Solution to Assignment 2

2012/2013 Semester I MA4264 Game Theory

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1. Question 7, in Tutorial set 3;
2. Question 5, Game 2, in Tutorial set 4;
3. Question 7, in Tutorial set 4.

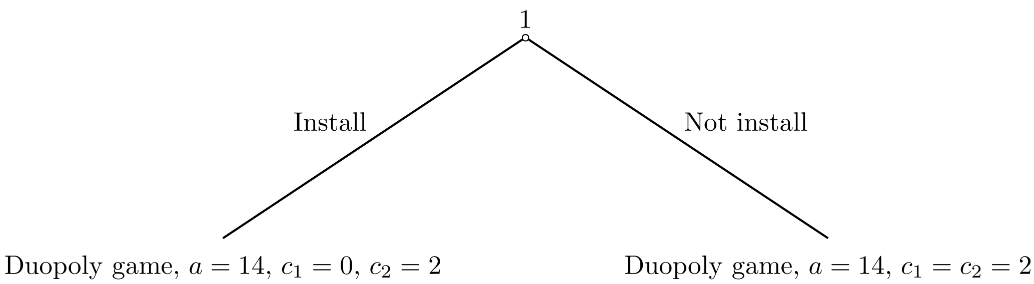

Figure 1

Solution for Question 1 Figure 1 is the extensive-form representation of the game. There are 2 stages:

- In the first stage, Player 1 choose "Install" or "Not install";
- In the second stage, Players 1 and 2 play the Cournot Duopoly Game.

1. If Player 1 chooses "Install" in the first stage, then $a = 14$, $c_1 = 0$, $c_2 = 2$. Since $0 \leq c_i < \frac{a}{2}$, by Question 3 in Tutorial 2, the unique Nash equilibrium is $(q_1^*, q_2^*) = (\frac{a-2c_1+c_2}{3}, \frac{a-2c_2+c_1}{3}) = (\frac{16}{3}, \frac{10}{3})$, and Player 1's payoff is $\frac{16}{3}(14 - \frac{16}{3} - \frac{10}{3}) - 8 = 20\frac{4}{9}$.
2. If Player 1 chooses "Not install" in the first stage, then $a = 14$, $c_1 = c_2 = 2 < a$, and the unique Nash equilibrium is $(q_1^*, q_2^*) = (\frac{a-c}{3}, \frac{a-c}{3}) = (4, 4)$. Player 1's payoff is $4(14 - 8 - 2) = 16$.

Since $16 < 20\frac{4}{9}$, the subgame-perfect outcome is: Player 1 chooses "Install" in the first stage, and Players 1 and 2 choose $\frac{16}{3}$ and $\frac{10}{3}$, respectively in the second stage.

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Solution for Question 2

(i) From Figure 2a, we have the subgame-perfect outcome: since M is strictly dominated by U, we only need to consider the reduced game, displayed in Figure 2b. Hence the subgame-perfect outcome is: Player 1 chooses U in the first stage, and Player 2 chooses L in the second stage.

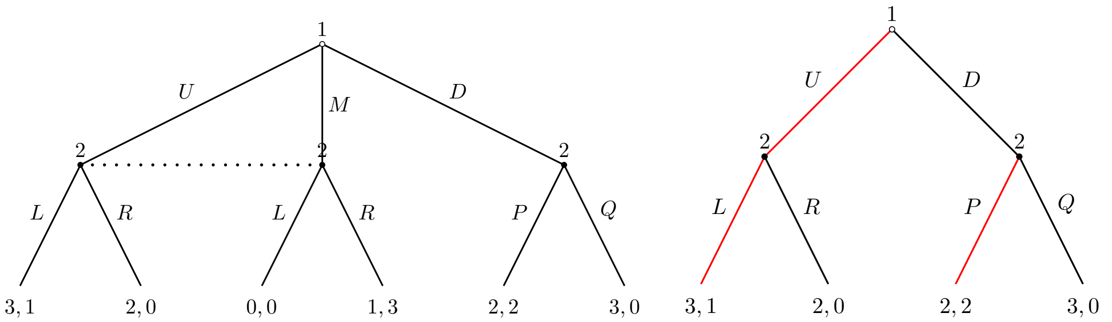

Figure 2: The extensive-form representation and subgame-perfect outcome

(ii,iii) Figure 3 is the normal-form representation, and it tells us the all Nash equilibria: (U, LP), (U, LQ) and (D, RP).

		Player 2			
		LP	LQ	RP	RQ
Player 1	U	3, 1	3, 1	2, 0	2, 0
	M	0, 0	0, 0	1, 3	1, 3
	D	2, 2	3, 0	2, 2	3, 0

Figure 3: The normal-form representation and Nash equilibria

(iv) There is only one subgame, in which Player 2 will choose P. Therefore the subgame-perfect Nash equilibria are (U, LP) and (D, RP).

Remark: there is no subgame-perfect outcome for Game 4.

