武汉大学 2016–2017 学年第二学期期末考试
经济与管理学院 2016 级 弘毅、数理经济、数理金融
中级微观经济学提高班试题 A 卷

Examiner: Xiang Sun

Instructions

- Time allowed: 3 hours
- This examination paper comprises 4 printed pages and contains 29 questions. Candidates must attempt all of them.
- The total mark for this paper is 100, which contributes 50% of aggregate mark. Marks for each question are indicated at the beginning of the question.
- This is a closed-book examination.
- Please answer clearly and precisely.
- Good luck and have fun.

Agreement

Before you proceed, write down your name and university ID. Please read all questions carefully. You will get full credit only if you provide complete answers. Note that questions vary in difficulty. If you get stuck on a question, skip it and come back it later.

I am familiar with Wuhan University regulations regarding academic dishonesty and pledge not to violate them.

- Name:
- University ID:
- Signature:

Part A Consider the following one-to-one two-sided matching problem: Let $S = \{s_1, s_2, s_3\}$, $C = \{c_1, c_2\}$, and preferences are given by

s_1	s_2	s_3	c_1	c_2
c_1	c_1	c_2	s_3	s_1
c_2		c_1	s_2	s_3
			s_1	

1. (3 marks) Calculate the matching that results from the college-proposing DA for this college admission problem.
2. (3 marks) Is the result of the college-proposing DA you found above the same as the matching produced by the student-proposing DA in this problem? Is the result of the college-proposing DA stable in this problem? (don't just say "stable" or "unstable". Instead, provide some explanation as well).
3. (3 marks) Calculate the matching that results from the Boston mechanism for this problem.
4. (4 marks) Is the result of the Boston mechanism you found above the same as the result of the (student-proposing or college proposing) DAs? Is the result of the Boston mechanism stable in this example (don't just say "stable" or "unstable". Instead, provide some explanation as well)?

Part B Consider the following house allocation problem: Let 1, 2, and 3 be agents, and h_1, h_2, and h_3 be houses. Preferences of agents are

1	2	3
h_1	h_1	h_2
h_2	h_3	h_1
h_3	h_2	h_3

Consider a priority order function f, which says that agent 1 is ordered first, agent 2 is ordered second, and then agent 3 is ordered as the last agent. Then consider a serial dictatorship with respect to the priority order f, SD^f. Now, answer the following questions.

5. (3 marks) Compute the result of the serial dictatorship SD^f above.
6. (3 marks) Verify that the resulting matching you found is in fact Pareto efficient.
7. (3 marks) Suppose that agent 2 misreports her preferences by reporting $\succ_2'\colon h_2, h_1, h_3$, that is, she declares that she prefers h_2 to h_1 to h_3, although her true preference is still $\succ_2\colon h_1, h_3, h_2$. Compute the result of the serial dictatorship SD^f under this misreporting (while agents 1 and 3 still report their true preferences). Is agent 2 made better off with her misreporting? Is the resulting matching Pareto efficient (with respect to the true preferences)?
8. (3 marks) Consider a matching $\mu = \begin{bmatrix} 1 & 2 & 3 \\ h_2 & h_1 & h_3 \end{bmatrix}$, that is, a matching in which agent 1 gets h_2, agent 2 gets h_1, and agent 3 gets h_3. Find a priority ordering f' over agents such that the serial dictatorship $\text{SD}^{f'}$ results in μ under the preference profile of agents $(\succ_1, \succ_2, \succ_3)$.
9. (4 marks) Consider a matching $\mu' = \begin{bmatrix} 1 & 2 & 3 \\ h_2 & h_3 & h_1 \end{bmatrix}$. Prove that there exists no priority ordering f'' over agents such that the serial dictatorship $\text{SD}^{f''}$ results in μ' under the preference profile of agents $(\succ_1, \succ_2, \succ_3)$.

