Monopolistic competition and product differentiation

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Outline

Section 1

Monopolistic competition

Characteristics of monopolistic competition

- Product differentiation
- Many firms

⇒ Each MC firm has the freedom to set prices without engaging in strategic decision making regarding the prices of other firms, and each firm's actions have a negligible impact on the market

- No entry and exit cost in the long run

⇒ Long-run profit is zero

- Independent decision making

Characteristics of monopolistic competition (cont.)

- Same degree of market power: Market power means that the firm has control over the terms and conditions of exchange

$\Rightarrow$ Face a downward sloping demand curve
The source of an MC firm's market power is not barriers to entry since they are low. Rather, an MC firm has market power because it has relatively few competitors, those competitors do not engage in strategic decision making and the firms sells differentiated product

- Buyers and Sellers do not have perfect information

Inefficiency in monopolistic competition

- An MC firm maximizes profits where marginal revenue = marginal cost

$\Rightarrow$ Since the MC firm's demand curve is downward sloping, this means that the firm will be charging a price that exceeds marginal costs

$\Rightarrow$ At its profit maximizing level of production, there will be a net loss of consumer (and producer) surplus

Inefficiency in monopolistic competition (cont.)

- An MC firm's demand curve is downward sloping

$\Rightarrow$ The MC firm's profit maximizing output is less than the output associated with minimum average cost

$\Rightarrow$ MC firms operate with excess capacity

Section 2

Product differentiation

Product differentiation

- Firms try to differentiate their products from their competitors' in order to make them less substitutable (Bertrand paradox) to increase demand
- Horizontal differentiation: consumers rank the products differently
- Example
  - Carrot vs. cabbage
  - Light beer vs. regular beer
  - The Rolling Stones vs. The Beatles
  - Thin-crust pizza vs. thick-crust pizza

Product differentiation (cont.)

- Vertical differentiation
    - Consumers rank the products similarly (homogeneous consumers)
    - Consumers have a different willingness to pay for quality
- Example
    - iPhone 6s vs. Nokia 1100
    - Ferrari 488 GTB vs. 长安之星 2
    - Burj Al Arab vs. Hostels
    - Qatar Airways vs. 春秋航空
    - Harvard University vs. 哈尔滨佛学院

Section 3

Horizontal differentiation

Horizontal differentiation

- Goods approach: assume that consumers have preferences over the goods themselves (rather than their characteristics)
- Location-based approach: assume that consumers have preferences over the characteristics of the product as represented by the location of the product on a line (Hotelling's model) or a circle (Salop's model)
- Discrete choice model

Subsection 1

Goods model of competition

Goods model of competition

- It is usual to assume that the preferences of consumers can be "aggregated" and represented by the preferences of a single "representative consumer."
- Suppose that the representative consumer has utility from two goods expressed as

$$u(q_1, q_2) = \alpha(q_1 + q_2) - \frac{1}{2}(\beta q_1^2 + \beta q_2^2 + 2\gamma q_1 q_2)$$

where $\beta > 0$

Inverse demand function

- Consumers solve

$$\max_{q_1, q_2} u(q_1, q_2) - p_1 q_1 - p_2 q_2$$

- Concave function (negative semi-definite) ⇒ Inverse demand function

$$p_1 = \alpha - \beta q_1 - \gamma q_2$$
$$p_2 = \alpha - \beta q_2 - \gamma q_1$$

Demand function

Demand function

$$q_1 = a - bp_1 + gp_2$$
$$q_2 = a - bp_2 + gp_1$$

where

$$a = \frac{\alpha(\beta - \gamma)}{\beta^2 - \gamma^2},\ b = \frac{\beta}{\beta^2 - \gamma^2},\ g = \frac{\gamma}{\beta^2 - \gamma^2}$$

Demand function (cont.)

- Case 1: $\gamma = \beta$
  $\Rightarrow u(q_1, q_2) = \alpha(q_1 + q_2) - \beta(q_1 + q_2)^2/2$
  $\Rightarrow$ consumer only cares about the total consumption of the two goods
  $\Rightarrow$ goods are homogenous
- Case 2: $\gamma = 0$
  $\Rightarrow$ marginal utility for good i does not depend on the consumption of good j
  $\Rightarrow$ goods are independent
  $\Rightarrow$ the firm's demand does not depend on the rival's price

Demand function (cont.)

Case 3: $\gamma < 0$

$\Rightarrow$ $\frac{\partial^2 u}{\partial q_i \partial q_j} = -\gamma > 0$

$\Rightarrow$ marginal utility for good *i* increases in the consumption of good *j*

$\Rightarrow$ goods are complements

$\Rightarrow$ firm's demand increases when the rival firm lowers its price

Demand function (cont.)

Case 4: $\beta > \gamma > 0$

- $\Rightarrow$ $\frac{\partial^2 u}{\partial q_i \partial q_j} = -\gamma < 0$
- $\Rightarrow$ marginal utility for good i decreases in the consumption of good j
- $\Rightarrow$ firm's demand decreases when the rival firm lowers its price

At the same time

- $\Rightarrow$ consumers like variety (consuming too much of one good lowers the marginal utility)
- $\Rightarrow$ firm's demand decreases more as a result of an increase in its own price than an equivalent reduction in its rival's price ($b > g$)
- $\Rightarrow$ the own price effect on demand will be stronger than the cross-price effect

- Using these demand structures, we can again solve for
  - NE with quantity competition (as we did for Cournot competition)
  - NE with price competition (as we did for Bertrand competition)
- We will restrict attention to $0 < \gamma < \beta$, where $\frac{\gamma}{\beta}$ is an inverse measure of product differentiation—the closer it is to one, the less differentiation there is between the goods
- Question: how about $\gamma > \beta$?