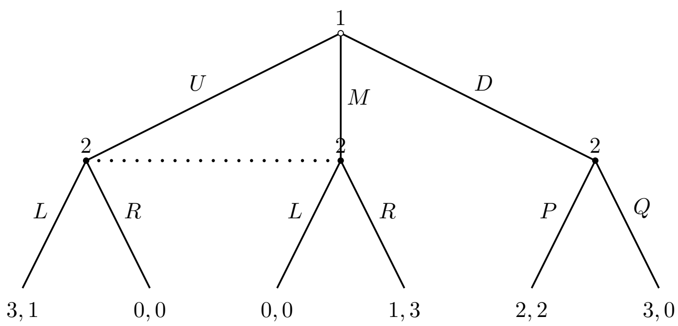

Figure 4: There is no subgame-perfect outcome

Solution for Question 3 Figure 5 is the extensive-form representation of the game. It is easy to see that Player 1's strategy space is $S_1 = \{0, 1, \ldots, 20\}$. Since a strategy is

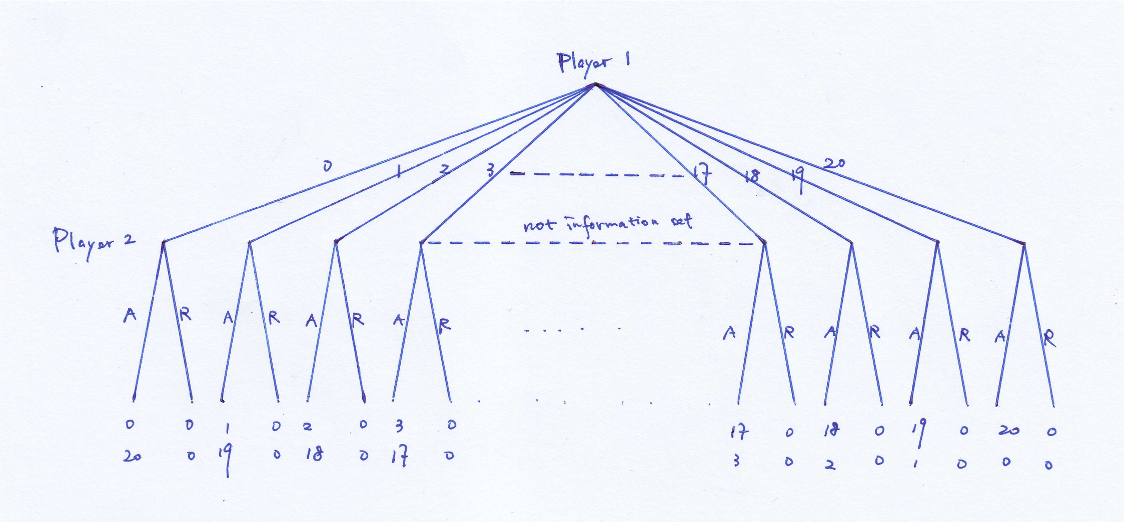

Figure 5: The extensive-form representation of the game

a complete plan of actions in every contingency when a player is called upon to make, a strategy for Player 2 can be represented as a function

$$f\colon S_1 \to \{A, R\}.$$

For example,

$$f(s_1) = \begin{cases} A, & \text{if } 0 \le s_1 \le 10; \\ R, & \text{otherwise} \end{cases}$$

is a strategy of Player 2 in which Player 2 will accept if Player 1 offers any $s_1 \le 10$ and otherwise she will reject.

Thus, the space of all strategies of Player 2 is the set of all functions from S_1 to $\{A, R\}$. We denote it by S_2.

(i)

- Player 1's best-response correspondence: Given a strategy f of Player 2, note that for any $s_1 \in f^{-1}(A)$, Player 2 will accept the offer. Hence, given f, Player 2 will choose the maximum in $f^{-1}(A)$. Since $f^{-1}(A)$ is a subset of S_1, the maximal always exists. Thus, Player 1's best-response correspondence is

$$B_1^*(f) = \begin{cases} S_1, & \text{if } f^{-1}(A) = \emptyset; \\ S_1, & \text{if 0 is the maximum of } f^{-1}(A); \\ \{s^*\}, & \text{if } f^{-1}(A) \text{ has a maximum } s^* \neq 0. \end{cases}$$

- Player 2's best-response correspondence: note that Player 2's strategy is a function

$$B_2^*(s_1) = \begin{cases} \{f \in S_2 \colon f(s_1) = A\}, & \text{if } s_1 < 20; \\ S_2, & \text{if } s_1 = 20. \end{cases}$$

That means for any $s_1 < 20$, Player 2 will accept. If $s_1 = 20$, Player 2 is indifferent between the two actions (accept or reject).

- We can use various combinations of the conditions in the expression of B_1^* and B_2^* to construct all the Nash equilibria:

- When ${f^*}^{-1}(A) \neq \emptyset$, (s_1^*, f^*) is a Nash equilibrium if and only if $s_1^* = \max {f^*}^{-1}(A)$;
- When ${f^*}^{-1}(A) = \emptyset$, (s_1^*, f^*) is a Nash equilibrium if and only if $s_1^* = 20$.

(ii) For each given s_1, we need to consider a corresponding subgame, displayed in Figure 6. We know if f^* is subgame-perfect, $f^*(s_1) = A$ for any $s_1 < 20$. Hence, if (s_1^*, f^*)

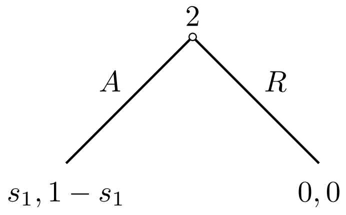

Figure 6

is subgame-perfect, f^* should be either f_1^* or f_2^*:

$$f_1^*(s_1) = \begin{cases} A, & \text{if } s_1 = 0, 1, \ldots, 19; \\ R, & \text{if } s_1 = 20. \end{cases} \quad \text{or } f_2^*(s_1) \equiv A \text{ for all } s_1.$$

It is easy to check that there are 2 subgame-perfect Nash equilibria: $(s_1^* = 19, f_1^*)$ and $(s_1^* = 20, f_2^*)$.