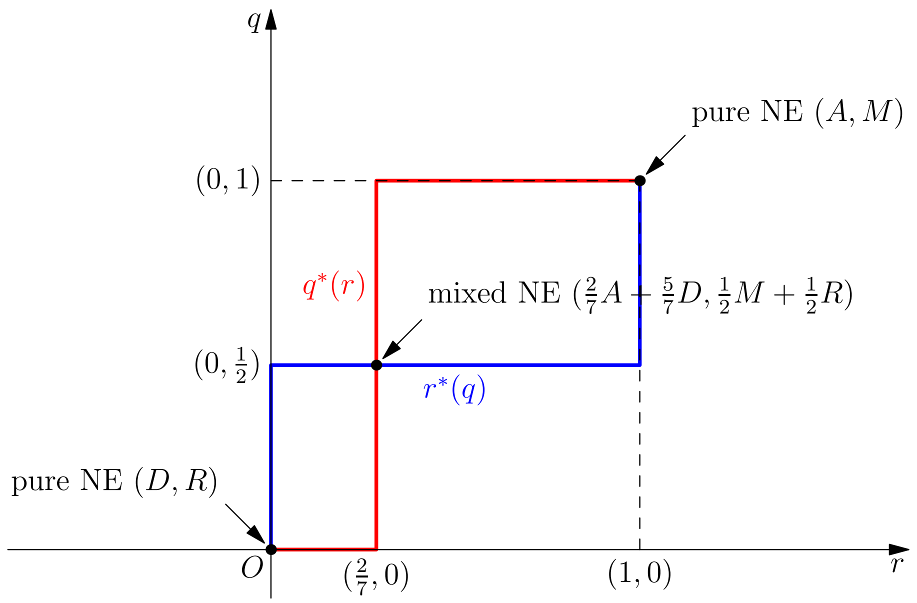

Figure 2: Intersection of best-response correspondences

The graphs of the best response correspondences $r^*(q)$ and $q^*(r)$ intersect at 3 points $(r = \frac{2}{7}, q = \frac{1}{2})$, $(0,0)$ and $(1,1)$. Hence, there are 3 mixed-strategy Nash equilibria:

- $(1A, 1M)$ (or $((1,0,0,0),(0,1,0))$) with expected payoff $(2,5)$,
- $(1D, 1R)$ (or $((0,0,0,1),(0,0,1))$) with expected payoff $(3,3)$,
- $(\frac{2}{7}A + \frac{5}{7}D, \frac{1}{2}M + \frac{1}{2}R)$ (or $((\frac{2}{7},0,0,\frac{5}{7}),(0,\frac{1}{2},\frac{1}{2}))$), with expected payoff $(2, \frac{15}{7})$.