Best response function

- Constant marginal costs c for both firms
- Profits

$$\pi_1 = (\alpha - \beta q_1 - \gamma q_2 - c) q_1$$
$$\pi_2 = (\alpha - \beta q_2 - \gamma q_1 - c) q_2$$

- FOC leads to best response functions

$$q_1^*(q_2) = \frac{\alpha - c - \gamma q_2}{2\beta} \text{ and } q_2^*(q_1) = \frac{\alpha - c - \gamma q_1}{2\beta}$$

- Best response functions are downward sloping
- The more a firm's rival produces, the less a firm will want to produce itself

Nash equilibrium

- NE

$$q_1^* = q_2^* = \frac{\alpha - c}{2\beta + \gamma}$$

- NE prices

$$p_1^* = p_2^* = \frac{\alpha\beta + (\beta + \gamma)c}{2\beta + \gamma}$$

- NE profits

$$\pi_1^* = \pi_2^* = \beta \left(\frac{\alpha - c}{2\beta + \gamma} \right)^2$$

the more differentiated the products, the smaller is γ and the higher are NE profits

Best response function

- Constant marginal costs c for both firms
- Profits

$$\pi_1 = (p_1 - c)(a - bp_1 + gp_2)$$
$$\pi_2 = (p_2 - c)(a - bp_2 + gp_1)$$

where $b > g$

- FOC leads to best response functions

$$p_1 = \frac{a + bc + gp_2}{2b} \text{ and } p_2 = \frac{a + bc + gp_1}{2b}$$

- Best response functions are upward sloping
- The higher a firm's rival price, the higher a firm will want to set its own price

Nash equilibrium

- NE

$$p_1^* = p_2^* = \frac{bc + a}{2b - g}$$

- NE outputs

$$q_1^* = q_2^* = b\frac{a - (b - g)c}{2b - g} = \frac{(\alpha - c)\beta}{(2\beta - \gamma)(\gamma + \beta)}$$

- NE profits

$$\pi_1^* = \pi_2^* = b\left(\frac{a - (b - g)c}{2b - g}\right)^2 = \beta\frac{(\alpha - c)^2(\beta - \gamma)}{(2\beta - \gamma)^2(\beta + \gamma)}$$

the more differentiated the products, the smaller is γ and the higher are NE profits

Price competition vs. quantity competition

- With price competition, a reduction in one firm's price makes the rival firm also want to decrease its price

$\Rightarrow$ When one firm acts more aggressively, the rival follows suit

- With quantity competition, an increase in one firm's output (which tends to lower the firm's price) makes the rival firm decrease its output (which tends to raise its price)

$\Rightarrow$ When one firm acts more aggressively, the rival becomes less aggressive

Strategic substitutes and strategic complements

- Two actions are strategic substitutes if an increase in firm 2's action, makes firm 1 prefer less of the same action

$\Rightarrow$ Provided goods are substitutes, quantities are strategic substitutes

$\Rightarrow$ With strategic substitutes, best response functions slope downwards

- Two actions are strategic complements if an increase in firm 2's action, makes firm 1 prefer more of the same action

$\Rightarrow$ Provided goods are substitutes, prices are strategic complements

$\Rightarrow$ With strategic complements, best response functions slope upwards

Subsection 2

Hotelling's model

Location

- Consumers' tastes vary continuously over some parameters which describe the nature of products
- Different consumers have different most preferred "locations" in this space
- Products are characterized by their locations in this space, θ
  - θ may be interpreted as a location of firms in a physical space—location in city
  - θ may be the characteristic of the good, *e.g.*, colour, sweetness
  - θ may be the time at which the service is delivered, *e.g.*, airline or TV scheduling
- Most often in the address approach, we focus on just a single characteristic, usually referred to as the consumer's location

Consumers' utility

- The utility of a consumer located at address x^* who purchases brand i is

$$u(x^*, x_i) = v - T(d) - p_i$$

  where the transportation cost T depends on $d = |x^* - x_i|$
- Consumer's utility from the good v is high enough
- Common specifications of $T(d)$ are $T = td$ and $T = td^2$
- Consumers purchase the brand which gives them the highest utility

Distribution of consumers

- To work out each firm's demand we must make some assumption about the distribution of consumers' locations
- The simplest case is when consumers are uniformly distributed along a unit interval between 0 and 1
- This gives rise to Hotelling's classic 1929 model of a linear city
- Each consumer purchases exactly one unit of the good

Principle of minimum differentiation

- Prices are fixed and two firms are considering where to locate in $[0, 1]$
- Suppose each firm's payoff is exactly the number of consumers they attract
- Assuming consumers are uniformly distributed over $[0, 1]$, each firm will try to maximise its market length by choosing its location optimally given the other firm's choice
- NE: both firms locating at the middle point of $[0, 1]$

$\Rightarrow$ Principle of minimum differentiation due to Boulding 1966

- when there is a non-uniform distribution of consumers, firms may also like to locate close to a 'pole'—a point where there is anyway a concentration of consumers

Fixed price

- Is the assumption that firms' prices are fixed (possible at zero) critical to the result of minimal differentiation?
- If firms do not differentiate their products

$\Rightarrow$ Price competition will result in no profits

- If firms can somehow commit to differentiate their products, then price competition will be relaxed, and they can make positive profits
- In general there is a tension between
  - firms trying to move closer together to steal each other's customers
  - firms trying to move further apart to reduce price competition

Minimum differentiation

- There are four firms
- Unique NE: Two firms choose location $\frac{1}{4}$ and the other choose location $\frac{3}{4}$ (check by yourself)

Efficient differentiation

- Given fixed prices, the social planner will seek to minimise transportation costs
- With uniformly distributed consumers, this implies locating the firms equidistantly on either side of the middle of the segment; that is, at $\frac{1}{4}$ and $\frac{3}{4}$
- The same (welfare) result holds allowing firms to set prices, as long as consumers all buy one unit of the good (in this case, the level of prices is irrelevant for welfare)

Efficient differentiation (cont.)

