Oligopoly

Xiang Sun

Wuhan University

March 23–April 6, 2016

Outline

Section 1

Introduction

Introduction

- Oligopoly differs from the other market structures we've examined so far because oligopolists are concerned with their rivals' actions
- A competitive firm potentially faces many rivals, but the firm and its rivals are price takers

$\Rightarrow$ No need to worry about rivals' actions

- A monopolist does not have to worry about how rivals will react to its actions simply because there are no rivals

Introduction (cont.)

- An oligopolist, however, operates in a market with few competitors and needs to anticipate and respond to rivals' actions (*e.g.*, prices, output, advertising) since they affect its own profit

⇒ Decisions are strategic

- To study oligopoly we'll rely extensively on game theory, a mathematical approach that formally models strategic behavior

Section 2

Game theory

Game theory

- A game is a formal representation of a situation in which individuals or firms interact strategically
- A game consists of:
  - Players (*e.g.*, 2 firms)
  - Set of strategies for all players. A strategy is a full specification of a player's behavior at each of his/her decision points
  - Payoffs for each player for all outcomes (combinations of strategies)

Game theory (cont.)

- A Nash Equilibrium is a set of strategies for which no player wants to change his/her strategy given the strategies played by everyone else
- Each player is playing his/her best response given the equilibrium actions of the other players

Game theory (cont.)

- The extensive form representation (game tree) specifies:
  - the players in the game
  - when each player has the move
  - what each player can do at each of his or her opportunities to move
  - what each player knows at each of his or her opportunities to move
  - the payoffs received by each player for each combination of moves that could be chosen by the players
- The normal form representation

Section 3

Oligopoly models

Oligopoly models

- In monopoly, we saw that choosing price is the same as choosing quantity
- But in oligopoly the strategic variable matters a great deal
- The nature of the competition and the outcome depends on whether firms compete in terms of quantities or in terms of price:
  - Cournot: quantity
  - Bertrand: price
- The timing of the decisions is also important: A sequential move game is called Stackelberg

Section 4

Cournot competition

Subsection 1

Two symmetric firms

The Cournot model

- Consider the case of duopoly (2 competing firms)
- Firms produce a homogenous product with marginal cost c
- Inverse market demand is

$$p = a - Q,$$

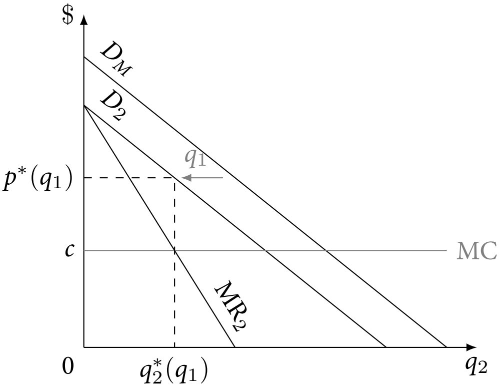
  where $Q = q_1 + q_2$ is total output, $a > c > 0$.
- The market price depends on the combined output of the two firms
- The market price isn't known until both firms have made their output choice
- each firm chooses output based on the expectation of the other firm's output

The Cournot model (cont.)

- Suppose firm 2 expected firm 1 to produce q_1 units
- The relationship between the market price and firm 2's output for a given amount of firm 1 output is given by the residual demand curve of firm 2:

$$q_1 + q_2 = a - p \Rightarrow q_2 = a - p - q_1$$

Best response

- Graphically, firm 2's residual demand curve is the market demand curve shifted left by q_1 units
- Firm 2 acts as a monopolist relative to the residual demand $\Rightarrow q_2^*(q_1)$ is firm 2's best response.
- 

Best response (cont.)

- By varying firm 1's output we could find $q_2^*(q_1)$ for all q_1. This is called the best response function
- Mathematically, we derive the best response function of firm 2 by setting the marginal revenue of firm 2 equal to marginal cost
- The inverse residual demand curve is $p = a - q_1 - q_2$:
  $\text{MR}_2 = a - q_1 - 2q_2$.
- Set $\text{MR}_2 = c$ and solve for q_2^*:

$$q_2^*(q_1) = \frac{a - c}{2} - \frac{q_1}{2}$$

Best response (cont.)

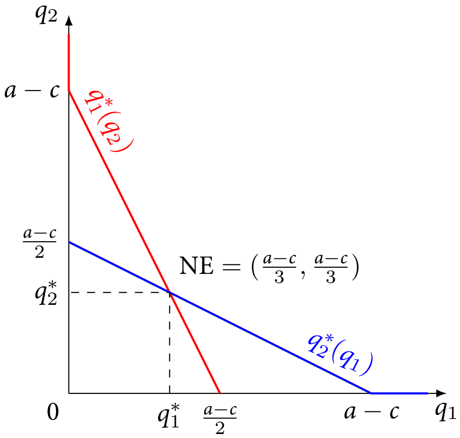
- Similarly we can find the reaction function of firm 1: $q_1^*(q_2)$

$$q_1^*(q_2) = \frac{a-c}{2} - \frac{q_2}{2}$$

- A Nash Equilibrium requires that each firm's output satisfies the best response functions
  - each firm's output must be a best response to its rival's output
  - neither firm has any after-the-fact reason to regret its output choice

Nash equilibrium

$$q_1^*(q_2) = \frac{a-c}{2} - \frac{q_2}{2} \text{ and } q_2^*(q_1) = \frac{a-c}{2} - \frac{q_1}{2}$$



Nash equilibrium

- NE

$$q_1^* = q_2^* = \frac{a-c}{3}$$

- Total output

$$Q^* = \frac{2(a-c)}{3}$$

- Price

$$p^* = \frac{a+2c}{3}$$

- Profit

$$\pi_1^* = \pi_2^* = \frac{(a-c)^2}{9}$$

Comparison with monopoly and perfect competition

- $$Q^{PC} = a - c > Q^* = \frac{2(a-c)}{3} > Q^M = \frac{a-c}{2}$$
- Cournot duopoly output is higher than under monopoly but lower than the competitive output
- $$p^{PC} = c < p^* = \frac{a+2c}{3} < p^M = \frac{a+c}{2}$$
- The price is lower than under monopoly but higher than in perfect competition

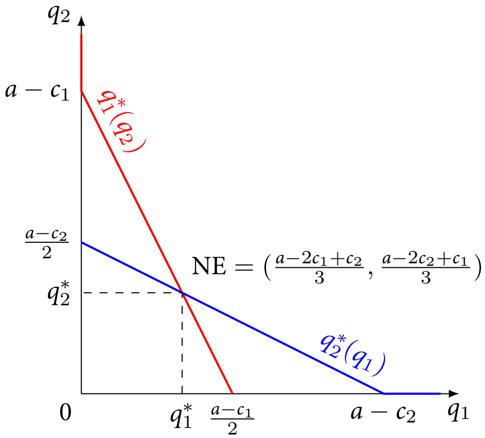
Subsection 2

Two asymmetric firms

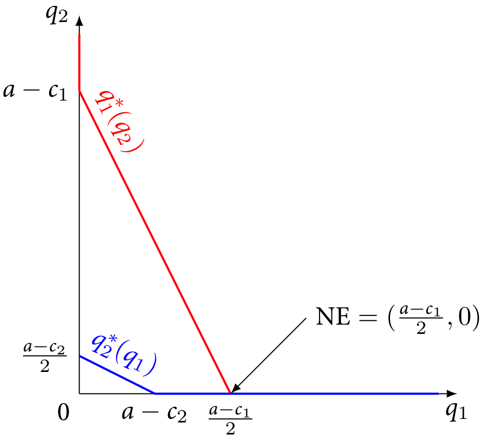
Two asymmetric firms

- The inverse demand is $p = a - Q$
- Firms have asymmetric marginal costs: c_1 for firm 1 and c_2 for firm 2
- $$q_i^*(q_j) = \begin{cases} \frac{a-c_i-q_j}{2}, & \text{if } q_j \leq a - c_i \\ 0, & \text{otherwise} \end{cases}$$

