Solution to Tutorial 7*

2011/2012 Semester I MA4264 Game Theory

Tutor: Xiang Sun†

October 12, 2011

1 Review

How to find Bayesian Nash equilibrium(a)?

1. Find all the type spaces $\{T_i\}_{i\in N}$;
2. Construct a new game (which is called "agent strategic game") as follows:
   - Player set: $N' = \{(i, t_i) : i \in N,\ t_i \in T_i\}$;
   - Action spaces: for any $(i, t_i) \in N'$, his action space is $A_{(i,t_i)} = A_i$;
   - Payoff functions: for any $(i, t_i) \in N'$, his payoff function is $u_{(i,t_i)} = u_i$.
3. Find all the Nash equilibrium(a) for the new game;
4. Pull back: each Nash equilibrium can be reformulated to a Bayesian Nash equilibrium in the original game.

2 Tutorial

Exercise 1. *Two individuals are involved in a synergistic relationship. If both individuals devote more effort to the relationship, they are both better off. Specifically, an effort level is a nonnegative number, and player 1's payoff function is* $e_1(1 + e_2 - e_1)$*, where* e_i *is player* i*'s effort level. For player 2 the cost of effort is either the same as that of player 1, and hence her payoff function is given by* $e_2(1 + e_1 - 2e_2)$*, or effort is very costly for her in which case her payoff function is given by* $e_2(1 + e_1 - e_2)$*. Player 2 knows player 1's payoff function and whether the cost of effort is high for herself or not. Player 1, however, is uncertain about player 2's cost of effort. He believes that the cost of effort is low with probability* p*, and high with probability* $1 - p$*, where* $0 < p < 1$*. Find the Bayesian Nash equilibrium of this game as a function of* p*.*

*Corrections are always welcome.

†Email: xiangsun@nus.edu.sg; Mobile: 9169 7677; Office: S17-06-14.

Solution.

- There are two players;
- Action spaces: $A_1 = A_2 = [0, \infty)$;
- Type spaces: $T_1 = \{\{H, L\}\}$, and $T_2 = \{H, L\}$;
- Strategy spaces: $S_1 = \{e_1 \colon e_1 \geq 0\}$, and $S_2 = \{(e_{2H}, e_{2L}) \colon e_{2H}, e_{2L} \geq 0\}$.

Let $(e_1^*, e_{2H}^*, e_{2L}^*)$ be a Bayesian Nash equilibrium, then we will have:

1. Player 1 does not know the exact type of the cost of effort, so he will choose q_1^* to maximize his expected payoff

$$p \times e_1(1 + e_{2L}^* - e_1) + (1 - p) \times e_1(1 + e_{2H}^* - e_1),$$

and hence

$$e_1^* = \frac{1 + pe_{2L}^* + (1 - p)e_{2H}^*}{2}. \tag{1}$$

2. For Player 2, if the cost of effort is high, then Player 2 will choose q_{2H}^* to maximize his payoff

$$e_{2H}(1 + e_1^* - 2e_{2H}),$$

and hence

$$e_{2H}^* = \frac{1 + e_1^*}{4}. \tag{2}$$

3. For Player 2, if the cost of effort is low, then Player 2 will choose q_{2L}^* to maximize his payoff

$$e_{2L}(1 + e_1^* - e_{2L}),$$

and hence

$$e_{2L}^* = \frac{1 + e_1^*}{2}. \tag{3}$$

Solving Equations (1), (2) and (3), we will have

$$e_1^* = \frac{5 + p}{7 - p}, \quad e_{2H}^* = \frac{3}{7 - p}, \quad e_{2L}^* = \frac{6}{7 - p}.$$

□

Exercise 2. *Two people are involved in a dispute. Person 1 does not know whether person 2 is strong or weak; she believes that person 2 is equally likely being strong and weak. Person 2 is fully informed. Each person can either fight or yield. Each person obtains a payoff of 0 is she yields (regardless of the other person's action) and a payoff of 2 if she fights and her opponent yields. If both people fight then their payoff are* $(-2, 1)$ *if person 2 is strong and* $(1, -2)$ *if person 2 is weak. Formulate the situation as a Bayesian game and find all Bayesian Nash equilibria of the game.*

