Advanced Microeconomics I: Lecture 9

1 When two parties engage in a relationship, it is often the case that they are uncertain about the value of some parameter that will affect their future gains from trade. This uncertainty is represented by assuming that the parameter can take several values, each value corresponding to different states of nature whose probability distribution is common knowledge.

Even though they will both learn the value of the parameter in the future, the trading partners cannot write *ex ante* contracts contingent on the state of nature, because this state of nature is not verifiable by a third party that could enforce their contract. That is the nonverifiability of the state of nature.

1 No contract

2 An owner (principal) wishes to hire a manager (agent) to run a one-time project.

If the agent's effort level is $e \in [0, \infty)$, then principal's income is $\pi(e)$, with $\pi(0) = 0$, $\pi'(e) > 0$, and $\pi''(e) < 0$ for all e.

If the principal pays wage w to the agent, the agent's utility/profit is $\pi(e) - w$.

3 The agent is an expected utility maximizer with utility $w - g(e, \theta)$.

- $\theta \in \{\theta_L, \theta_H\}$ represents agent's ability. Here, $\theta_H > \theta_L$ and $\text{Prob}(\theta_H) = \lambda \in (0, 1)$.
- $g(e, \theta)$ measures the disutility of effort.
- $g(0, \theta) = 0$, $g_e(e, \theta) \begin{cases} > 0, & \text{if } e > 0 \\ = 0, & \text{if } e = 0 \end{cases}$, $g_{ee} > 0$, $g_\theta < 0$, $g_{e\theta}(e, \theta) \begin{cases} < 0, & \text{if } e > 0 \\ = 0, & \text{if } e = 0 \end{cases}$.

$\Rightarrow$ The agent's indifference curves have single-crossing property.

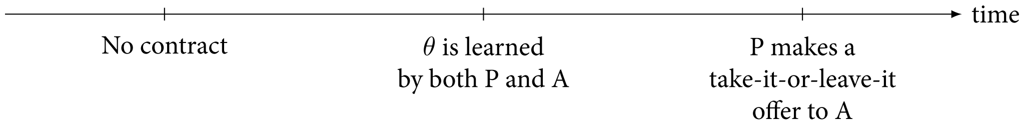
- The agent is risk neutral. The agent has a reservation utility 0.

1.1 Principal has full bargaining power

4 We assume that the principal has all the bargaining power.

5 After being informed about θ, the principal can make a take-it-or-leave-it offer to the agent under complete information.

6 The sequence of play is as follows:



7 The offer can implement the first-best outcome:

- If agent is of high ability, then he will make effort e_H^* such that $\pi'(e_H^*) = g_e(e_H^*, \theta_H)$, receive wage $w_H^* = g(e_H^*, \theta_H)$.
- If agent is of low ability, then he will make effort e_L^* such that $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$, receive wage $w_L^* = g(e_L^*, \theta_L)$.

1.2 Bargaining

8 We assume that the principal and the agent have equal weights in the negotiation.

9 The sequence of play is as follows:

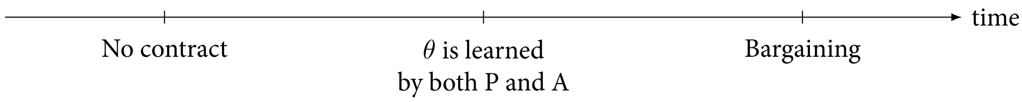


Figure 1

10 We use Nash bargaining solution.

11 When the agent is of high ability, they shall agree on effort e and wage w, which are solutions to the problem

$$\max_{(e,w)} \big(\pi(e) - w\big)\big(w - g(e, \theta_H)\big).$$

12 Solution:

- Effort is first-best effort e_H^* such that $\pi'(e_H^*) = g_e(e_H^*, \theta_H)$;
- Wage $w_H^{\text{NB}} = \frac{\pi(e_H^*)+g(e_H^*,\theta_H)}{2}$.

Both principal and agent receive an equal share of the first-best gains.

13 Similarly for the low ability case.

- Effort is first-best effort e_L^* such that $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$;
- Wage $w_L^{\text{NB}} = \frac{\pi(e_L^*)+g(e_L^*,\theta_L)}{2}$.

Both principal and agent receive an equal share of the first-best gains.