- Compared to this socially optimal differentiation, the private choice of differentiation involves too much bunching or too little differentiation
- The monopolist (with two stores) will locate the stores at the socially optimal positions since this allows him to charge the highest prices (*i.e.* there is no reason to distort the location choices)

Efficient differentiation (cont.)

- The monopolist (or the social planner) would choose n stores (location evenly) to minimise the sum of the fixed costs and the transportation costs
- What is the optimal n?
- Fixed cost f for each store
- The transportation costs is

$$2nt \int_0^{\frac{1}{2n}} x \, \mathrm{d}x = \frac{t}{4n}$$

- $$\min_n nf + \frac{t}{4n}$$

$\Rightarrow$ $n^s = \frac{1}{2}\sqrt{\frac{t}{f}}$

Free-entry equilibrium

- Fixed cost f for entering
- Fixed price p
- Fixed marginal cost c, where $p = c + 1$ for simplicity
- Firms enter the market sequentially—firms enter and select their locations in order, with firm 1 first, firm 2 second, *etc*.
- SPE outcome: $n^e = \frac{1}{2f}$, the first n^e firms choose the following locations sequentially
$$\frac{1}{2n^e}, \frac{3}{2n^e}, \dots, \frac{2n^e - 1}{2n^e}$$
(check by yourself)
- Profit: f
- Question: How about when the prices are strategically set by firms?

Price competition with given locations

- If firms locate at the same point

⇒ the Betrand result with homogeneous products, resulting in zero profits for the firms

- If firms locate at extremes of the unit line and transportation costs are linear

⇒ a consumer located at x pays $p_1 + tx$ if she buys from firm 1, while she pays $p_2 + t(1 - x)$ if she buys from firm 2

Consumers' behaviors

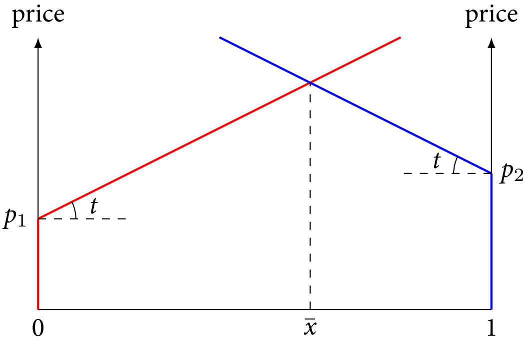
- All those consumers to the left of $\bar{x}$ will buy from firm 1 and to the right of $\bar{x}$ will buy from firm 2

$$\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

- Consumers are uniformly distributed on the unit interval

$\Rightarrow$ Firm 1's market share is $s_1 = \bar{x}$, which is also firm 1's demand function (as a function of its own price and the rival's price)

Consumers' behaviors (cont.)



Optimal pricing

- Firm 1 sets p_1 to maximise

$$(p_1 - c)s_1$$

while firm 2 sets p_2 to maximise

$$(p_2 - c)(1 - s_1)$$

- FOC leads to

$$p_i^*(p_j) = \frac{p_j}{2} + \frac{c+t}{2}$$

- NE:

$$p_i^* = c + t$$

⇒ Market shares $s_i^* = \frac{1}{2}$

⇒ Profits $\pi_i^* = \frac{t}{2}$

Limit case

- In the limit as $t \to 0$, there is no transportation costs, and consumers treat the two firms' products as homogenous

$\Rightarrow$ Prices tend to marginal cost, and profits tend to zero

$\Rightarrow$ The result is the normal Bertrand competition model with homogenous goods

Extension

- Suppose that consumers face quadratic transportation costs of the form $T(d) = td^2$
- A consumer located at x that buys from firm 1 pays $p_1 + tx^2$ while if she buys from firm 2 she pays $p_2 + t(1 - x)^2$
- Equating these two, the consumer that is indifferent between 1 and 2 is characterized by the same x as above
- With the same market share equation, the resulting equilibrium prices and profits will be same as before

Price discrimination

- Suppose N consumers are uniformly distributed between 0 and 1, and there is a monopoly located in the middle of the unit interval
- Consumers all value the good at v, but face cost of transportation td when the consumer is located a distance d away from the monopolist

Single price

- If the monopolist sets the maximal (uniform) price it can set to ensure all consumers purchase, it will set

$$p = v - \frac{t}{2}$$

- Profit

$$N(v - \tfrac{t}{2} - c)$$

Price discrimination

- If monopolist knows each consumer's location, the firm could price discriminate setting a different price depending on each consumer's location
- Set price

$$p(x) = v - t|\tfrac{1}{2} - x|$$

⇒ It ensures the monopolist extracts all the surplus from each consumer

- Every consumer just buys and the monopolists makes a profit of $N(v - \frac{t}{4} - c)$, which is higher than if its sets a uniform price
- The monopolist could achieve the same outcome by offering to sell the good at a set price $p = v$, but paying each consumer's transportation cost td

⇒ This corresponds to offering free delivery (包邮)

Competition with price discrimination

- Measure one of consumers are located at each of two cities, with distance s between them
- A firm is located at each city
- Each consumer gets utility v from a purchase, where $s < v < 2s$
- Each consumer has unit demand
- No cost of production
- Consumers purchase from the firm that is closer

Single price

If firms have to set a single price, the equilibrium is to set price $p = v$

- If any firm is to attract customers from the rival, they have to undercut by more than s (set price $v - s - \epsilon$ for some small ϵ)

$\Rightarrow$ This will give them all customers

$\Rightarrow$ The firm will earn $2(v - s - \epsilon)$ for some small ϵ

$\Rightarrow$ Since $v < 2s$, $2(v - s - \epsilon) < v$

$\Rightarrow$ Firms will not want to undercut the price v

Price discrimination

- Suppose that firms can price discriminate by offering different prices to their local consumers and those that are far away
- They could do this by offering a coupon for consumers who are far away, *i.e.*, only delivering it to consumers in the other city

Price discrimination (cont.)