Interior equilibrium

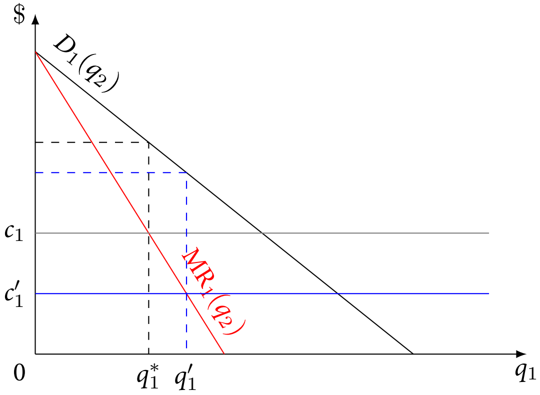


Boundary equilibrium

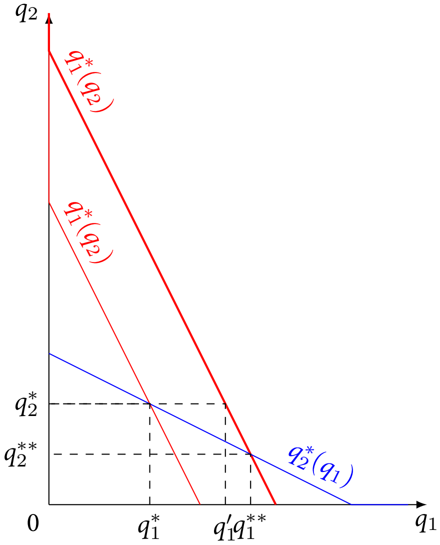


Comparative statics

Decrease firm 1's marginal cost from c_1 to c_1' (fix q_2)



Comparative statics (cont.)



Comparative statics (cont.)

- The direct effect of the decrease in marginal costs is to increase firm 1's output from q_1^* to q_1'
- There is also an indirect effect. In response to the increase by firm 1, firm 2 reduces its output, providing firm 1 with an incentive to further increase its output

Comparative statics (cont.)

The decrease in firm 1's marginal cost results in the following changes

- an increase in q_1
- a decrease in q_2
- an increase in market output
- an increase in firm 1's profits
- a decrease in firm 2's profits

Subsection 3

Many symmetric firms

Many symmetric firms

- There are n firms in the Cournot oligopoly model
- Let q_i denote the quantity produced by firm i, and let $Q = q_1 + \cdots + q_n$ denote the aggregate quantity on the market
- Let the inverse demand is given by $p(Q) = a - Q$ (assuming $Q < a$, else $p = 0$)
- Assume that the marginal cost of firm i is c

Best response

$$q_i^*(q_{-i}) = \begin{cases} \frac{a-c-q_{-i}}{2}, & \text{if } q_{-i} \leq a - c \\ 0, & \text{otherwise} \end{cases}$$

Only interior equilibrium

There does not exist a Nash equilibrium in which some players choose 0

- Assume there is a Nash equilibrium $(q_1^*, \ldots, q_n^*)$, such that $J \triangleq \{i \colon q_i^* = 0\} \neq \emptyset$
- For any $i \in J$, $q_i^* = 0$, and hence $q_{-i}^* \geq a - c$. Thus, $\sum_{j \in J^c} q_j^* \geq a - c$

Only interior equilibrium (cont.)

For any $i \in J$, $q_i^* = 0$, and hence

$$q_{-j}^* = \sum_{k \in J^c, k \neq j} q_k^* \text{ for each } j \in J^c$$

which implies

$$q_j^* = \frac{a - c - q_{-j}^*}{2} = \frac{a - c - \sum_{k \in J^c, k \neq j} q_k^*}{2} \text{ for each } j \in J^c$$

Summing this $|J^c|$ equations, we have

$$\sum_{j \in J^c} q_j^* = \frac{a-c}{2}|J^c| - \frac{|J^c| - 1}{2} \sum_{j \in J^c} q_j^*$$

which implies

$$\sum_{j \in J^c} q_j^* = \frac{|J^c|}{|J^c| + 1}(a - c) < a - c$$

Contradiction

Nash equilibrium

- $q_i^* = \frac{a-c-q_{-i}^*}{2}$ for each i
- $q_i^* = \frac{a-c}{n+1}$
- $Q^* = \frac{n}{n+1}(a-c)$
- $p^* = a - Q^* = \frac{a+nc}{n+1}$
- Profit of each firm $\pi^c = \frac{(a-c)^2}{(n+1)^2}$

Approximation

As $n \to \infty$

- $Q^* \to a - c$ (perfect competition output)
- $p^* \to c$ (perfect competition price)

Many asymmetric firms

- Marginal cost c_i, not distinct too much

⇒ Interior solution

- $$q_1^* = \frac{a - c_i + n(\bar{c} - c_i)}{n+1}$$

  (check by yourself)

- $$p^* = \frac{a + n\bar{c}}{n+1}$$

- $$\pi_i^* = \frac{\big(a - c_i + n(\bar{c} - c_i)\big)^2}{(n+1)^2}$$

- $$s_i = \frac{a - c_i}{a - \bar{c}} \frac{1}{n} + \frac{\bar{c} - c_i}{a - \bar{c}}$$

Subsection 4

Market concentration

Market concentration

- Consider now the case of n firms with different marginal costs
- Recall that the demand for firm i is $p = a - q_{-i} - q_i$
- Equating MR to MC: $a - q^*_{-i} - 2q^*_i = c_i$
- $p^* - c_i = q^*_i$
- $$\frac{p^* - c_i}{p^*} = \frac{q^*_i}{Q^*}\frac{Q^*}{p^*} = \frac{Q^*}{p^*} s^*_i$$

  where s^*_i is the market share of firm i.
- Since the elasticity of demand is $\epsilon = \frac{\mathrm{d}Q}{\mathrm{d}p}\frac{p}{Q}$,

$$\frac{p^* - c_i}{p^*} = -\frac{s^*_i}{\epsilon}$$

Market concentration (cont.)

- The Lerner index, or market power, of each firm is determined by its cost and the elasticity of demand
- What about market power at the industry level?
- Multiply each firm's Lerner index by its market share and then sum them to find the weighted-average Lerner index for the industry

Market concentration (cont.)

- LHS is
$$\sum_{i=1}^{n} s_i^* \frac{p^* - c_i}{p^*} = \frac{p^* - \bar{c}}{p^*}$$
where $\bar{c}$ is the weighted average of marginal costs
- RHS is
$$-\sum_{i=1}^{n} \frac{(s_i^*)^2}{\epsilon} = -\frac{\text{HHI}}{\epsilon}$$
- The industry Lerner index is then
$$\frac{p^* - \bar{c}}{p^*} = -\frac{\text{HHI}}{\epsilon}$$
where HHI is the Herfindahl-Hirschman index
- This tells us that as a market becomes more concentrated the average-price margin increases

Subsection 5

Free-entry equilibrium

Free-entry equilibrium

- There is entry cost f
- Short-run $\rightarrow$ long-run

$\Rightarrow$ Profit is zero

$\Rightarrow$ The equilibrium number of firms is endogenous

Free-entry equilibrium (cont.)

- Let q_i denote the quantity produced by firm i, and let $Q = q_1 + \cdots + q_n$ denote the aggregate quantity on the market
- Let the inverse demand is given by $p(Q) = a - Q$ (assuming $Q < a$, else $p = 0$)
- Assume that the total cost of firm i from producing quantity q_i is $cq_i + f$

Free-entry equilibrium (cont.)

Let the profit of firm be equal to the fixed cost

$$\frac{(a-c)^2}{(n+1)^2} = f$$

or

$$n^e = \frac{a-c}{\sqrt{f}} - 1$$

Free entry equilibrium (cont.)