Solution (first method). Let Game 1 and Game 2 be as follows:

Game 1, $t_2 = S$

		Person 2: F	Person 2: Y
Person 1	F	$-2, 1$	$2, 0$
	Y	$0, 2$	$0, 0$

Game 2, $t_2 = W$

		Person 2: F	Person 2: Y
Person 1	F	$1, -2$	$2, 0$
	Y	$0, 2$	$0, 0$

- There are two players;
- Action spaces: $A_1 = A_2 = \{F, Y\}$, where F and Y stand for "Fight" and "Yield", respectively;
- Type spaces: $T_1 = \{\{S, W\}\}$, and $T_2 = \{S, W\}$;
- Strategy spaces: $S_1 = \{F, Y\}$, and $S_2 = \{FF, FY, YF, YY\}$.

let a be Person 1's action, and b_1 and b_2 Person 2's actions in Game 1 and Game 2, respectively.

1. Person 1 does not know the exact type of the cost of effort, so he will choose a to maximize her expected payoff. The following table is Person 2's expected payoff table: Thus we get Person 1's best-response correspondence:

		Person 2: FF	FY	YF	YY
Person 1	F	-0.5	0	1.5	2
	Y	0	0	0	0

$$a^*(b_1, b_2) = \begin{cases} \{Y\}, & \text{if } b_1b_2 = FF; \\ \{F, Y\}, & \text{if } b_1b_2 = FY; \\ \{F\}, & \text{if } b_1b_2 = YF; \\ \{F\}, & \text{if } b_1b_2 = YY. \end{cases}$$

2. If Game 1 is drawn, then Person 2's best-response correspondence is

$$b_1^*(a) = \begin{cases} \{F\}, & \text{if } a = F; \\ \{F\}, & \text{if } a = Y. \end{cases}$$

3. If Game 2 is drawn, then Person 2's best-response correspondence is

$$b_2^*(a) = \begin{cases} \{Y\}, & \text{if } a = F; \\ \{F\}, & \text{if } a = Y. \end{cases}$$

Therefore, by definition, we will get all the Bayesian Nash equilibria: (F, FY) and (Y, FF). The reason is as follows:

- If Person 1 chooses F, then Person 2 will choose F and Y in Game 1 and Game 2, respectively. Note that F is the unique best response of Person 1 when Person 2 chooses FY;

  So, given that Person 1 plays F, the only possible pure-strategy Bayesian Nash equilibrium is (FY, F) in this case.

- If Person 1 chooses Y, then Person 2 will choose F in each game. Note that Y is the unique best response of Person 1 when Person 2 chooses YY;

  So, given that Person 1 plays Y, the only possible pure-strategy Bayesian Nash equilibrium is (FF, Y) in this case.

$\square$

Solution (second method). The related agent strategic game is as follows:

- Player set: $N' = \{1, (2, S), (2, W)\}$;
- Action sets: $A_1 = A_{(2,S)} = A_{(2,W)} = \{F, Y\}$;
- Payoff functions: $u_{(2,S)} = u_{(2,W)} = u_2$.

It is a three-person static game with complete information, and the payoff table is as follows: There are 2 Nash equilibria: (F, F, Y) and (Y, F, F). Thus, all the

		(2, S) and (2, W)			
		FF	FY	YF	YY
1	F	$-0.5, 1, -2$	$0, 1, 0$	$1.5, 0, -2$	$2, 0, 0$
	Y	$0, 2, 2$	$0, 2, 0$	$0, 0, 2$	$0, 0, 0$

Bayesian Nash equilibria of the original game are (F, FY) and (Y, FF). $\square$

Exercise 3. *A firm and a worker play a double auction. The firm knows the worker's marginal product (m) and the worker knows his or her outside opportunity (v), respectively. In this context, trade means that the worker is employed by the firm. A wage w is preset by the union. If there is trade, then the firm's payoff is $m - w$ and the worker's is w; if there is no trade then the firm's payoff is zero and the worker's is v. Suppose that m and v are independent draws from a uniform distribution on $[0, 1]$. The both players simultaneously announce either that they Accept the wage w or that they Reject that wage. The worker will be employed by the firm if and only if both of them accept the wage. Given an arbitrary value of w from $[0, 1]$, what is the Bayesian Nash equilibrium of this game? Draw a diagram showing the type-pairs that trade. Find the value of w that maximizes the sum of the players' expected payoff and compute this maximized sum.*

Solution.

