Advanced Microeconomics I: Lecture 8

1 Moral hazard models hidden action, where asymmetric information forms after the parties enter into a relationship. A moral hazard is a situation in which a party is more likely to take risks because the costs that could result will not be borne by the party taking the risk. Moral hazard arises because an individual or institution does not take the full consequences and responsibilities of its actions, and therefore has a tendency to act less carefully than it otherwise would, leaving another party to hold some responsibility for the consequences of those actions.

In a principal-agent problem, one party, called an agent, acts on behalf of another party, called the principal. The agent usually has more information about his or her actions or intentions than the principal does, because the principal usually cannot completely monitor the agent. The agent may have an incentive to act inappropriately (from the viewpoint of the principal) if the interests of the agent and the principal are not aligned.

In particular, consider that a firm (the principal) hires a worker (the agent) to work on a project, which succeeds with probability p if the worker exerts effort. The firm may only observe the outcome of the project but not the agent's effort level. In such a situation, the firm's payment contract can only depend on the outcome, which is an imperfect indicator of the worker's effort level. If the worker is paid fixed wage or if the payment conditional on success is not high enough, since effort is costly, the worker will shirk—moral hazard arises.

1 The principle-agent problem

2 A principal (employer) hires an agent (employee) for production. The agent can exert a costly effort $e \in \{0, 1\}$. Exerting effort e implies a disutility for the agent that is equal to $g(e)$ with the normalizations $g(0) = 0$ and $g(1) = g > 0$. The agent receives a wage w from the principal.

The agent's utility is assumed to be

$$u(w) - g(e),$$

where u is increasing and concave, and $u(0) = 0$. Denote $h = u^{-1}$, which is increasing and convex. We normalize the agent's reservation utility at zero.

3 Profit is stochastic, and effort affects the profit level as follows: the stochastic profit level π can only take two values $\{\pi_L, \pi_H\}$ with $\pi_H - \pi_L > 0$, and the stochastic influence of effort on profit is characterized by the probabilities

$$\text{Prob}(\pi = \pi_H \mid e = 0) = \lambda_0 \text{ and } \text{Prob}(\pi = \pi_H \mid e = 1) = \lambda_1,$$

with $\lambda_1 - \lambda_0 > 0$.

Effort improves profit in the sense of first-order stochastic dominance.

4 The principal can only offer a contract based on the observable profit level, i.e., $w(\pi)$. Let w_H (resp. w_L) be the wage received by the agent if the profit is π_H (resp. π_L).

5 The risk-neutral principal's expected utility is

$$V_1 = \lambda_1(\pi_H - w_H) + (1 - \lambda_1)(\pi_L - w_L)$$

if the agent makes a positive effort $e = 1$, and

$$V_0 = \lambda_0(\pi_H - w_H) + (1 - \lambda_0)(\pi_L - w_L)$$

if the agent makes no effort $e = 0$.

6 The problem of the principal is to decide whether to induce the agent to exert effort or not and, if he chooses to do so, then to decide which contract should be used.

7 The timing is as follows:

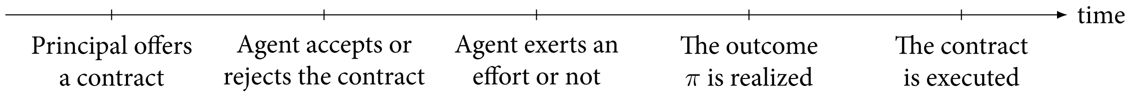


Figure 1

2 Complete information

8 First assume that the principal can observe effort.

9 In this situation, a contract is of the form (e, w_L, w_H). That is, the agent exerts the effort ($e = 1$) or not ($e = 0$) and he will receive w_L when the profit is low and w_H when the profit is high.

10 It is convenient to think of this problem in two steps:

- For each $e \in \{0, 1\}$ that might be specified in the contract, what is the best (w_L, w_H)?
- What is the best choice of e?

11 To induce the agent to exert effort ($e = 1$), the principal's problem is:

$$\begin{aligned} \underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_1(\pi_H - w_H) + (1 - \lambda_1)(\pi_L - w_L) \\ \text{subject to} \quad & \lambda_1 u(w_H) + (1 - \lambda_1)u(w_L) - g \geq 0. \end{aligned}$$

Indeed, only the agent's individual rationality matters for the principal, because the agent can be forced to exert a positive level of effort. If the agent were not choosing positive effort, his action could be perfectly detected by the principal, and hence the agent could be heavily punished.