Parameter: $a = 10$, $c = 2$, and $f = 3$

Number of firms	q_i^c	Q^c	p^c	Profit
1	4	4	6	13
2	2.67	5.33	4.67	4.11
3	2	6	4	1
4	1.6	6.4	3.6	-0.44

Subsection 6

Socially optimal number of firms

Socially optimal number of firms

- Consider a general demand function, the total welfare with n firms is

$$\int_0^{Q(n)} (P(Q) - c)\, \mathrm{d}Q - fn$$

where f is the fixed cost

- FOC:

$$(P(Q) - c)\frac{\mathrm{d}Q}{\mathrm{d}n} = f$$

⇒ Efficient entry requires firms enter until the additional surplus from greater output just equals the additional fixed setup costs

Socially optimal number of firms (cont.)

- Firms will enter the market provided there is non-negative profit from doing so
- This implies entry will occur until

$$(P(Q) - c)q(n) = f$$

- Comparing the two expressions, we can see there will be too much entry if

$$\frac{\mathrm{d}Q}{\mathrm{d}n} < q(n)$$

- By symmetry, $Q(n) = nq(n)$, so that

$$\frac{\mathrm{d}Q}{\mathrm{d}n} = q(n) + n\frac{q(n)}{n} < q(n)$$

- There is excessive entry because each firm that enters does not take account of its entry decision on the output of all other firms
- This business-stealing effect means that there are socially excessive incentives for entry

Socially optimal number of firms (cont.)

- Consider the NE output $Q^* = \frac{n}{n+1}(a-c)$ and NE price $P^* = \frac{a+nc}{n+1}$
- So

$$\frac{a-c}{n^s+1}\frac{a-c}{(n^s+1)^2} = f$$

-

$$n^s = \frac{(a-c)^{\frac{2}{3}}}{f^{\frac{1}{3}}} - 1 < \frac{a-c}{\sqrt{f}} - 1 = n^e$$

Section 5

Bertrand competition

Bertrand competition

- We discuss oligopoly price setting
- We'll rework the basic Cournot model into a Bertrand model and see how dramatically the results change
- Besides the strategic variable changing from quantity to price, all other assumptions are the same
  - one shot
  - two firms sell an identical product
  - each firm has constant marginal cost c
  - direct demand: $Q = D(p)$
  - no capacity constraint

Subsection 1

Bertrand paradox

Bertrand paradox

- We assume that consumers will buy from the low-price firm (efficient rationing)
- In the event firms charge the same price, we assume that demand will be split evenly
- Summarizing, demand for firm 1 is

$$D_1(p_1, p_2) = \begin{cases} D(p_1), & \text{if } p_1 < p_2 \\ \frac{1}{2}D(p_1), & \text{if } p_1 = p_2 \\ 0, & \text{if } p_1 > p_2 \end{cases}$$

The demand for firm 2 is similar

Bertrand paradox (cont.)

Case 1: $p_1 > p_2 > c$

- At these prices firm 1's sales and profits are both zero. Firm 1 could profitably deviate by setting $p_1 = p_2 - \epsilon$, where ϵ is very small. Firm 1's profits would increase to $\pi_1 = D(p_2 - \epsilon)(p_2 - \epsilon - c) > 0$ for small ϵ
- Firm 2 could profitably deviate by setting $p_2 = p_1 - \epsilon$, where ϵ is very small. Firm 2's profits would increase
- This is not an equilibrium

Bertrand paradox (cont.)

Case 2: $p_1 > p_2 = c$

- Firm 2 captures the entire market, but its profits are zero. Firm 2 could profitably deviate by setting $p_2 = p_1 - \epsilon$, where ϵ is very small
- Firm 2's profits would increase to $\pi_2 = D(p_1 - \epsilon)(p_1 - \epsilon - c) > 0$ for small ϵ
- This is not an equilibrium

Bertrand paradox (cont.)

Case 3: $p_1 = p_2 > c$

- This is not an equilibrium since either firm (say, firm 1) could profitably deviate by setting $p_1 = p_2 - \epsilon$
- Then, instead of sharing the market equally with firm 2 and earning profits of $\pi_1 = \frac{1}{2}D(p_1)$, firm 1 would capture the entire market, with sales of $D(p_1 - \epsilon)$ and profits of $\pi_1 = D(p_1 - \epsilon)(p_1 - \epsilon - c)$
- For small ϵ this almost doubles firm 1's sales and profits

Bertrand paradox (cont.)

Case 4: $p_1 = p_2 = c$

- These are the Nash equilibrium strategies
- Neither firm can profitably deviate and earn greater profits even though in equilibrium, profits are zero
- If a firm raises its price, its sales fall to zero and its profits remain at zero
- Charging a lower price increases sales and ensures a market share of 100%, but it also reduces profits since price falls below unit cost

Bertrand paradox (cont.)

The Nash equilibrium to this simple Bertrand game has two significant features

- Two firms are enough to eliminate market power
- Competition between two firms results in complete dissipation of profits

Two possible extensions that softens this outcome

- So far firms set prices and quantities adjust $\Rightarrow$ what if firms had capacity constraints?
- What happens if products are differentiated?

Subsection 2

Empirical evidence

Empirical evidence: Airline industry

- Consistent with Bertrand pricing behavior, many airlines follow a policy of reduced pricing on routes on which they face competition, especially from low-price airlines
- The carriers' rationale for this behavior is consistent with the Bertrand model: each carrier fears that if its fares are even slightly higher than the competition it will lose a large part of the market

Empirical evidence: Airline industry (cont.)

Consider the following fares offered on the internet for two pairs of routes of almost identical distances (so costs are similar)

Route	China Southern	Scoot	Vietnam Airlines
Guangzhou 2 Singapore	1,597	638	–
Haikou 2 Singapore	2,577	No flight	–
Guangzhou 2 Siem Reap	1,317	–	1,337
Haikou 2 Siem Reap	2.567	–	No flight

Empirical evidence: Airline industry (cont.)

Subsection 3

Extension

Bertrand competition with sunk cost

- Suppose that production required not only a marginal cost c, but also a fixed and sunk cost f
- Duopoly with Bertrand competition results in marginal-cost pricing
- With economies of scale, average cost is greater than marginal cost, so the two firms will each incur losses
- In the long run, one of the firms would exit and the free-entry equilibrium would be monopoly (destructive competition)

Bertrand competition with distinct marginal costs

- Suppose there are two firms with unit costs c_1 and c_2, where $c_1 < c_2$
- If the profit-maximizing monopoly price of firm 1 is less than c_2, then firm 1 sets $p_1 = p^m(c_1)$ and monopolizes the market
- If $p^m(c_1) > c_2$, then firm 1 cannot charge its monopoly price in equilibrium, since firm 2 can undercut it and reduce its sales to zero. The (ϵ) Nash equilibrium is $p_2 = c_2$ and $p_1 = c_2 - \epsilon$ where ϵ is very small. Firm 1 charges just slightly below the cost of firm 2 and monopolizes the market
- If we modify the demand function such that firm 1 has the total demand if $p_1 = p_2$, then $p_1 = p_2 = c_2$ is an equilibrium in the case $p^m(c_1) = c_2$

Subsection 4

Capacity constraints

Capacity constraints

- For the $p_1 = p_2 = c$ to be an equilibrium, both firms need enough capacity to satisfy all demand at $p_1 = p_2 = c$
- With enough capacity each firm has a big incentive to undercut each other until price is equal to marginal cost
- Without sufficient capacity each firm knows it can raise prices without losing the entire market

$\Rightarrow$ $p_1 = p_2 = c$ is no longer an NE

- Capacity constraints can affect the equilibrium

Capacity constraints (cont.)