- There are two players: firmer and worker;
- Type spaces: $T_f = \{m \colon m \in [0, 1]\}$, and $T_w = \{v \colon v \in [0, 1]\}$;
- Action spaces: $A_f = A_w = \{A, R\}$;

- Strategy spaces: $S_f = S_w = \{\text{function from } [0,1] \text{ to } \{A, R\}\}$;
- Payoff functions:

$$u_f(s_f(w), s_w(v); m, v) = \begin{cases} m - w, & \text{if } s_f(w) = s_w(v) = A; \\ 0, & \text{otherwise.} \end{cases}$$

$$u_w(s_f(w), s_w(v); m, v) = \begin{cases} w, & \text{if } s_f(w) = s_w(v) = A; \\ v, & \text{otherwise.} \end{cases}$$

(i) For any $w \in [0,1]$, it is easy to see $(s_f^*(m), s_w^*(v))$ is a Bayesian Nash equilibrium, where

$$s_f^*(m) = \begin{cases} A, & \text{if } m \geq w \\ R, & \text{otherwise} \end{cases}, \qquad s_w^*(v) = \begin{cases} A, & \text{if } w \geq v \\ R, & \text{otherwise} \end{cases}.$$

(ii) There is trade when (m, v) is drawn if and only if $s_f^*(m) = s_w^*(v) = A$, and thus T is the trading area in Figure 1.

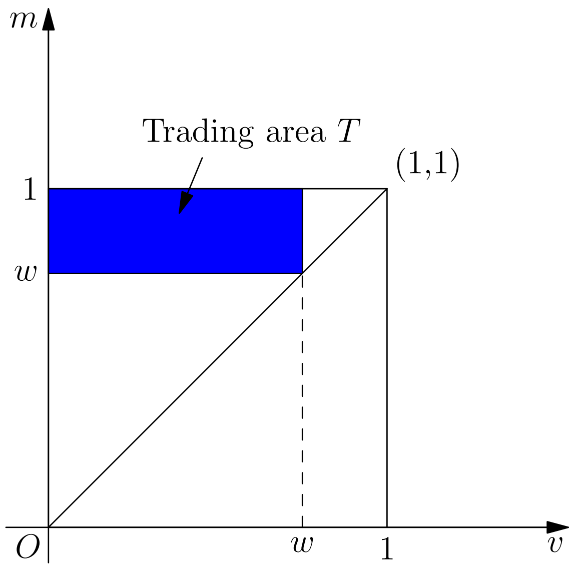

Figure 1: Trading area T

(iii) In the Bayesian Nash equilibrium, the payoff are as follows:

$$u_f(m, v) = \begin{cases} m - w, & \text{if } (m, v) \in T \\ 0, & \text{otherwise} \end{cases}, \qquad u_w(m, v) = \begin{cases} w, & \text{if } (m, v) \in T \\ v, & \text{otherwise} \end{cases}.$$

Since m and v are uniformly distributed on $[0,1]$, we have:

$$\mathbb{E}[u_f] = \int_0^1 \int_0^1 u_f(m, v)\, \mathrm{d}v\, \mathrm{d}m = \iint_T (m - w)\, \mathrm{d}v\, \mathrm{d}m$$

$$\mathbb{E}[u_w] = \int_0^1 \int_0^1 u_w(m, v)\, \mathrm{d}v\, \mathrm{d}m = \iint_T w\, \mathrm{d}v\, \mathrm{d}m + \iint_{T^c} v\, \mathrm{d}v\, \mathrm{d}m$$