- Starting from the point where $p = v$, either firm would want to offer a price of $v - s - \epsilon$ to the rival firm's customers

⇒ This would give it more profit: $v + (v - s - \epsilon) > v$

- The rival will respond by lowering its price to its own customers by ϵ and of course, cutting its price to the first firm's customers
- The equilibrium occurs when $p = s$ to a firm's own customers and $p = 0$ to the rival's customers

Price discrimination (cont.)

- Each firm makes s on its own customers, and does not sell to the rival's customers (so in equilibrium coupons would not actually be used)
- The introduction of coupons provides another instrument of competition for the firms
- Although this allows for targeted price discrimination, the result is lower prices in equilibrium

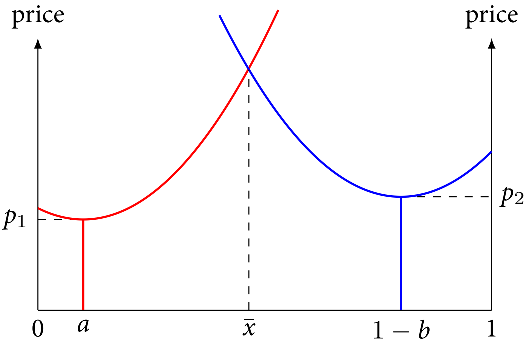
Price discrimination (cont.)

- How does a monopolist (with two stores) set price?
- Charge v for consumers in both cities
- Assumption: $v > st$

Price competition with endogenous locations

- In period 1, firms choose location; in period 2, firms set prices $\Rightarrow$ SPE
- We will assume quadratic transportation costs, and endogenize the choice of location
- Suppose firm 1 is located at a and firm 2 is located at $1-b$, where $0 \leq a \leq 1-b \leq 1$
- The cost of buying from firm 1 is $p_1 + t(x-a)^2$ and the cost of buying from firm 2 is $p_2 + t(1-b-x)^2$

Consumers' behaviors



Firms' market shares

- The consumer that is just indifferent between buying from the two firms is located at

$$\bar{x} = \frac{p_2 - p_1}{2t(1-a-b)} + \frac{(1-b)^2 - a^2}{2(1-a-b)} = \frac{p_2 - p_1}{2t(1-a-b)} + a + \frac{1-a-b}{2}$$

- Given that consumers are uniformly distributed, firm 1's market share s_1 is just $\bar{x}$ and firm 2's market share is $1 - \bar{x}$

Optimal pricing

- Firm 1 sets p_1 to maximise

$$(p_1 - c)s_1$$

while firm 2 sets p_2 to maximize

$$(p_2 - c)(1 - s_1)$$

- (Subgame) NE:

$$p_1^*(a,b) = c + t(1-a-b)\left(1 + \frac{a-b}{3}\right)$$
$$p_2^*(a,b) = c + t(1-a-b)\left(1 + \frac{b-a}{3}\right)$$

Optimal location

Firm 1's profit function

$$\pi_1(a,b) = [p_1^*(a,b) - c] \cdot s_1\left(a, b, p_1^*(a,b), p_2^*(a,b)\right)$$

The effect on firm 1's (subgame equilibrium) profit of any change in its location is (envelop theorem)

$$\frac{\mathrm{d}\pi_1}{\mathrm{d}a} = \underbrace{\frac{\partial \pi_1}{\partial a}}_{\text{demand (market share) effect}} + \underbrace{\frac{\partial \pi_1}{\partial p_2}\frac{\mathrm{d}p_2}{\mathrm{d}a}}_{\text{strategic effect}} = (p_1^* - c)\left(\frac{\partial s_1}{\partial a} + \frac{\partial s_1}{\partial p_2}\frac{\mathrm{d}p_2^*}{\mathrm{d}a}\right)$$

Optimal location (cont.)

- If a and b are less than $\frac{1}{2}$, then $\frac{\partial s_1}{\partial a} > 0$

$\Rightarrow$ Both firms will want to move towards the centre to increase market share

- $\frac{\mathrm{d}p_2}{\mathrm{d}a} < 0$ and $\frac{\partial s_1}{\partial p_2} > 0$

$\Rightarrow$ Moving towards the centre will reduce product differentiation and so cause its rival to lower its price

$\Rightarrow$ A lower price by the rival will lower the firm's market share and so its profits

Optimal location (cont.)

- $$\frac{\partial s_1}{\partial a} = \frac{1}{2} + \frac{p_2^* - p_1^*}{2t(1-a-b)^2} = \frac{3-5a-b}{6(1-a-b)}$$
- $$\frac{\partial s_1}{\partial p_2}\frac{\mathrm{d}p_2^*}{\mathrm{d}a} = \frac{-2+a}{3(1-a-b)}$$
- $\frac{\mathrm{d}\pi_1}{\mathrm{d}a} < 0$

$\Rightarrow$ Firm 1 always wants to move leftward, and similarly for firm 2

$\Rightarrow$ Maximal differentiation

- SPE: firms locate at opposite ends of the unit interval, and set prices as $p_1^*(a,b)$ and $p_2^*(a,b)$ in the second period

$\Rightarrow$ Prices: $p_1^* = p_2^* = c + t$

$\Rightarrow$ Profits: $\pi_1^* = p_2^* = \frac{t}{2}$

Optimal location (cont.)

- How about linear cost function td?