- Daily demand for a product: $Q = 6000 - 60p$
- Suppose there are two firms. Firm 1 has daily capacity of 1,000 and firm 2 has daily capacity of 1,400
- Marginal cost for both firms is $c = 10$
- Is $p_1 = p_2 = c$ still an equilibrium?

$\Rightarrow$ Quantity demanded at 10 is 5,400, far exceeding the total capacity of the two firms

Capacity constraints (cont.)

Consider firm 2's reasoning:

- Normally raising price decreases quantity demanded
- But where can consumers go? Firm 1 is already at capacity
- Some buyers will still buy from firm 2 even if $p_2 > p_1$
- So firm 2 can price above MC and make profit on the buyers who remain

Capacity constraints (cont.)

- We will show that in the NE both firms use all their capacity and the price is the market-clearing price
- $2400 = 6000 - 60p \Rightarrow$ both firms set $p_1 = p_2 = 60$ in the NE

Capacity constraints (cont.)

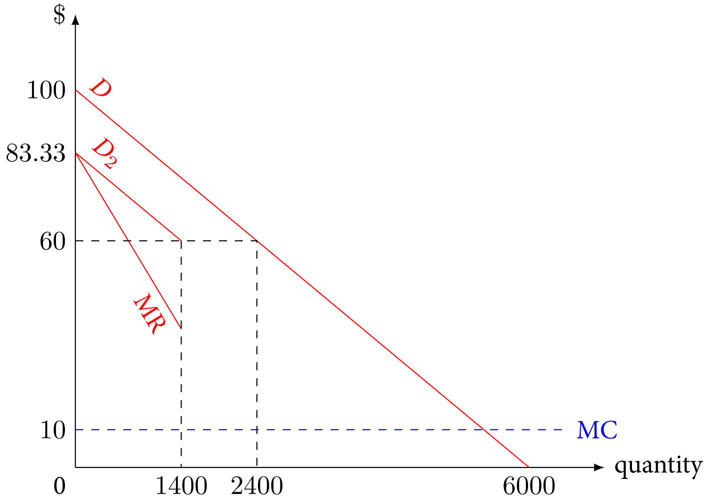
- Assume that there is efficient rationing: Buyers with the highest willingness to pay are served first
  Proportional-rationing rule: randomized rationing

$$D_2(p_2) = D(p_2) \quad \underbrace{\frac{D(p_1) - \bar{q}_1}{D(p_1)}}_{\text{fraction of consumers that cannot buy at } p_1}$$

- Suppose $p_1 = 60$
  - Total demand = 2,400 = total capacity
  - Firm 1 sells 1,000 units
  - Residual demand of firm 2 with efficient rationing: $q_2 = 5000 - 60p$ or $p = 83.33 - q_2/60$
  - Marginal revenue is then $\text{MR} = 83.33 - q_2/30$

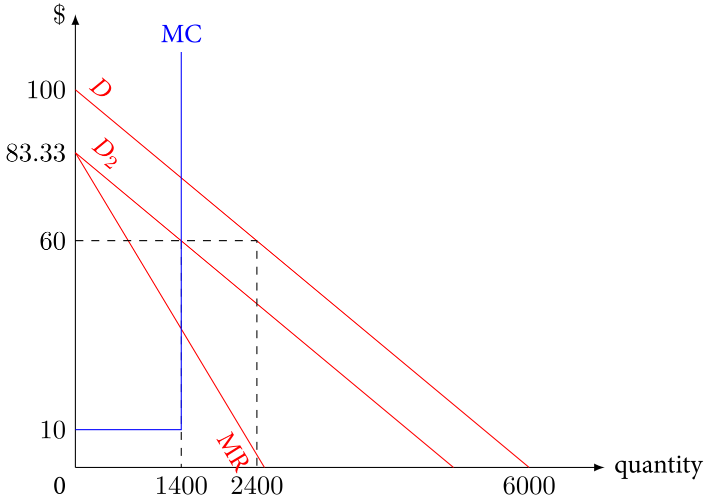
Capacity constraints (cont.)

Firm 2's residual demand



Capacity constraints (cont.)

Firm 2's residual demand



Capacity constraints (cont.)

- Does firm 2 want to deviate from $p = 60$?
- Lowering its price does not lead to any more customers since it is at capacity
- Raising price and losing customers will decrease profits because MR $>$ MC
- It is not profitable for firm 2 to deviate
- Same logic applies to firm 1 so $p_1 = p_2 = 60$ is an NE

Capacity constraints (cont.)

- Firms are unlikely to choose sufficient capacity to serve the whole market when price equals marginal cost since they get only a fraction of the market in equilibrium
- So the capacity of each firm is less than needed to serve the whole market
- But then there is no incentive to cut the price to marginal cost

Subsection 5

Capacity constraints: Cournot to Betrand

Capacity constraint: Cournot to Betrand

- Consider two firms producing homogeneous good
- Linear demand: $D(p) = 1 - p$ or $p = 1 - q_1 - q_2$
- Investment: Firm i pay $c_0 \geq \frac{3}{4}$ for per unit capacity
- Capacity constraint: firm i has marginal cost 0 for $q \leq \bar{q}_i$ and ∞ after $\bar{q}_i$
- Result: Equilibrium price is

$$p^* = 1 - (\bar{q}_1 + \bar{q}_2)$$

  and profits are

$$\pi_i(\bar{q}_i, \bar{q}_j) = [1 - (\bar{q}_1 + \bar{q}_2)]\bar{q}_i$$

- This reduced form profit functions are the exact Cournot forms

Capacity constraint: Cournot to Betrand (cont.)

- Price that maximizes (gross) monopoly profit

$$\max_p p(1-p)$$

  is $p^m = \frac{1}{2}$ and thus $\pi^m = \frac{1}{4}$
- Thus the (net) profit of firm i is at most $\frac{1}{4} - c_0\bar{q}_i$ and is negative for $\bar{q}_i > \frac{1}{3} \Rightarrow \bar{q}_i \leq \frac{1}{3}$

Capacity constraint: Cournot to Betrand (cont.)

- Is it worth charging a lower price? NO, because of the capacity constraints
- Is it worth charging a higher price? Profit of i if price $p \geq p^*$ is

$$\pi_i = p(1 - p - \bar{q}_j) = q_i(1 - q_i - \bar{q}_j),$$

where q_i is the quantity sold by firm i at price p. This profit is concave in q_i
Furthermore, $\frac{\partial \pi_i}{\partial q_i} = 1 - 2q_i - \bar{q}_j \geq 0$ Hence, lowering q_i below $\bar{q}_i$ is not optimal, that is, increasing p above p^* is not optimal

Capacity constraint with proportional-rationing rule

- Assume that $c_0 \geq 1$.
- Price that maximizes (gross) monopoly profit

$$\max_p p(1-p)$$

  is $p^m = \frac{1}{2}$ and thus $\pi^m = \frac{1}{4}$
- Thus the (net) profit of firm i is at most $\frac{1}{4} - c_0\bar{q}_i$ and is negative for $\bar{q}_i > \frac{1}{4} \Rightarrow \bar{q}_i \leq \frac{1}{4}$

$\Rightarrow$ $p^* = 1 - \bar{q}_1 - \bar{q}_2 \geq \frac{1}{2}$

Capacity constraint with proportional-rationing rule (cont.)

- Is it worth charging a lower price? NO, because of the capacity constraints
- Is it worth charging a higher price?
- Suppose that i charges $p > p^*$

⇒ Residual demand of i is

$$(1-p)\frac{1-p^*-\bar{q}_j}{1-p^*}$$

⇒ Profit is

$$p(1-p)\frac{1-p^*-\bar{q}_j}{1-p^*}$$

⇒ Optimal solution is to charge $p = p^*$ since $p(1-p)$ obtain maximum at $p = \frac{1}{2}$ and monotonic decreasing beyond that

Subsection 6

Product differentiation

Product differentiation

- The analysis so far assumes that firms sells homogenous products
- Another extension that removes the Bertrand paradox is production differentiation
- When firms differentiate their products the firm doesn't lose all demand when it raises price above its rival's price
- We will discuss this in detail when we talk more about product differentiation under competition

Product differentiation (cont.)