$$= \iint_T (w - v)\, \mathrm{d}v\, \mathrm{d}m + \int_0^1 \int_0^1 v\, \mathrm{d}v\, \mathrm{d}m$$

and thus

$$\begin{aligned}
\mathbb{E}[u_f] + \mathbb{E}[u_w] &= \iint_T (m - v)\, \mathrm{d}v\, \mathrm{d}m + \int_0^1 \int_0^1 v\, \mathrm{d}v\, \mathrm{d}m \\
&= \int_w^1 \int_0^w (m - v)\, \mathrm{d}v\, \mathrm{d}m + \int_0^1 \int_0^1 v\, \mathrm{d}v\, \mathrm{d}m \\
&= \frac{w - w^2}{2} + \frac{1}{2}
\end{aligned}$$

Therefore, $w^* = \frac{1}{2}$ is the maximizer of the sum of the expected payoff.

□

Exercise 4. *Consider the double auction where the seller's and buyer's valuations, v_s and v_b, are uniformly distributed on $[\alpha_s, \beta_s]$ and $[\alpha_b, \beta_b]$, respectively. Find the linear Bayesian Nash equilibrium of the game.*

Solution.

- There are two players: seller (s) and buyer (b);
- Type spaces: $T_s = [\alpha_s, \beta_s]$ and $T_b = [\alpha_b, \beta_b]$;
- Action spaces: $A_s = A_b = [0, \infty)$;
- Strategy spaces: $S_b = \{\text{function from } T_b \text{ to } A_b\}$, and $S_s = \{\text{function from } T_s \text{ to } A_s\}$;
- Payoff:

$$u_s(p_s, p_b; v_s, v_b) = \begin{cases} \frac{p_s+p_b}{2} - v_s, & p_b \geq p_s \\ 0, & p_b < p_s \end{cases},$$

$$u_b(p_s, p_b; v_s, v_b) = \begin{cases} v_b - \frac{p_s+p_b}{2}, & p_b \geq p_s \\ 0, & p_b < p_s \end{cases}.$$

Suppose (p_s^*, p_b^*) is a linear Bayesian Nash equilibrium, where

$$p_s^*(v_s) = a_s + c_s v_s, \quad p_b^*(v_b) = a_b + c_b v_b.$$

Note that a_s, c_s, a_b, c_b are to be determined. Here we should assume $c_s, c_b > 0$.

- For seller, when v_s is drawn, given buyer's strategy p_b^*, $p_s^*(v_s)$ will maximize his expected payoff

$$\begin{aligned}
&\mathbb{E}[u_s(p_s, p_b^*; v_s, v_b)] \\
&= \frac{1}{\beta_b - \alpha_b} \int_{p_s \le p_b^*(v_b) \le p_b^*(\beta_b)} \frac{p_s + p_b^*(v_b)}{2} - v_s \, \mathrm{d}v_b + \frac{1}{\beta_b - \alpha_b} \int_{p_b^*(\alpha_b) \le p_b^*(v_b) < p_s} 0 \, \mathrm{d}v_b \\
&= \frac{1}{\beta_b - \alpha_b} \int_{\frac{p_s - a_b}{c_b}}^{\beta_b} \frac{p_s + a_b + c_b v_b}{2} - v_s \, \mathrm{d}v_b \\
&= \frac{1}{\beta_b - \alpha_b} \left[\left(\frac{p_s + a_b}{2} - v_s \right) \left(\beta_b - \frac{p_s - a_b}{c_b} \right) + \frac{c_b}{2} \int_{\frac{p_s - a_b}{c_b}}^{\beta_b} v_b \, \mathrm{d}v_b \right] \\
&= \frac{1}{\beta_b - \alpha_b} \left[\left(\frac{p_s + a_b}{2} - v_s \right) \left(\beta_b - \frac{p_s - a_b}{c_b} \right) + \frac{c_b}{4} \left(\beta_b - \frac{p_s - a_b}{c_b} \right) \left(\beta_b + \frac{p_s - a_b}{c_b} \right) \right] \\
&= \frac{1}{\beta_b - \alpha_b} \left(\beta_b - \frac{p_s - a_b}{c_b} \right) \left[\left(\frac{p_s + a_b}{2} - v_s \right) + \frac{c_b}{4} \left(\beta_b + \frac{p_s - a_b}{c_b} \right) \right] \\
&= \frac{c_b}{\beta_b - \alpha_b} (c_b \beta_b - p_s + a_b) \left[-v_s + \frac{3}{4} p_s + \frac{1}{4} (a_b + c_b \beta_b) \right]
\end{aligned}$$