14 We can also use bargaining game with alternative offers to model the bargaining process.

2 *Ex ante* contract

15 Instead of waiting for the realization of the state of nature, the principal can offer to the agent, at the *ex ante* stage, a menu of contracts.

16 The contract can only be written in terms of the verifiable variables. θ is not verifiable and cannot be written into a contract.

A menu $\{(e_H, w_H), (e_L, w_L)\}$ is a feasible instrument.

17 The sequence of play is as follows:

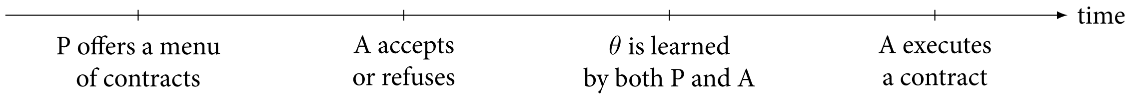


Figure 2

18 Agent's problem:

- IR: $\lambda(w_H - g(e_H, \theta_H)) + (1 - \lambda)(w_L - g(e_L, \theta_L)) \geq 0$.
- IC: $w_H - g(e_H, \theta_H) \geq w_L - g(e_L, \theta_H)$ and $w_L - g(e_L, \theta_L) \geq w_H - g(e_H, \theta_L)$.

19 Principal's problem:

$$\begin{aligned} \underset{(e_L, w_L), (e_H, w_H)}{\text{maximize}} \quad & \lambda\big(\pi(e_H) - w_H\big) + (1 - \lambda)\big(\pi(e_L) - w_L\big) \\ \text{subject to} \quad & \text{IR and IC.} \end{aligned}$$

20 IR should be binding. Otherwise, principal can lower w_H and w_L simultaneously.

21 The first-best outcome can be implemented: $\pi'(e_H^*) = g_e(e_H^*, \theta_H)$ and $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$,

$$\begin{aligned} w_H^* &= \pi(e_H^*) - \lambda(\pi(e_H^*) - g(e_H^*, \theta_H)) - (1 - \lambda)(\pi(e_L^*) - g(e_L^*, \theta_L)), \\ w_L^* &= \pi(e_L^*) - \lambda(\pi(e_H^*) - g(e_H^*, \theta_H)) - (1 - \lambda)(\pi(e_L^*) - g(e_L^*, \theta_L)). \end{aligned}$$

22 If agent is risk-averse, *ex ante* contracting fails to achieve efficiency. (Bonus question)

3 Nash implementation

23 The principal and agent can achieve *ex post* efficiency through an *ex ante* contract when they are both risk neutral.

This contract uses only agent's message but fails to achieve efficiency when the agent is risk-averse.

24 Consider the following mechanism:

- If both principal and agent report that θ_H has realized, the contract (e_H^*, w_H^*) is enforced.
- If both principal and agent report that θ_L has realized, the contract (e_L^*, w_L^*) is enforced.
- If they disagree, then nothing is enforced.

		Principal	
		θ_H	θ_L
Agent	θ_H	e_H^*, w_H^*	$0, 0$
	θ_L	$0, 0$	e_L^*, w_L^*

Note that the same game form must be played by the agent and the principal, whatever the true θ.

The goal of this mechanism is to ensure that there exists a truthful Nash equilibrium in each θ that implements the first-best outcome.

25 Proposition: The first-best outcome can be implementable in Nash equilibrium.

Proof. First consider θ_H.

- Given that agent reports θ_H, principal gets $\pi(e_H^*) - g(e_H^*, \theta_H)$ by reporting the truth and zero otherwise.
- By assumption, the delegation is valuable: $\pi(e_H^*) - g(e_H^*, \theta_H) = \pi(e_H^*) - w_H^* \geq 0$.
- Telling the truth is a best response for principal.
- Agent is indifferent telling the truth or not when principal reports θ_H.

Next consider θ_L.

- Given that agent reports θ_L, principal gets $\pi(e_L^*) - g(e_L^*, \theta_L)$ by reporting the truth and zero otherwise.
- By assumption, the delegation is valuable: $\pi(e_L^*) - g(e_L^*, \theta_L) = \pi(e_L^*) - w_L^* \geq 0$.
- Telling the truth is a best response for principal.
- Agent is indifferent telling the truth or not when principal reports θ_L.