12 Denoting the multiplier of the individual rationality constraint by μ and optimizing with respect to w_H and w_L yields, respectively, the following first-order conditions:

$$-\lambda_1 + \mu\lambda_1 u'(w_H^*) = 0,$$
$$-(1 - \lambda_1) + \mu(1 - \lambda_1)u'(w_L^*) = 0,$$

where w_H^* and w_L^* are the first-best wages.

We immediately derive that $\mu = \frac{1}{u'(w_L^*)} = \frac{1}{u'(w_H^*)} > 0$, and finally that $w^* = w_H^* = w_L^*$.

13 Remark:

- The wage w^* the agent receives is the same whatever the state of nature.
- Because the IR constraint is binding we also obtain the value of this wage, which is just enough to cover the disutility of effort, namely $w^* = u^{-1}(g)$. It is called the first-best cost C^* of implementing the positive effort level.

14 For the principal, inducing effort yields an expected payoff equal to

$$V_1 = \lambda_1 \pi_H + (1 - \lambda_1)\pi_L - u^{-1}(g).$$

15 Had the principal decided to let the agent exert no effort ($e = 0$), his problem is

$$\begin{aligned} \underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_0(\pi_H - w_H) + (1 - \lambda_0)(\pi_L - w_L) \\ \text{subject to} \quad & \lambda_0 u(w_H) + (1 - \lambda_0)u(w_L) \geq 0. \end{aligned}$$

He would make a zero payment (it is optimal) to the agent whatever the realization of profit. In this scenario, the principal would instead obtain a payoff equal to

$$V_0 = \lambda_0 \pi_H + (1 - \lambda_0)\pi_L.$$

16 Inducing effort is optimal from the principal's point of view when $V_1 \geq V_0$, i.e.,

$$(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq u^{-1}(g). \tag{1}$$

17 The left-hand side of Equation (1) captures the gain of increasing effort from $e = 0$ to $e = 1$. This gain comes from the fact that the return π_H, which is greater than π_L, arises more often when a positive effort is exerted. The right-hand side of Equation (1) is instead the first-best cost of inducing the agent's acceptance when he exerts a positive effort.

18 Summary:

- The first-best outcome (effort level) will be achieved:
  - The first-best outcome calls for $e^* = 1$ if and only if $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq u^{-1}(g)$.
  - When $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq u^{-1}(g)$, to implement the first-best outcome $e^* = 1$, the principal offers a contract $(1, u^{-1}(g), u^{-1}(g))$ and the agent will accept.
- The agent gets full insurance.

3 Incomplete information with risk-neutral agent

19 In this situation, a contract is of the form (w_L, w_H). That is, the agent will receive w_L when the profit is low and w_H when the profit is high, regardless of his effort level.

20 If the agent is risk-neutral, we can assume that (up to an affine transformation) $u(w) = w$ for all w.

21 To induce the agent to exert effort, the principal's problem is

$$\begin{aligned} \underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_1(\pi_H - w_H) + (1-\lambda_1)(\pi_L - w_L) \\ \text{subject to} \quad & \lambda_1 w_H + (1-\lambda_1)w_L - g \geq \lambda_0 w_H + (1-\lambda_0)w_L \\ & \lambda_1 w_H + (1-\lambda_1)w_L - g \geq 0. \end{aligned}$$

22 IR condition should be binding; otherwise the principal can decrease w_L without breaking IR condition.

23 The expected profit of principal is always $\lambda_1\pi_H + (1-\lambda_1)\pi_L - g$, if the above problem has a solution.

24 To find a solution, we let IC condition be binding. Then we have

$$w_H^{\text{SB}} = g + \frac{1-\lambda_1}{\lambda_1 - \lambda_0} g \text{ and } w_L^{\text{SB}} = g - \frac{\lambda_1}{\lambda_1 - \lambda_0} g. \tag{2}$$

- The agent is rewarded if profit is high, and his utility is $w_H^{\text{SB}} - g = \frac{1-\lambda_1}{\lambda_1-\lambda_0} g > 0$.
- The agent is punished if profit is low, and his utility is $w_L^{\text{SB}} - g = -\frac{\lambda_1}{\lambda_1-\lambda_0} g < 0$.

The principal makes an expected payment

$$\lambda_1 w_H^{\text{SB}} + (1-\lambda_1) w_L^{\text{SB}} = g,$$

which is equal to the disutility of effort he would incur if he could control the effort level perfectly or if he was carrying the agent's task himself.