$\Rightarrow$ Best response function is not continuous in the subgame initiated by (a, b) when $a > 0$ and $b > 0$ (check by yourself)

$\Rightarrow$ NE in these subgames may not exist (check by yourself)

$\Rightarrow$ SPE may not exist (check by yourself)

- What is the solution when v is not high enough?

Subsection 3

Salop's circular city model

Motivation

- Hotelling's model: study price competition with differentiated goods, and the choice of product in duopoly
- Salop's model: study entry and location when there are no "barriers to entry" other than fixed costs or entry costs

Model: consumers

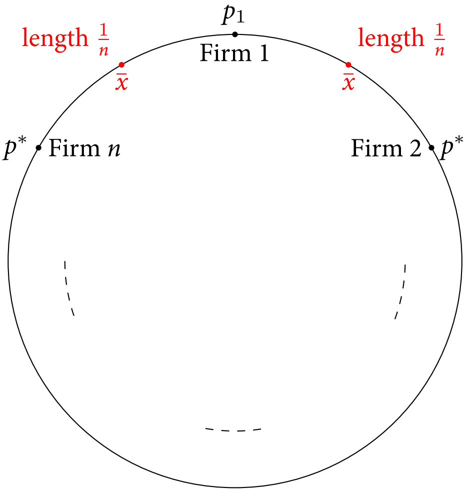
- Consumers are uniformly distributed around the perimeter of the circle (of length 1)
- Each consumer has unit demand
- Marginal transportation cost t
- v is high enough so that all consumers to buy in equilibrium

Model: firms

- The product space is completely homogeneous (no location is a priori better than another)
- Marginal production cost c
- Fixed cost of entry f
- Firm i's profit, if it enters, is

$$(p_i - c)s_i - f$$

Model: illustration



Model: game

- In period 1, firms choose whether or not to enter (let n denote the number of entering firms)
- In period 2, firms do not choose their location, but rather are automatically located equidistant from one another on the cycle

$\Rightarrow$ Maximal differentiation is exogenously imposed

$\Rightarrow$ Each firm only directly competes with its two nearest neighbours

- In period 3, price competition

Consumers' behaviors

- Consider the symmetric NE: all firms choose p^*
- Suppose all other firms' price is p^*
- A consumer located at a distance $x \in (0, \frac{1}{n})$ from firm i is indifferent between purchasing from i and purchasing i's closest neighbour if

$$p_i + tx = p^* + t(\frac{1}{n} - x)$$

$\Rightarrow$

$$\bar{x} = \frac{p^* + \frac{t}{n} - p_i}{2t}$$

Firms' optimal pricing

- Firm i's demand

$$D_i(p_i, p^*) = 2\bar{x} = \frac{p^* + \frac{t}{n} - p_i}{t}$$

- Firm i's profit

$$(p_i - c)\frac{p^* + \frac{t}{n} - p_i}{2t} - f$$

- p^* maximizes firm i's profit:

$$p^* = c + \frac{t}{n}$$

$\Rightarrow$ Profit

$$\pi^* = \frac{t}{n^2} - f$$

Number of firms

- The number of firms is determined by the zero profit condition at period 1

$$\frac{t}{n^2} - f = 0$$

$\Rightarrow$

$$n^e = \sqrt{\frac{t}{f}}$$

$\Rightarrow$ $n^e \to \infty$ as $f \to 0$

$\Rightarrow$

$$p^e = c + \sqrt{tf} \text{ and } \pi^e = 0$$

- Price is above marginal cost, but zero profit

IR condition

- In this model, we assumed implicitly that the equilibrium is such that

$$p^e + \frac{t}{2n^e} \leq v$$

IR condition

$\Leftrightarrow$

$$f \leq \frac{4}{9t}(v-c)^2 \triangleq \bar{f}$$

- This assumption does not create any problem for small fixed costs
- When fixed costs increase, the number of firms decreases, the distances between firms increase, and the prices increase
- Question: How about when $f \geq \bar{f}$?

Socially optimal number of firms

- Social planner would choose n to minimise the sum of the fixed costs and the transportation costs
- The transportation costs is

$$2nt \int_0^{\frac{1}{2n}} x \, \mathrm{d}x = \frac{t}{4n}$$

- $$\min_n nf + \frac{t}{4n}$$

$\Rightarrow$ $n^s = \frac{1}{2}\sqrt{\frac{t}{f}} = \frac{1}{2} n^e$

- Market generates too many firms

Remark

- The private and social incentive to enter have no reason to coincide
- Entry is socially justified by the saving in transportation costs
- The private incentive to enter is linked with "stealing the business"

Quadratic transportation cost

- Transportation costs are quadratic rather than linear
- Equilibrium price $p = c + \frac{t}{n^2}$ (check by yourself)
- Free-entry number of firms $n^e = (\frac{t}{f})^{\frac{1}{3}}$ (check by yourself)
- Socially optimal number of firms $n^s = (\frac{t}{6f})^{\frac{1}{3}}$ (check by yourself)

Remark

- The assumption that firms locate equidistantly is appealing given the maximal-differentiation result for quadratic transportation costs in a linear city
- It has been justified in the context of the circular city in the case of quadratic transportation costs (check by yourself; Economides, 1984)

Subsection 4

Discrete choice model

Discrete choice model

- In address models, consumers just vary in one characteristic—location

⇒ In practice consumers vary in their tastes for lots of different characteristics of products

- In address models, firms only competed (directed) with their nearest neighbours

⇒ Firms are likely to compete (directly) with all other firms to some extent

- Consumers make "discrete choices"—they typically choose only one of the competing products

Discrete choice model (cont.)