□

26 When θ_H realizes, (θ_H, θ_H) is not the unique Nash equilibrium.

27 Consider the following mechanism:

		Principal	
		θ_H	θ_L
Agent	θ_H	e_H^*, w_H^*	$\hat{e}_2, \hat{w}_2$
	θ_L	$\hat{e}_1, \hat{w}_1$	e_L^*, w_L^*

28 The conditions for having a truthful Nash equilibrium in θ_H are:

$$\pi(e_H^*) - w_H^* \geq \pi(\hat{e}_2) - \hat{w}_2 \text{ and } 0 = w_H^* - g(e_H^*, \theta_H) > \hat{w}_1 - g(\hat{e}_1, \theta_H).$$

Similarly, the conditions for having a truthful Nash equilibrium in θ_L are:

$$\pi(e_L^*) - w_L^* \geq \pi(\hat{e}_1) - \hat{w}_1 \text{ and } 0 = w_L^* - g(e_L^*, \theta_L) > \hat{w}_2 - g(\hat{e}_2, \theta_L).$$

29 When θ_H, to ensure (θ_L, θ_L) not to be a Nash equilibrium, we must have

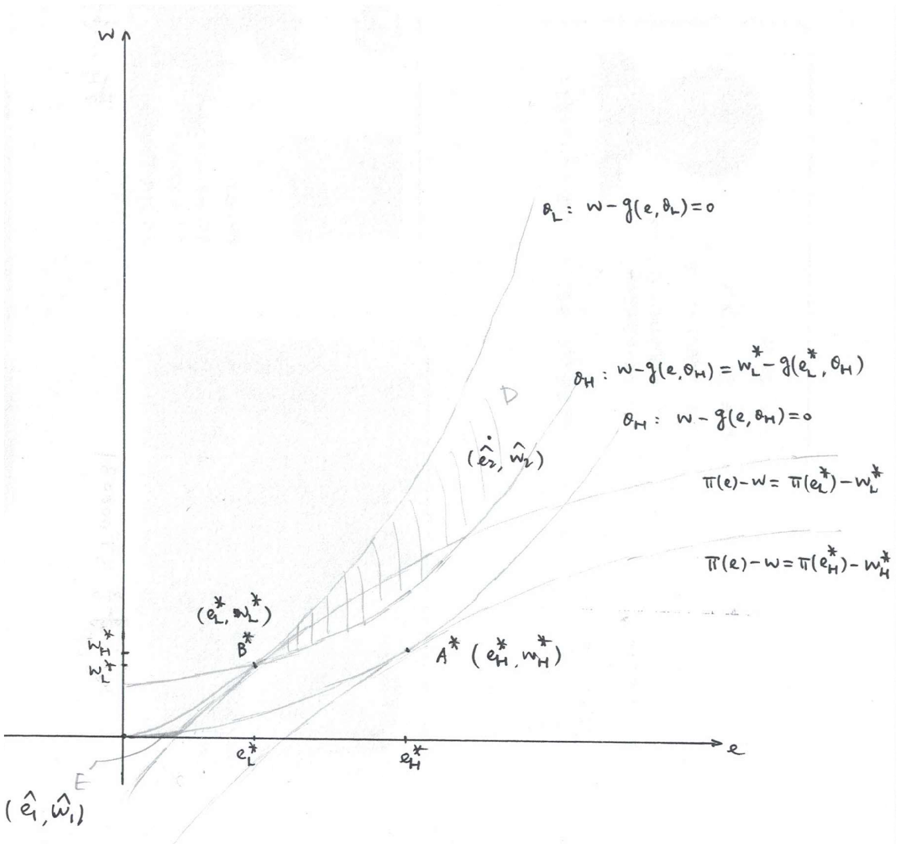
$$\hat{w}_2 - g(\hat{e}_2, \theta_H) > w_L^* - g(e_L^*, \theta_H).$$

When θ_L, to ensure (θ_H, θ_H) not to be a Nash equilibrium, we must have

$$\hat{w}_1 - g(\hat{e}_1, \theta_L) > w_H^* - g(e_H^*, \theta_L).$$

30 Proposition: The first-best outcome can be uniquely implementable in Nash equilibrium.

Proof. Consider the following graph.



Pick $(\hat{e}_1, \hat{w}_1)$ in region E and $(\hat{e}_2, \hat{w}_2)$ in region D. $\square$

4 Homework

- Reading: Chapter 6 in *The Theory of Incentives*