Gasmi, Vuong, and Laffont (1992) estimated the demand system and marginal costs for Coke and Pepsi

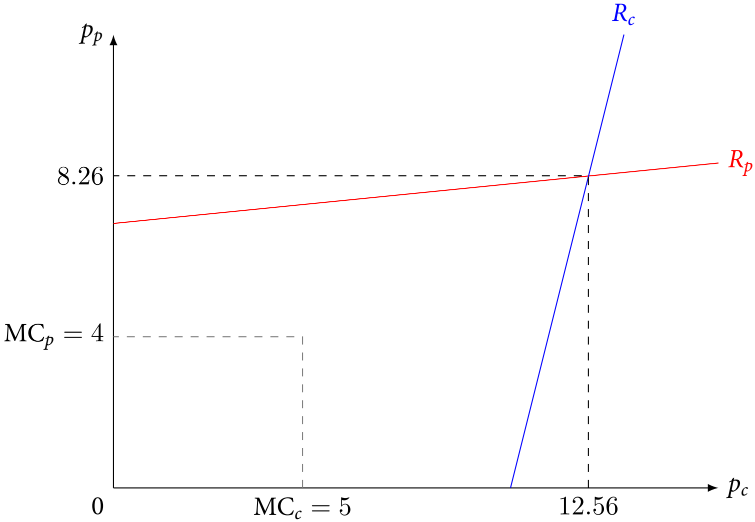
- $q_c = 64 - 4p_c + 2p_p$, $\text{MC}_c = 5$
- $q_p = 50 - 5p_p + p_c$, $\text{MC}_p = 4$
- Profits are

$$\pi_c = (p_c - 5)(64 - 4p_c + 2p_p)$$
$$\pi_p = (p_p - 4)(50 - 5p_p + p_c)$$

- Differentiate with respect to price and solve for the best response of each firm:

$$p_c = 10.5 + 0.25p_p \text{ and } p_p = 7 + 0.1p_c$$

Product differentiation (cont.)



Product differentiation (cont.)

- Equilibrium prices: $p_p^* = 8.26$ and $p_c^* = 12.56$
- Prices are greater than MC $\Rightarrow$ product differentiation "softens" competition
- Price cutting is less effective when products are differentiated

Subsection 7

Cournot vs. Bertrand

Cournot vs. Bertrand

- Which of the two modelling assumptions is more realistic? Do firms set prices or quantities?
- The answer depends, not surprisingly, on what industry we are studying
- Most industries involve firms directly setting prices, so perhaps Bertrand (price-setting) competition is the more realistic approach

Cournot vs. Bertrand (cont.)

- However, if the firms' capacities are fixed, then a firm's price really may be determined by its available capacity
- In such situations it is typical to model firms as competing in quantities (Cournot) since choosing capacity determines how much is produced which then in turn determines the price firms have to set to clear the market
- Examples might include industries such as airlines, hotels, cars, computers

Cournot vs. Bertrand (cont.)

- There are other situations in which output is not capacity constrained, or is easily adjusted to meet the quantity demanded at whatever price is set
- For instance, a software provider, a publisher, an insurance company, or a bank can easily handle any increase in quantity demanded when it lowers its price
- Summary: The standard approach is to adopt the Cournot modelling assumption if prices are easier to adjust than quantities, and the Bertrand modelling assumption if quantities are easier to adjust than prices

Section 6

Stackelberg competition

Introduction

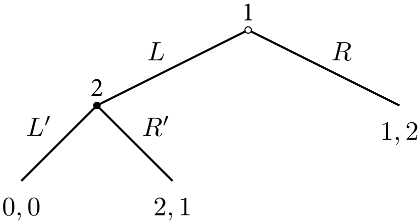
- In a wide variety of markets firms compete sequentially
- One firm (leader/incumbent) takes an action
- The second firm (follower/potential entrant) observes the action and responds

Subsection 1

Subgame perfect equilibrium

Subgame perfect equilibrium

- Eliminating non-credible threats



- Two NE: (L, R') and (R, L')
- Consider the Nash equilibrium (R, L'): L' is not credible for player 2 since R' is strictly better than L' for him

Subgame perfect equilibrium (cont.)

- A subgame is part of the game tree including a decision node (not part of an information set) and everything branching below it
- A strategy profile is a SPE if it induces a NE in each subgame
- To find SPE: backwards induction
- SPE vs. SP outcome

Subsection 2

Stackelberg competition with quantity

Stackelberg competition with quantity

- We'll look first at the Stackelberg model with quantity choice (1934)
- Firms choose output sequentially
- The leader/incumbent (firm 1) sets output first
- The follower/potential entrant (firm 2) observes the output choice and chooses its own output in response
- We solve by backwards induction to find the SPE

Stackelberg competition with quantity (cont.)

- Demand:
$$p = a - Q = a - (q_1 + q_2)$$
- Marginal cost c
- Firm 1 is the leader and chooses $q_1 \Rightarrow$ the second stage is firm 2's decision
- Demand for firm 2 for any choice output q_1 is
$$p = (a - q_1) - q_2,$$
and marginal revenue is
$$\mathrm{MR}_2 = (a - q_1) - 2q_2.$$

Stackelberg competition with quantity (cont.)

- Setting $MR_2 = MC$, we find firm 2's best response:

$$q_2^*(q_1) = \frac{a-c}{2} - \frac{q_1}{2}.$$

- Firm 1 knows firm 2's best response and can therefore anticipate firm 2's behavior
- Demand for firm 1 is then

$$p = a - q_1 - q_2^*(q_1) = \frac{a+c}{2} - \frac{q_1}{2}.$$

- Solving $MR_1 = MC$, we find firm 1's optimal choice:

$$q_1^* = \frac{a-c}{2}.$$

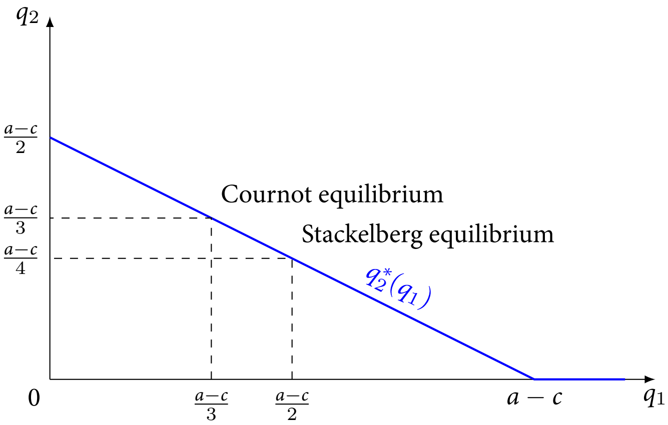
- Substituting q_1^* into firm 2's reaction function we have:

$$q_2^* = \frac{a-c}{4}.$$

Stackelberg competition with quantity (cont.)

- The reaction function of firm 2 is the same as in Cournot
- The leader chooses the location on R_2 by its choice of output
- The leader chooses a higher output than in Cournot, the follower reacts by producing less

Stackelberg competition with quantity (cont.)



Stackelberg competition with quantity (cont.)

The first-mover advantage

- The leader obtains a higher profit by limiting the size of the follower's entry
- The leader gets a greater market share and a larger profit than the follower

Stackelberg competition with quantity (cont.)

