- matrix
  - Elementary row operations
    G.E. ⟶ REF. RREF ⟶ linear system
    - consistence : $Ax=b$ consistent $\iff$ rank$(A|b)$ = rank(A)
    - structure
      - relation between the general solw of homo-linear system & inhomo linear sys.
      - # parameters = # columns − rank (A).
  - REF. RREF ↓ inverse (Definition, Adjoint), rank, determinant (cofactor expansion, REF)
    - A is invertible $\iff$ A is full rank $\iff$ $\det A \neq 0$.
  - row space, column space ⟶ vector space (basis, dimension, entending)
  - eigenvalue, eigenvector, eigenspace $\xrightarrow{\text{multiplicity}}$ diagonalization.
    - symmetric matrix, yes.
    - orthogonal matrix, not necessary.
- vector space
  - definition ($\mathbb{R}^n$, 8 axions) ⟶ subspace (basis, dimension, entending)
    - $V = \text{span}\{u_1, \cdots, u_k\}$ $\Updownarrow$ $\forall u, v \in V,\ a, b \in \mathbb{R},\ a\vec{u} + b\vec{v} \in V$.
    - subspace ↓ $W^{\perp}$.
  - linearly independent – rank – dimension (def)
  - linearly independent ↓ basis ⟶ dimension (def) $\xrightarrow[\text{Gram-Schmidt}]{\text{inner product}}$ orthogonal, orthonormal ⟶ projection, Least squares solution
    - $A^TAx = A^Tb$ consistent
    - if $Ax=b$ consistent then solution sets are same
    - projection – linear transformation standard matrix
  - basis ↓ S. V
    1. $S \subseteq V$
    2. 1 S. LI
    2. 2 S spans V
    2. 3 $|S| = \dim V$
  - basis ⟶ coordinate vectors; orthonormal ⟶ coordinate vectors
  - transition matrix ⟶ coordinate vectors
  - transition matrix (rotation, reflection, …) – orthogonal matrix – symmetrix matrix
- linear transformation
  - definition
    - standard matrix $T(u) = Au$ $\forall u$ — eigenvalue — diagonalization
    - $\forall \vec{u}, \vec{v} \in V,\ a, b \in \mathbb{R},\ \Rightarrow T(a\vec{u} + b\vec{v}) = aT(\vec{u}) + bT(\vec{v})$.
  - range, kernal — space (basis, dimension, entension)
    - rank
    - nullity