- There are a large number of consumers (say N) and n firms each producing a unique good
- Firms are assumed to produce at marginal cost c and fixed cost f
- Each consumer attains utility $u_i = v_i + \mu\epsilon_i$ from buying from firm i, where v_i is the indirect utility of good i (we will take it to be simply $v - p_i$), and ϵ_i is a random variable that reflects idiosyncratic taste differences for firm i's good
- Consumers may also choose an outside alternative (*i.e.*, do not buy one of the goods), which gives utility $u_0 = v_0 + \mu\epsilon_0$
- ϵ_i can be interpreted as an uncertainty due to the lack of knowledge

Discrete choice model (cont.)

- The parameter μ represents the consumers' taste for variety
- As $\mu \to 0$, the alternatives become homogeneous and if $v_i = v - p_i$, consumers would simply buy the good with the lowest price (a'la Bertrand competition with homogenous goods)
- As $\mu \to \infty$, the differentiation becomes very high and consumers behave as if they make their choices randomly (*i.e.*, they do not act on any price differences)

Logit type

- ϵ_i are i.i.d. according to the double exponential distribution, which has the cumulative density function

$$F(x) = \text{Prob}(\epsilon_i \leq x) = e^{-e^{-\frac{x}{\rho}-\gamma}}$$

where $\gamma \approx 0.5772$ is Euler's constant and ρ is a positive constant

Demand

- Consumers choose the alternative that gives the highest utility
- The market share firm i expects is given by

$$s_i = \text{Prob}\left(u_i = \max_{j=0,1,\ldots,n} u_j\right)$$

- The expected market share for firm i is given by

$$s_i = \frac{e^{v_i/\mu}}{e^{v_0/\mu} + e^{v_1/\mu} + \cdots + e^{v_n/\mu}}$$

(check by yourself)

Demand (cont.)

- The case where everyone buys a good (for example, $v_0 \to -\infty$) implies

$$s_i = \frac{e^{v_i/\mu}}{e^{v_1/\mu} + e^{v_2/\mu} + \cdots + e^{v_n/\mu}}$$

- Alternatively, the case where the outside good has expected surplus normalized to zero implies

$$s_i = \frac{e^{v_i/\mu}}{1 + e^{v_1/\mu} + e^{v_2/\mu} + \cdots + e^{v_n/\mu}}$$

Price competition

- Then firm *i*'s profit is simply

$$\pi_i = (p_i - c)s_i N - f$$

- $$\frac{\mathrm{d}s_i}{\mathrm{d}p_i} = \frac{-s_i(1 - s_i)}{\mu} \text{ and } \frac{\mathrm{d}s_i}{\mathrm{d}p_j} = \frac{s_i s_j}{\mu}$$

- Negative semi-definite $\Rightarrow$ FOC
- NE:

$$p^* = c + \frac{\mu}{1 - s^*}$$

where

$$s^* = \frac{1}{n + e^{\frac{v_0 - v + p^*}{\mu}}}$$

Market are fully covered

- If everyone buys one of the goods (that is, $v_0 \to -\infty$), then $s^* = \frac{1}{n}$
- In this case, the equilibrium price becomes simply

$$p^* = c + \frac{n\mu}{n-1}$$

- As $\mu \to 0$, equilibrium prices tend to marginal cost
- In this case, profits are

$$\pi^* = \frac{\mu N}{n-1} - 1$$

Positive profit

- With positive μ (differentiation), firms make positive margins even as n gets large (each firm offers a differentiated product, so it will not drive prices exactly to cost)
- With just two firms competing, the equilibrium prices are $c + 2\mu$
- As $n \to \infty$, the equilibrium prices tends to $c + \mu$

Free-entry equilibrium

- In the case everyone buys a good ($v_0 \to -\infty$), then free-entry firms make zero profits, so

$$\frac{\mu N}{n-1} = f$$

$\Rightarrow$

$$n^e = 1 + \frac{\mu N}{f}$$

Inefficiency

- It is possible to have too much or too little entry in this model compared to the socially efficient level
- While there is still a business-stealing effect in the model, so that firms do not take into account the negative effect their entry has on other firms' profits (resulting in an excessive incentive to enter), there is also a strong variety effect in which greater entry means consumers are more likely to be able to buy a product that suits their particular tastes (that is, with more firms consumers are more likely to obtain a high draw of ϵ_i)
- Firms that are deciding to enter do not fully internalise the benefit to consumers from the added variety they offer to consumers' choice, and so there can be too little entry

Section 4

Vertical differentiation

Vertical differentiation

- Consumers rank the products similarly
- Consumers have a different willingness to pay for quality

Model

- Let consumers' preferences be described by

$$U = \theta s - p$$

  if they buy one unit of quality s and pay price p; otherwise they get utility of 0
- Here θ measures the consumers' taste for quality, and s indicates the level of quality
- Assume θ is uniformly distributed between $\underline{\theta} \geq 0$ and $\bar{\theta} = \underline{\theta} + 1$ with density 1,
- Two firms. Firm i produces a good of quality s_i, where $s_2 > s_1$
- Marginal production cost c
- Denote $\Delta s = s_2 - s_1$, $\bar{\Delta} = \bar{\theta}\Delta s$ and $\underline{\Delta} = \underline{\theta}\Delta s$

Assumptions

- Assumption 1

$$\bar{\theta} \geq 2\underline{\theta}$$

⇒ The amount of consumer heterogeneity is sufficient for what follows

- Assumption 2

$$c + \frac{\bar{\theta} - 2\underline{\theta}}{3}(s_2 - s_1) \leq \underline{\theta} s_1$$

⇒ Ensure that in the price equilibrium the market is covered (each consumer buys one of the two brands)

Subsection 1

Price competition

Price competition

- A consumer with parameter θ is indifferent between the two brands iff

$$\theta s_1 - p_1 = \theta s_2 - p_2$$

$\Rightarrow$

$$\theta_0 = \frac{p_2 - p_1}{\Delta s}$$

$\Rightarrow$ Demands

$$D_1(p_1, p_2) = \frac{p_2 - p_1}{\Delta s} - \underline{\theta} \text{ and } D_2(p_1, p_2) = \bar{\theta} - \frac{p_2 - p_1}{\Delta s}$$

Price competition (cont.)