- General profit function:
  - $\pi^i_{ij} < 0$: quantity levels are strategic substitutes
  - $\pi^i_j < 0$: each firm dislikes quantity accumulation by the other firm
- By raising q_1, firm 1 reduces the marginal profit from investing for firm 2 ($\pi^2_{21} < 0$)
- Thus firm 2 invest less, which benefits its rival ($\pi^1_2 < 0$)

Stackelberg vs. Cournot

- Aggregate output and price:

$$Q^* = q_1^* + q_2^* = \frac{3(a-c)}{4} \text{ and } p^* = \frac{a+3c}{4}$$

- Profit

$$\pi_1 = \frac{(a-c)^2}{8} \text{ and } \pi_2 = \frac{(a-c)^2}{16}$$

- Recall in the Cournot equilibrium:

$$q_1^c = q_2^c = \frac{a-c}{3},\ Q^c = \frac{2(a-c)}{3},\ p^c = \frac{a+c}{3}$$

Profit:

$$\pi_1^c = \pi_2^c = \frac{(a-c)^2}{9}$$

Commitment

- We assume implicitly that firm 1 can commit to its output level
- Another situation: two firms choose quantities simultaneously, but firm 1 gets the opportunity to announce to firm 2 the output that it intends to produce
- Would the NE be (q_1^s, q_2^s)?
  $\Rightarrow$ No! This quantities involves firm 1 makeing a noncredible threat to produce q_1^s since q_1^s is not optimal ($\frac{3(a-c)}{8}$) for it if really thinks that firm 2 is going to produce q_2^s
  $\Rightarrow$ NE is (q_1^c, q_2^c)
- A natural reinterpretation of Stackelberg model is that the firms do not choose quantities sequentially, but capacities

Subsection 3

Stackelberg price competition

Stackelberg price competition

- We've seen that in a Stackelberg model with quantity choice there is a first-mover advantage. But is moving first always better than moving second?
- Consider price competition with a homogenous product and identical marginal costs

Stackelberg price competition

Is $p = \text{MC}$ still the outcome?

- Would the leader raise the price above MC? The follower would undercut to earn all profits
- Would the leader lower the price below MC? The follower wouldn't match or undercut price in order to avoid losses
- There is no incentive for the leader to deviate from $p = \text{MC}$ and therefore the Stackelberg outcome is the same as in the simultaneous-move model

Subsection 4

Entry deterrence

Entry deterrence

- In the previous discussion of Stackelberg we implicitly assumed that the leader would accommodate entry by the follower
- However, the leader may be able to deter entry
- Entry is deterred if firm 2 expects that postentry its profits will be nonpositive
- The minimum level of output for firm 1 that deters entry by firm 2 is called the limit output. Denote the limit output by q_1^l:

$$\pi_2\left(q_2(q_1^l), q_1^l\right) = 0$$

Constant returns to scale

- Firms have identical cost functions given by cq_i
- No fixed costs
- Can firm 1 deter entry of an equally efficient rival and still exercise market power?

Constant returns to scale (cont.)

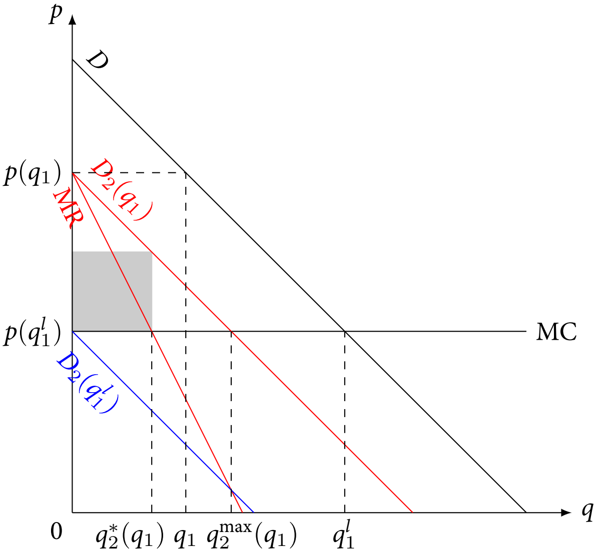
- In order for firm 2 not to have an incentive to produce

⇒ Any output by the entrant would reduce price below average cost and result in negative profits

- With constant returns to scale, there is no cost disadvantage associated with small-scale production
- Provided price exceeds average cost, firm 2 can always enter, perhaps on a very small scale, and earn positive profits

⇒ Firm 1's limit output is such that price equals average and marginal cost, c

Constant returns to scale (cont.)



Constant returns to scale (cont.)

Firm 1 will compare the profitability of its two options:

- deterring entry
- optimally accommodating entry (Stackelberg equilibrium)

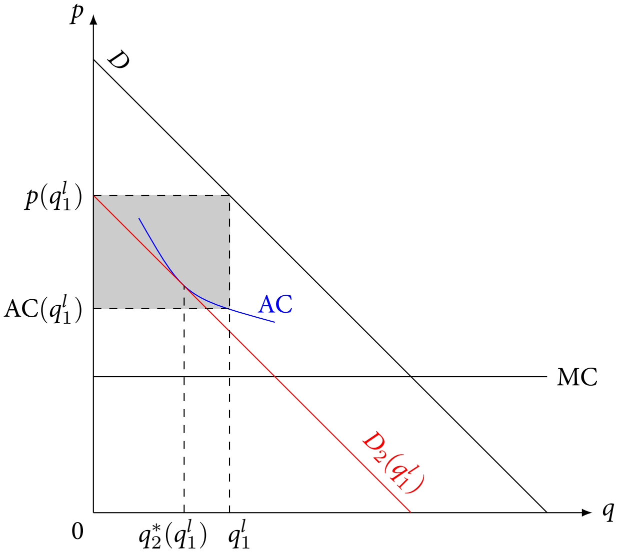
Solution: entry deterrence is not profitable, but the Stackelberg solution is, so the latter will be chosen

$\Rightarrow$ With constant returns to scale, it is not possible for firm 1 to deter entry of firm 2, exercise market power, and earn profits

Economics of scale

- The cost function of both firms is $cq_i + f$
- The fixed cost (f) might correspond to setup or entry costs. The greater f the greater the extent of economies of scale
- When firm 2 considers entering it will compare its postentry profits or quasi-rents ($(p - c)q_2$) with the cost of entering (f)

Economics of scale (cont.)



Economics of scale (cont.)

- Firm 2's residual demand curve is tangent to the average cost curve, firm 1 is producing the limit output.
- The shaded area is the profit of firm 1 from deterring entry by producing q_1^l.
- If firm 2 enters and tries to realize economies of scale, it must produce a substantial amount of output

$\Rightarrow$ Reduce price sufficiently $\Rightarrow$ it falls below its average cost

- If firm 2 enters on a small scale to avoid depressing the price, then its costs are too high.

Economics of scale (cont.)

- Demand: $p = a - Q$
- Cost: $C = cq_i + f$
- Firm 2's profit $\pi_2(q_1, q_2) = (a - q_1 - q_2)q_2 - cq_2 - f$
- Firm 2's best response

$$q_2^*(q_1) = \frac{a - q_1 - c}{2}$$

- Let $\pi_2(q_1, q_2^*(q_1)) = 0$

$\Rightarrow$

$$q_1^l = a - c - \sqrt{4f}$$

- Firm 1's profit

$$\pi_1^l = (a - c - \sqrt{4f})\sqrt{4f}$$

Economics of scale (cont.)