Therefore, by the first order condition,

$$p_s^*(v_s) = \frac{2}{3} v_s + \frac{1}{3} a_b + \frac{1}{3} c_b \beta_b,$$

and hence

$$c_s = \frac{2}{3}, \quad a_s = \frac{1}{3}(a_b + c_b \beta_b). \tag{4}$$

- For buyer, when v_b is drawn, given seller's strategy p_s^*, $p_b^*(v_b)$ will maximize his expected payoff

$$\begin{aligned}
&\mathbb{E}[u_b(p_s^*, p_b; v_s, v_b)] \\
&= \frac{1}{\beta_s - \alpha_s} \int_{p_s^*(\alpha_s) \le p_s^*(v_s) \le p_b} v_b - \frac{p_s^*(v_s) + p_b}{2} \, \mathrm{d}v_s + \frac{1}{\beta_s - \alpha_s} \int_{p_b < p_s^*(v_s) \le p_s^*(\beta_s)} 0 \, \mathrm{d}v_s \\
&= \frac{1}{\beta_s - \alpha_s} \int_{\alpha_s}^{\frac{p_b - a_s}{c_s}} v_b - \frac{a_s + c_s v_s + p_b}{2} \, \mathrm{d}v_s \\
&= \frac{1}{\beta_s - \alpha_s} \left[\left(v_b - \frac{a_s + p_b}{2} \right) \left(\frac{p_b - a_s}{c_s} - \alpha_s \right) - \frac{c_s}{2} \int_{\alpha_s}^{\frac{p_b - a_s}{c_s}} v_s \, \mathrm{d}v_s \right] \\
&= \frac{1}{\beta_s - \alpha_s} \left[\left(v_b - \frac{a_s + p_b}{2} \right) \left(\frac{p_b - a_s}{c_s} - \alpha_s \right) - \frac{c_s}{4} \left(\frac{p_b - a_s}{c_s} - \alpha_s \right) \left(\frac{p_b - a_s}{c_s} + \alpha_s \right) \right] \\
&= \frac{1}{\beta_s - \alpha_s} \left(\frac{p_b - a_s}{c_s} - \alpha_s \right) \left[\left(v_b - \frac{a_s + p_b}{2} \right) - \frac{c_s}{4} \left(\frac{p_b - a_s}{c_s} + \alpha_s \right) \right] \\
&= \frac{c_s}{\beta_s - \alpha_s} (p_b - a_s - c_s \alpha_s) \left[v_b - \frac{3}{4} p_b - \frac{1}{4} (a_s + c_s \alpha_s) \right]
\end{aligned}$$

Therefore, by the first order condition,

$$p_b^*(v_b) = \frac{2}{3} v_b + \frac{1}{3} a_s + \frac{1}{3} c_s \alpha_s,$$

and hence

$$c_b = \frac{2}{3}, \quad a_b = \frac{1}{3}(a_s + c_s \alpha_s). \tag{5}$$

Solving Equations (4) and (5), we will have

$$a_s = \frac{\alpha_s}{12} + \frac{\beta_b}{4}, \quad a_b = \frac{\beta_b}{12} + \frac{\alpha_s}{4}.$$

□

Exercise 5. *Consider the following first-price sealed-bid auction. Suppose there are two bidders, $i = 1, 2$. The bidders' valuations v_1 and v_2 for a good are independently and uniformly distributed on $[0, 1]$. The bidders have preferences represented by the utility functions $u_i(x) = x^{\alpha_i}$ where $0 < \alpha_i \leq 1$, $i = 1, 2$. Bidders submit their bids b_1 and b_2 simultaneously. The higher bidder wins the good and pays her bidding price, so that $x = v_i - b_i$; the other bidder gets and pays nothing, so that $x = 0$. In the case that $b_1 = b_2$, the winner is determined by a flip of a coin. Find a Bayesian Nash equilibrium (b_1, b_2) in which b_i is a linear function of v_i, $i = 1, 2$.*

Solution.