Part C Consider the following housing market: There are four agents a_1, a_2, a_3, a_4, and four houses h_1, h_2, h_3, h_4, where a_i is the initial owner of house h_i for each $i = 1, 2, 3, 4$, and preferences of agents are given as:

a_1	a_2	a_3	a_4
h_2	h_1	h_2	h_2
h_3	h_2	h_1	h_1
h_1	h_3	h_4	h_3
h_4	h_4	h_3	h_4

10. (3 marks) Compute the matching resulting from TTC under the above preference profile.
11. (3 marks) Verify that the resulting matching you found is in fact Pareto efficient.
12. (3 marks) Suppose that agent a_1 misreports her preferences by declaring that she prefers h_3 to h_2 to h_1 to h_4, although her true preference is still given as in the above table. Compute the result of TTC under this misreporting (while all other agents still report their true preferences). Is agent 1 made better off with her misreporting? Is the resulting matching Pareto efficient (with respect to the true preferences)?
13. (3 marks) Consider a matching $\mu = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ h_3 & h_1 & h_4 & h_2 \end{bmatrix}$. Show that μ is Pareto efficient.
14. (4 marks) Show that μ in the last question is not in the core.
15. Now assume that no agent has a house initially: That is, the problem reduces to the house allocation problem.
    (i) (3 marks) Compute the result of the serial dictatorship associated with priority order $f = (a_1, a_2, a_3, a_4)$.
    (ii) (3 marks) Compute the result of the serial dictatorship associated with priority order $f' = (a_4, a_3, a_2, a_1)$.
16. Now assume that not all agents have a house initially. Rather, agents a_1 and a_2 are existing tenants (occupying h_1 and h_2 respectively) while a_3 and a_4 are newcomers.
    (i) (3 marks) Compute the result of YRMH-IGYT mechanism associated with priority order $f = (a_1, a_2, a_3, a_4)$.
    (ii) (3 marks) Compute the result of YRMH-IGYT mechanism associated with priority order $f' = (a_4, a_3, a_2, a_1)$.

Part D Consider the random assignment problem as in the class. Let there be two real goods a and b with one unit each, plus the outside option $\emptyset$ with infinite supply. There are 4 agents, $N = \{1, 2, 3, 4\}$, and

1	2	3	4
a	a	b	b
b	$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$			

17. (3 marks) Compute the result of PS if 1 reports true preferences. Then prove that the result is ordinally efficient.
18. (3 marks) Compute the result of PS when 1 mis-reports her preferences, claiming that she prefers b to a to $\emptyset$ (while the true preferences are still the same as before).
19. (4 marks) Let agent 1 be endowed with the standard expected utility function, that is, her utility from a random assignment is given by an expected value of the utility from the assigned object. Assume that her utility from a, b, and $\emptyset$ are 1, u, and 0, respectively (assume that u is a number in an open interval $(0, 1)$). Characterize the values of u such that a misreporting by the agent 1 in the last question gives strictly higher expected utility to her than truthtelling does.

20. (4 marks) Find a lottery over feasible allocation that implements the random assignment you found in question 18.

Part E Consider the following school choice problem: Let 1, 2, and 3 be students, and A, B, C be schools. Assume that each school has one seat. Preferences of students and Schools' priorities are

1	2	3	A	B	C
B	B	C	1	3	1
C	A	B	2	2	3
A	C	A	3	1	2

21. (3 marks) Compute the result of the student-proposing DA.
22. (3 marks) Is the result you obtained in the last question the only stable matching? Is it Pareto efficient for students?
23. (3 marks) Does any student have incentives to mis-report her preferences?
24. (3 marks) Use the TTC algorithm to find a matching that is Pareto efficient for students. Is the matching you found stable? (Don't just say "stable" or "unstable". Explain your answers and prove your assertion).
25. (3 marks) Are there other assignments that are Pareto efficient for students?

Part F Answer the following questions.

26. (3 marks) State the basic criterions for designing algorithms.
27. (3 marks) State the TTCC algorithm.
28. (4 marks) For school choice problems with weak priorities, state the sources of efficiency loss for DA.
29. (4 marks) Based on the question above, state the efficiency improvement methods for different sources and discuss the intuitions behind them.

End of paper