Firm 1's profitability of accommodation and deterrence ($a = 28$, $c = 4$)

Fixed cost	Stacklberg	Entry deterrence
1	72	44
4	72	80
9	72	108

For values of f less than approximately 3, accommodation is more profitable, while for values of f greater than 3, deterrence is more profitable

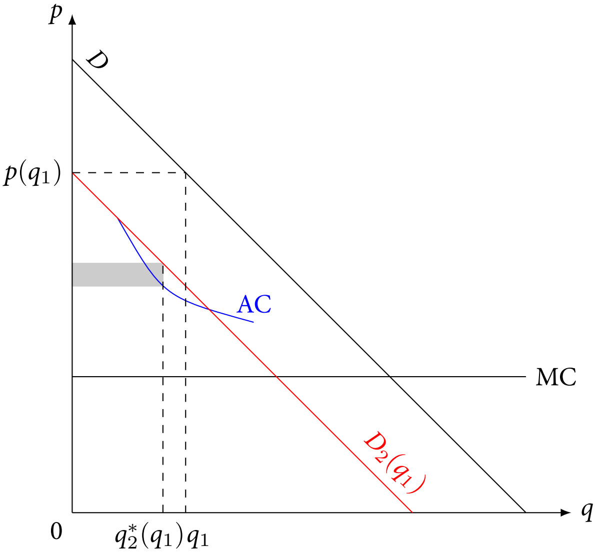
Economics of scale (cont.)

- Take $a = 1$ and $c = 0$.
- To prevent entry, firm 1's payoff is $(1 - 2\sqrt{f})2\sqrt{f}$
- If there is entry, firm 1's best payoff is $\frac{a-c}{8} = \frac{1}{8}$
- So that entry is preferred if

$$(1 - 2\sqrt{f})2\sqrt{f} \geq \frac{1}{8}$$

Hence, there is no entry if $0.00536 < f < 0.182$

Economics of scale (cont.)



Limit pricing

- The incumbent could increase its output above the monopoly level to the limit output.

$\Rightarrow$ This lowers the price below the monopoly price

$\Rightarrow$ The monopolist limits its price and profits in order to deter entry

- The trade off between
  - maximizing short-run profits by charging the monopoly price
  - limiting entry to preserve some profits in the long run by charging the limit price

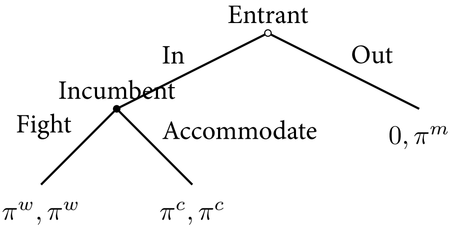
Limit pricing (cont.)

- No mechanism allows the incumbent to commit to the limit output in the future
- Producing the limit output today does not change the incentives or the choice set of the incumbent tomorrow if there is entry
- Postentry, the entrant should expect that the incumbent will maximize its profits given that the market structure is now a duopoly

$\Rightarrow$ This will typically involve some accommodation: a reduction in the incumbent's output below the limit output

Stylized entry game

- The entrant has two strategies: enter or stay out
- The incumbent has two strategies: fight the entrant if it enters, which involves a price war, or to accommodate entry, which involves sharing the market
- The value of the payoffs satisfies $\pi^m > \pi^c > 0 > \pi^w$, where π^m is monopoly profits, π^c Cournot profits, and π^w the profits from a price war

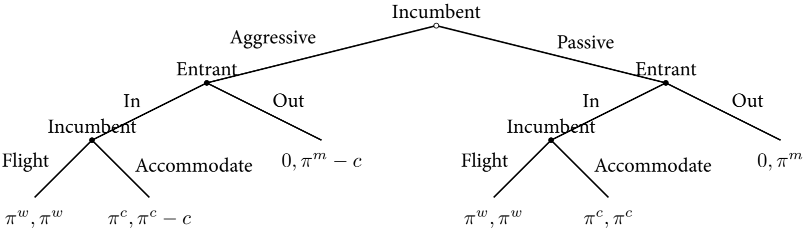


-

Stylized entry game (cont.)

- 2 NEs: (Fight, Out) and (Accommodate, In)
- (Fight, Out) is not subgame perfect, Out is noncredible

Stylized entry game (cont.)

- If the incumbent can invest in the price war prior to entry, it may be able to transform its threat of a price war into a commitment and credibly deter entry
- 

- Suppose that launching a price war (the fighting strategy) involves some sort of cost, c

Stylized entry game (cont.)

- If $\pi^w > \pi^c - c$, fighting is optimal when facing with entry

$\Rightarrow$ aggressive strategy changes the noncredible threat to fight into a commitment to fight

- If $\pi^m - c > \pi^c$, then SPE is (Aggressive, Fight, Accommodate), (Out, In)
- If either one of these inequalities does not hold, then SPE is (Passive, Accommodate, Accommodate), (In, In)

Dixit's model

- To produce 1 unit of output requires 1 unit of capacity and 1 unit of labor
- The cost of a unit of capacity is r and the cost of a unit of labor w. The cost of production per unit equals $w + r$
- Economies of scale arise from the presence of a startup cost, or entry fee, equal to f

Dixit's model (cont.)

This is a two-stage game

- In the first stage, the incumbent is able to invest in capacity k_1
- In the second stage, the entrant observes k_1, and then makes its entry decision
    - If it enters it incurs the entry cost of f
    - The entrant is assumed to choose the cost-minimizing capacity level for its level of output: $k_2 = q_2$

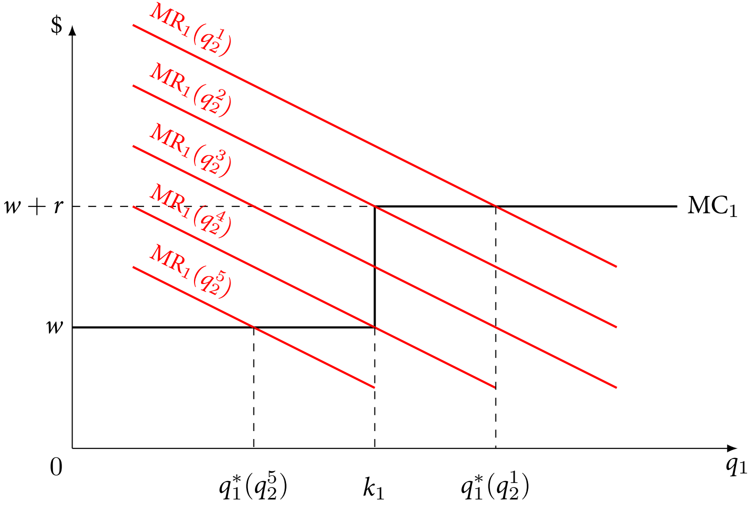
Dixit's model (cont.)

Given k_1

- For $q_1 \leq k_1$, the marginal cost of firm 1 is only w, since it has already incurred the necessary capacity cost
- For $q_1 > k_1$, the marginal cost for firm 1 is $w + r$, since it has to acquire additional capacity
- Profit

$$\pi_1 = \begin{cases} q_1(a - q_1 - q_2 - w) - rk_1 - f, & \text{if } q_1 \leq k_1 \\ q_1(a - q_1 - q_2 - w - r) - f, & \text{if } q_1 > k_1 \end{cases}$$

Dixit's model (cont.)



Dixit's model (cont.)