- There are two players;
- Type spaces: $T_1 = \{v_1 : v_1 \in [0, 1]\}$, and $T_2 = \{v_2 : v_2 \in [0, 1]\}$;
- Action spaces: $A_1 = A_2 = [0, \infty)$;
- Strategy spaces: $S_1 = \{\text{function from } T_1 \text{ to } A_1\}$ and $S_2 = \{\text{function from } T_2 \text{ to } A_2\}$;
- Payoff:

$$u_i(b_i, b_j; v_i, v_j) = \begin{cases} (v_i - b_i)^{\alpha_i}, & \text{if } b_i > b_j; \\ 0, & \text{if } b_i < b_j. \end{cases}$$

Suppose (b_1^*, b_2^*) is a linear Bayesian Nash equilibrium, where

$$b_i^*(v_i) = a_i + c_i v_i, \quad i = 1, 2,$$

where a_i, c_i are to be determined. Here we should assume $c_i > 0$.

- For Bidder 1, when v_1 is drawn, given Bidder 2's strategy b_2^*, $b_1^*(v_1)$ will maximize his expected payoff

$$\begin{aligned} \mathbb{E}[u_1(p_1, p_2^*(v_2); v_1, v_2)] &= (v_1 - b_1)^{\alpha_1} \operatorname{Prob}\{b_2^*(0) \leq b_2^*(v_2) < b_1\} \\ &= (v_1 - b_1)^{\alpha_1} \operatorname{Prob}\left\{0 \leq v_2 < \frac{b_1 - a_2}{c_2}\right\} \\ &= (v_1 - b_1)^{\alpha_1} \frac{b_1 - a_2}{c_2}. \end{aligned}$$

Note that when Bidder 1 chooses b_1, the probability that $b_1 = b_2^*(v_2)$ is 0, and thus we do not need to consider that.

Therefore

$$b_1^*(v_1) = \frac{\alpha_1}{1 + \alpha_1} a_2 + \frac{1}{1 + \alpha_1} v_1,$$

and hence

$$a_1 = \frac{\alpha_1}{1+\alpha_1} a_2, \quad c_1 = \frac{1}{1+\alpha_1}. \tag{6}$$

- For Bidder 2, when v_2 is drawn, given Bidder 1's strategy b_1^*, $b_2^*(v_2)$ will maximize his expected payoff

$$\begin{aligned}\mathbb{E}[u_2(p_1^*(v_1), p_2; v_1, v_2)] &= (v_2 - b_2)^{\alpha_2} \operatorname{Prob}\{b_1^*(0) \le b_1^*(v_1) < b_2\} \\ &= (v_2 - b_2)^{\alpha_2} \operatorname{Prob}\left\{0 \le v_1 < \frac{b_2 - a_1}{c_1}\right\} \\ &= (v_2 - b_2)^{\alpha_2} \frac{b_2 - a_1}{c_1}.\end{aligned}$$

  Note that when Bidder 2 chooses b_2, the probability that $b_2 = b_1^*(v_1)$ is 0, and thus we do not need to consider that.

  Therefore

$$b_2^*(v_2) = \frac{\alpha_2}{1+\alpha_2} a_1 + \frac{1}{1+\alpha_2} v_2,$$

  and hence

$$a_2 = \frac{\alpha_2}{1+\alpha_2} a_1, \quad c_2 = \frac{1}{1+\alpha_2}. \tag{7}$$

Solving Equations (6) and (7), we will have $a_1 = a_2 = 0$.