- Firm i's profit

$$(p_i - c)D_i(p_i, p_j)$$

- FOC leads to the best response functions

$$p_2^*(p_1) = \frac{p_1 + c + \bar{\Delta}}{2} \text{ and } p_1^*(p_2) = \frac{p_2 + c - \underline{\Delta}}{2}$$

- NE:

$$p_1^* = c + \frac{\bar{\theta} - 2\underline{\theta}}{3}\Delta s > c \text{ and } p_2^* = c + \frac{2\bar{\theta} - \underline{\theta}}{3}\Delta s > p_1^*$$

- Profits

$$\pi_1^* = \frac{(\bar{\theta} - 2\underline{\theta})^2}{9}\Delta s \text{ and } \pi_2^* = \frac{(2\bar{\theta} - \underline{\theta})^2}{9}\Delta s$$

IR condition

- IR condition:

$$\underline{\theta}s_1 \geq p_1^* \text{ and } \theta_0 s_2 \geq p_2^*$$

- The second condition holds automatically if the first one holds (check by yourself)
- Assumption 2 guarantees the first condition to hold (check by yourself)

$\Rightarrow$ The market is covered for (s_1, s_2)

Comparison with Bertrand competition

- The firms' profits are proportional to the quality difference Δs
- If there is no quality difference ($\Delta s = 0$), the firms will both set prices equal to marginal cost (Bertrand competition with homogenous goods)

$\Rightarrow$ firms make profits because they are differentiated by their quality (vertical differentiation)

Remark

- The higher quality firm charges a higher price than the lower quality firm
- The higher quality firm makes more profit than the lower quality firm, although this conclusion depends on quality being costless to produce
- Even if the low quality firm were to charge marginal cost, the higher quality firm can charge more and make a profit since it has a superior product

Subsection 2

Quality competition

Quality competition

- Two-stage game: firms choosing their quality level in the first stage, and then competing price in the second stage
- Suppose firms have to choose quality levels between s_L and s_H, and quality is costless to produce
- Firms will never want to choose the same quality levels in the first stage, as then they will make no profit in stage 2

Quality competition (cont.)

- Suppose that for some reason firm 1 has to set the lower quality level
- Firm 1's profit is increasing in quality difference

⇒ Firm 1 will always set quality $s_1 = s_L$ (as low as possible)

- Firm 2's profit is increasing in quality difference

⇒ Firm 2 will always set quality $s_2 = s_H$ (as high as possible)

- NE: Firm 1 sets minimal product quality and firm 2 sets maximal product quality (maximal vertical differentiation)
- Modified assumption 2

$$c + \frac{\bar{\theta} - 2\underline{\theta}}{3}(s_H - s_L) \leq \underline{\theta} s_L$$

⇒ Guarantee assumption 2 to hold for any pair (s_1, s_2) where $s_L \leq s_1 \leq s_2 \leq s_H$

- Question: How about the case $2\underline{\theta} > \bar{\theta}$?

Subsection 3

Natural oligopoly

Natural oligopoly

- There may be no incentive for additional firms to enter, even though entry is almost costless and existing firms make positive profit
- Shaken and Sutton (1983) show that for vertically differentiated markets there is a tendency to get a natural oligopoly
- There will only be a fixed number of firms in the market, even though they all earn positive profits
- The competing highest quality firms will drive the price down to a point where further entrants would not generate any positive demand (at a price above cost) if they offer a lower quality, and they will not obtain any profit if they set the same quality level as the existing firms
- There will be a finite number of firms in a market—no matter how large it becomes

Natural oligopoly (cont.)

- n of firms offer distinct substitute goods which vary in quality
- Consumer buy 1 unit or zero
- Zero costs
- Label goods $k = 1, \ldots, n$, firm k sells product k at p_k
- Continuum of consumers of different incomes uniformly distributed, with a density of 1, along a line segment $\underline{\theta} > 0$ to $\overline{\theta}$
- Utility

$$U(t,k) = \begin{cases} u_k \cdot (\theta - p_k), & \text{purchase good } k \\ u_0 \cdot \theta, & \text{not purchase} \end{cases}$$

where $0 < u_0 < u_1 < \cdots < u_n$, and θ denotes the income

Natural oligopoly (cont.)

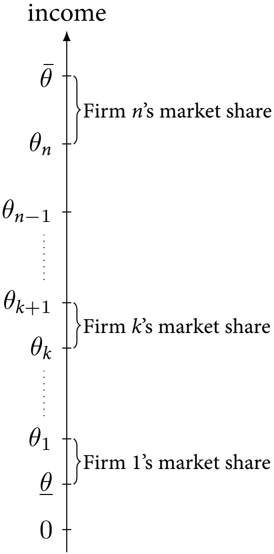
- Define $r_k = \frac{u_k}{u_k - u_{k-1}} > 1$
- Equation of the indifferent consumer (income θ_k) between goods $k-1$ and k

$$u_k \cdot (\theta_k - p_k) = u_{k-1} \cdot (\theta_k - p_{k-1})$$

$\Rightarrow$ $\theta_k = p_{k-1}(1 - r_k) + p_k r_k$

- Market share: $\theta_{k+1} - \theta_k$ for firm k, $\theta_1 - a$ for firm 1, $b - \theta_n$ for firm n

Natural oligopoly (cont.)



Natural oligopoly (cont.)