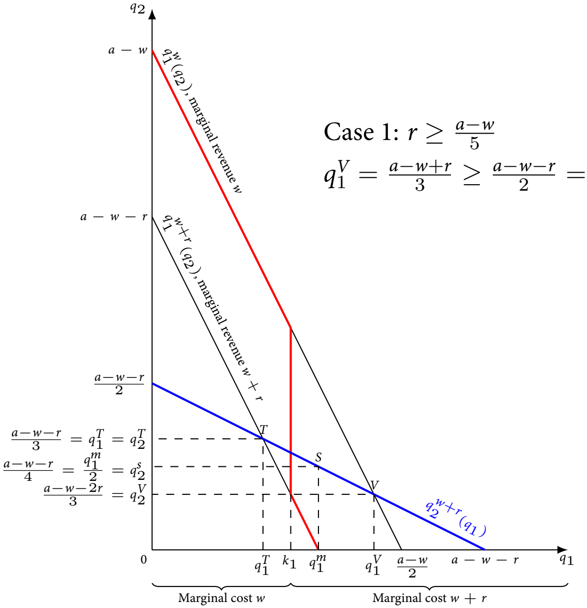
Given k_1

- Firm 1's best response

$$q_1^*(q_2) = \begin{cases} q_1^w(q_2) \triangleq \frac{a-q_2-w}{2} < k_1 & \text{if } q_2 > a - w - 2k_1 \\ q_1^{w+r}(q_2) \triangleq \frac{a-q_2-w-r}{2} > k_1 & \text{if } q_2 < a - w - r - 2k_1 \\ k_1 & \text{otherwise} \end{cases}$$

- Firm 2's best response

$$q_2^*(q_1) = q_2^{w+r}(q_1) \triangleq \frac{a - q_1 - w - r}{2}$$

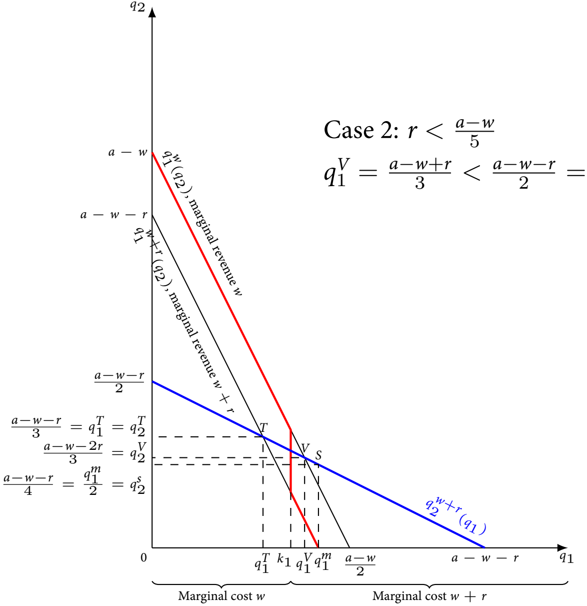


Case 1: $r \geq \frac{a-w}{5}$

$$q_1^V = \frac{a-w+r}{3} \geq \frac{a-w-r}{2} = q_1^m$$

Case 2: $r < \frac{a-w}{5}$

$q_1^V = \frac{a-w+r}{3} < \frac{a-w-r}{2} = q_1^m$



Quantity subgame

Type 1 of subgames: $k_1 \leq q_1^T$

- ⇒ In both cases, NE is the symmetric Cournot outputs at T, given firm 2 has nonnegative profit at T
- ⇒ It is profitable for firm 1 to expand its output beyond k_1
- ⇒ Capacity expansion

Quantity subgame (cont.)

Type 2 of subgames: $k_1 \geq q_1^V$

$\Rightarrow$ In both cases, NE is at V, given firm 2 has nonnegative profit at V

- Producing to capacity involves producing units of output for which marginal cost exceeds marginal revenue

$\Rightarrow$ It is not profitable for firm 1 to utilize all of its capacity

$\Rightarrow$ Excess capacity

Quantity subgame

Type 3 of subgames: $q_1^V > k_1 > q_1^T$

- ⇒ In both cases, NE is $(k_1, q_2^{w+r}(k_1))$, given firm 2 has nonnegative profit at this point
- ⇒ It is profitable for firm 1 to utilize its capacity, but will not expand its capacity
- ⇒ Full utilization

Observations

- If firm 2 can not get a positive profit at T, it will not have positive profit between T and V (since the total output is minimal at T, and the price is maximal at T)

$\Rightarrow$ It will not enter the market

- Let L be the point such that firm 2's profit is zero

Optimal capacity investment for case 1

Case 1a: L is to the left of T

- Firm 2 can not get positive profit at T
- Firm 2 will not enter
- Firm 1 chooses $k_1 = q_1^m$ in stage 1, and produces q_1^m in stage 2

⇒ Blockaded monopoly, $k_1 = q_1^m$, and equilibrium output is at $(q_1^m, 0)$

Optimal capacity investment for case 1 (cont.)

Case 1b: L is to the right of V

- Firm 2 has a positive profit at V
- Firm 2 will always enter

⇒ Stackelberg outcome, firm 1 will choose $k_1 = q_1^s (= q_1^m)$, and equilibrium output is at S

Optimal capacity investment for case 1 (cont.)

Case 1c: L is between T and S

- Firm 1 can choose capacity $k_1 = q_1^m$ in stage 1, and produces q_1^m in stage 2
- Firm 2 will have a non-positive profit if it follows $q_2^{w+r}(q_1)$

$\Rightarrow$ Blockaded monopoly, $k_1 = q_1^m$ and equilibrium output is at $(q_1^m, 0)$

Optimal capacity investment for case 1 (cont.)

Case 1d: L is between S and V

- Firm 1 has two options
  - Optimally accommodating entry: $k_1 = q_1^s$, and Stackelberg equilibrium output
  - Deterring entry: $k_1 = q_1^l$, and equilibrium is at $\left(q_1^l, q_2^*(q_1^l)\right) \Rightarrow$ expanding its output beyond the monopoly level
- It depends

Example 1

- Demand $p = 68 - Q$, $r = 38$, $w = 2$, and $f = 4$
- Limit output $q_1^l = 24$
- Monopoly output $q_1^m = \frac{a-w-r}{2} = 14$
- $q_1^T = q_2^T = \frac{a-w-r}{3} = \frac{28}{3}$
- $q_1^V = \frac{a-w+r}{3} = \frac{104}{3}$, $q_2^V = \frac{a-w-2r}{3} = -\frac{10}{3}$

$\Rightarrow$ L is between S and V

Example 1 (cont.)

Option 1: accommodation

- $k_1 = q_1^s$, and equilibrium is at (q_1^s, q_2^s)

⇒ Firm 1's profit is

$$(a - w - r - q_1^s - q_2^s)q_1^s - f = (68 - 38 - 2 - 14 - 7)14 - 4 = 94$$

Option 2: deter entry

- $k_1 = q_1^l$, and equilibrium is at $(q_1^l, 0)$

⇒ Firm 1's profit is

$$(a - w - r - q_1^l)q_1^l - f = (68 - 38 - 2 - 24)24 - 4 = 92$$

Accommodation is optimal

Example 2

- Demand $p = 120 - Q$, $r = w = 30$, and $f = 200$
- Limit output $q_1^l = 60 - 20\sqrt{2} \approx 31.7$
- Monopoly output $q_1^m = \frac{a-w-r}{2} = 30$
- $q_1^T = q_2^T = \frac{a-w-r}{3} = 20$
- $q_1^V = \frac{a-w+r}{3} = 40$, $q_2^V = \frac{a-w-2r}{3} = 10$

$\Rightarrow$ L is between S and V

Example 2 (cont.)

Option 1: accommodation

- $k_1 = q_1^s$, and equilibrium is at (q_1^s, q_2^s)

$\Rightarrow$ Firm 1's profit is

$$(a - w - r - q_1^s - q_2^s)q_1^s - f = (120 - 30 - 30 - 30 - 15)30 - 200 = 250$$

Option 2: deter entry

- $k_1 = q_1^l$, and equilibrium is at $(q_1^l, 0)$

$\Rightarrow$ Firm 1's profit is

$$(a - w - r - q_1^l)q_1^l - f = (120 - 30 - 30 - 31.7)31.7 - 200 = 697.11$$

Deterring entry is optimal

Optimal capacity investment for case 2

Case 2a: L is to the left of S

- Firm 1 chooses $k_1 = q_1^m$ in stage 1, and produces q_1^m in stage 2

⇒ Blockaded monopoly (can be regarded as natural monopoly), $k_1 = q_1^m$, and equilibrium output is at $(q_1^m, 0)$

Optimal capacity investment for case 2 (cont.)

Case 2b: L is to the right of S

- Firm 2 has a positive profit at S
- Firm 2 will always enter

⇒ Get as close to Stackelberg outcome as possible, firm 1 will choose $k_1 = q_1^V$, and equilibrium output is at V