□

Exercise 6. *There are $n \ge 2$ players. Each player i must simultaneously decide whether to join a team ($x_i = 1$) or not ($x_i = 0$); hence $z = \sum_{i=1}^n x_i$ is the size of the team. If player i does not join (so that $x_i = 0$) then i receives a payoff of zero. If player i joins the team (so that $x_i = 1$) then i pays a cost of c_i. If all n players join the team (so that $z = n$) then each player enjoys a benefit of v. Hence player i's payoff is $u_i = v - c_i$ when $z = n$, and $u_i = -x_i c_i$ when $z < n$. Suppose that $v > c_i > 0$.*

(i) Suppose that the costs $c_1, \dots, c_n$ are common knowledge. Find all pure-strategy Nash equilibria.

(ii) Now, suppose that information is incomplete. Player i's cost realization c_i is known only to i; players' costs are drawn independently from the same uniform distribution: $c_i \sim U[0, \bar{c}]$. Find the symmetric Bayesian Nash equilibrium.[1]

Solution. (i) For Player i, given other players' strategies, his best-response correspondence is

$$x_i^*(x_{-i}) = \begin{cases} 0, & \text{if } x_{-i} \neq n-1 \\ 1, & \text{if } x_{-i} = n-1 \end{cases}, \quad \text{where } x_{-i} = \sum_{j \neq i} x_i.$$

It is easy to see that there are two pure-strategy Nash equilibria $(0, 0, \dots, 0)$ and $(1, 1, \dots, 1)$. The reason is as follows:

[1] Symmentric Bayesian Nash equilibrium is a strategy profile, in which each Player chooses the same strategy.

- If Player 1 chooses 0, then each of other Player should choose 0. Note that 0 is Player 1's best response when each of other players chooses 0;

  So, given that Player 1 chooses 0, the only possible pure-strategy Nash equilibrium is $(0, 0, \ldots, 0)$ in this case.

- If Player 1 chooses 1. Note that 1 is Player 1's best response only when each of other players chooses 1;

  So, given that Player 1 chooses 1, the only possible pure-strategy Nash equilibrium is $(1, 1, \ldots, 1)$ in this case.

(ii)
- There are $n \geq 2$ players;
- Type spaces: $T_i = \{c_i : c_i \in [0, \bar{c}]\}$;
- Action spaces: $A_i = \{0, 1\}$;
- Strategy spaces: $S_i = \{\text{function from } T_i \text{ to } A_i\}$.

Suppose $x(c_i) : T_i \to A_i$ for each Player i constitutes a symmetric Bayesian Nash equilibrium. Since we know that when the cost becomes larger, the more possibility player will choose 0. So x can be characterized by $y \in [0, \bar{c}]$, that is,

$$x(c_i) = \begin{cases} 1, & \text{if } c_i \in [0, y]; \\ 0, & \text{otherwise.} \end{cases}$$

For Player i, when c_i is drawn, given other players' strategies $x(c_j)$, Player i's expected payoff is

$$\begin{cases} (y/\bar{c})^{n-1}(v - c_i) + [1 - (y/\bar{c})^{n-1}](-c_i), & \text{if } x(c_i) = 1; \\ 0, & \text{if } x(c_i) = 0. \end{cases}$$

Thus Player i chooses 1 if and only if $(y/\bar{c})^{n-1}(v - c_i) + [1 - (y/\bar{c})^{n-1}](-c_i) \geq 0$, that is

$$(y/\bar{c})^{n-1} v \geq c_i.$$

On the other hand, we know that

$$x(c_i) = \begin{cases} 1, & \text{if } c_i \in [0, y]; \\ 0, & \text{otherwise.} \end{cases}$$

therefore, $y = (y/\bar{c})^{n-1} v$, that is

$$y = \bar{c}^{\frac{n-1}{n-2}} v^{2-n}.$$

To summarize, $(x(c_i))_i$ is a symmetric Bayesian Nash equilibrium, where

$$x(c_i) = \begin{cases} 1, & \text{if } c_i \in [0, y] \\ 0, & \text{otherwise} \end{cases}, \quad y = \bar{c}^{\frac{n-1}{n-2}} v^{2-n}.$$

□