- More than one goods survive $\Rightarrow \theta_n > \underline{\theta}$
- Firm n maximizes its profit $p_n(\bar{\theta} - \theta_n)$

$\Rightarrow$ $\bar{\theta} - \theta_n - p_n r_n = 0$

- Since $\theta_n = p_{n-1}(\underline{\theta} - r_n) + p_n r_n$, we have

$$\bar{\theta} - 2\theta_n + p_{n-1}\underbrace{(1 - r_n)}_{<0} = 0$$

$\Rightarrow$ $\theta_n < \frac{\bar{\theta}}{2}$

- Assumption: $\bar{\theta} > 2\underline{\theta}$ (same as the original model)

Natural oligopoly (cont.)

- Firm k's profit

$$p_k(\theta_{k+1} - \theta_k)$$

- FOC (concave):

$$\theta_{k+1} - \theta_k - p_k[(r_{k+1} - 1) + r_k] = 0$$

- Since $\theta_k = p_{k-1}(1 - r_k) + p_k r_k$, we have

$$\theta_k < \frac{\theta_{k+1}}{2}$$

$\Rightarrow$ $\theta_n < \frac{\bar{\theta}}{2}, \theta_{n-1} < \frac{\bar{\theta}}{4}, \theta_{n-2} < \frac{\bar{\theta}}{8}, etc$

$\Rightarrow$ There exists a bound independent of product qualities and consumer density to the number of firms which can survive with positive prices at a NE in prices

Natural oligopoly (cont.)

- Number of firms depends on lower bound to income $\underline{\theta}$
- Can have lots of firms if $\underline{\theta} \to 0$ and $\bar{\theta} \to \infty$
- A firm that sets price equal to zero will NOT win all the market
- Pattern of Market shares will be independent of the density of consumers

Section 5

Horizontal and vertical product differentiation

Market segmentation

- Horizontal product differentiation increases firms' profits in equilibrium

$\Rightarrow$ It corresponds to the marketing strategy of market segmentation

- Product differentiation means firms' trade-off captive customers and those on the margin between two firms
- They would like to be able to price-discriminate across the different types of customers, charging more to their captive or inframarginal customers and less to their marginal customers

Horizontal and vertical product differentiation

- When one firm has more inframarginal consumers due to some difference in average preferences for the two products (say due to higher quality), then we would not expect equilibrium prices to be symmetric
- This suggests a very simple way to introduce both horizontal and vertical product differentiation, in which prices are no longer symmetric across the firms

Model

- Standard Hotelling model with firms located at either ends of the unit interval, and linear transportation costs, but suppose consumers value the good produced by firm 1 at $v + \beta$ and the good produced by firm 2 at just v, where $0 < \beta < 3t$ (to guarantee IR condition)
- Firm 1's good is better than firm 2's good, although not all consumers agree on the ranking due to their relative distances away from the two goods
- Consumers are uniformly distributed on the unit interval

Consumers' behaviors

- A consumer located at x gets utility $v + \beta - p_1 - tx$ if she buys from firm 1, while she gets utility $v - p_2 - t(1 - x)$ if she buys from firm 2

$\Rightarrow$

$$\bar{x} = \frac{1}{2} + \frac{\beta + p_2 - p_1}{2t}$$

$\Rightarrow$ Firm 1's market share $s_1 = \bar{x}$, and firm 2's market share $1 - \bar{x}$

- As $\beta \to t$, firm 2 will have to undercut firm 1 to get any demand

Price competition

- Firm 1 sets price p_1 to maximize

$$(p_1 - c)s_1$$

while firm 2 sets price p_2 to maximize

$$(p_2 - c)(1 - s_1)$$

- NE

$$p_1^* = c + t + \frac{\beta}{3} \text{ and } p_2^* = c + t - \frac{\beta}{3} > c$$

- Profits

$$\pi_1 = \frac{t}{2}\left(1 + \frac{\beta}{3t}\right)^2 \text{ and } \pi_2 = \frac{t}{2}\left(1 - \frac{\beta}{3t}\right)^2$$

Remark

- Firm 1 enjoys brand loyalty and so will have a greater percentage of captive customers
- This simply reflects the fact it has a greater share of inframarginal consumers relative to marginal consumers
- Firm 1 will compete less aggressively to steal customers from the rival
- Its equilibrium market share is

$$s_1^* = \frac{1}{2} + \frac{\beta}{6t}$$

$\Rightarrow$ Model behaves as one of horizontal differentiation but allowing for asymmetric shares

Section 6

Summary

Summary

- $U \to D_i \to p^* (\xrightarrow{f} n^e, n^s)$
- $\frac{\partial D_i}{\partial p_i} < 0$ and $\frac{\partial D_i}{\partial p_j} > 0$
- The differentiation is measured by several parameters

$\Rightarrow$ When differentiation is approaching zero, $p^* \to c$ and $\pi^* \to 0$ (Bertrand competition)

Section 7

Maximal or minimal differentiation?

The principle of differentiation

Firms want to differentiate to soften price competition
$\Rightarrow$ Maximal differentiation

Be where the demand is

- Although firms like to differentiate for strategic purposes, they also all want to locate where the demand is, *e.g.*, near the center of the linear city

$\Rightarrow$ Partial differentiation (Economides, 1986)

- Evidence: Abundance of ice cream parlors and bookstores near universities

Positive externalities between firms

- There may be externalities that induce firms to locate near one another
- Cost side: common installations and trade centers
- Evidence: fishermen converge to the same harbor to sell their fish, even if this means more intense competition
- Demand side: lowering search costs and increasing aggregate demand
- Evidence: 丹阳国际眼镜城, 义乌小商品市场

Absence of price competition

- Principle of minimum differentiation in Hotelling model with fixed price
- Evidence: political platforms tend to cluster around the center
- Evidence: similar TV shows compete in the same time slots of the major networks in many countries