25 The wages $(w_H^{\text{SB}}, w_L^{\text{SB}})$ yield one possible implementation of the first-best outcome, where IC binds.

Let us consider another pair of wages

$$w_H^{\text{SB}'} = g + 2\frac{1-\lambda_1}{\lambda_1 - \lambda_0} g \text{ and } w_L^{\text{SB}'} = g - 2\frac{\lambda_1}{\lambda_1 - \lambda_0} g.$$

Clearly, IR binds and IC is strictly satisfied.

Indeed, there are infinitely many solutions.

26 Graphic illustration:

(1) $w - g$ is the agent's utility function when he exerts effort.

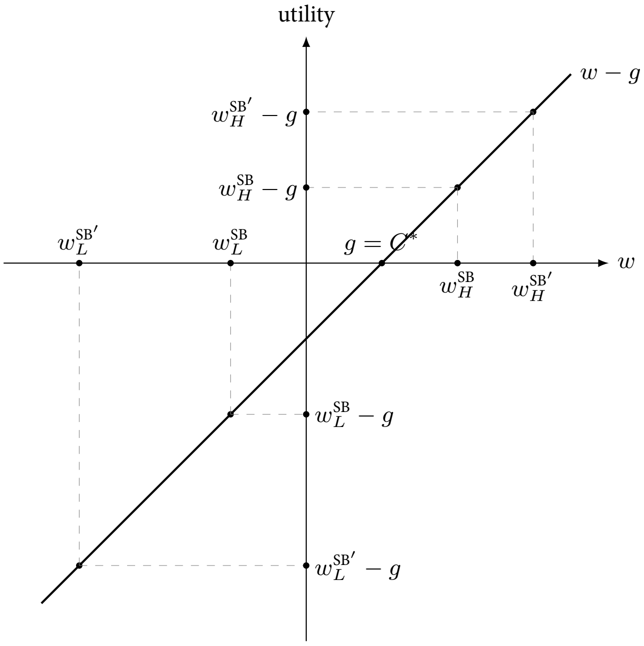
(2) In the complete information case, the agent's utility is zero, and the wage is always g.

(3) Since IR binds, the contract $(w_H^{\text{SB}}, w_L^{\text{SB}})$ makes the agent's expected utility be zero, shown as in the graph. That is, $\lambda_1 w_H^{\text{SB}} + (1-\lambda_1) w_L^{\text{SB}} - g = 0$.

(4) The expected wage should be $\lambda_1 w_H^{\text{SB}} + (1-\lambda_1) w_L^{\text{SB}} = g$.

(5) To induce the agent to exert effort, the principal needs to set w_H and w_L to satisfy $(\lambda_1 - \lambda_0)(w_H - w_L) \geq g$. That is, $w_H - w_L$ should be at least $\frac{g}{\lambda_1 - \lambda_0}$.

(6) IC could not bind: the principal can increase w_H^{SB} to $w_H^{\text{SB}'}$ and decrease w_L^{SB} to $w_L^{\text{SB}'}$ such that the expected wage $\lambda_1 w_H^{\text{SB}'} + (1-\lambda_1) w_L^{\text{SB}'} = g$.



27 Had the principal decided to let the agent exert no effort ($e = 0$), the principal's problem is

$$\begin{aligned}
\underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_0(\pi_H - w_H) + (1 - \lambda_0)(\pi_L - w_L) \\
\text{subject to} \quad & \lambda_0 w_H + (1 - \lambda_0)w_L \geq \lambda_1 w_H + (1 - \lambda_1)w_L - g \\
& \lambda_0 w_H + (1 - \lambda_0)w_L \geq 0.
\end{aligned}$$

Thus, principal would make the following payment:

- $w_H^{\mathrm{SB}} = g + \frac{1-\lambda_1}{\lambda_1 - \lambda_0} g$ and $w_L^{\mathrm{SB}} = g - \frac{\lambda_1}{\lambda_1 - \lambda_0} g$, or
- zero payment to the agent whatever the realization of profit.

The expected profit is $\lambda_0 \pi_H + (1 - \lambda_0)\pi_L$.

28 The optimal outcome calls for $e^* = 1$ if and only if $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq g = u^{-1}(g)$.

Therefore, we have shown: Moral hazard is not an issue with a risk-neutral agent despite the nonobservability of effort. The first-best level of effort is still implemented.

29 The principal can costlessly structure the agent's payment so that the agent has the right incentives to exert effort. Indeed, by increasing effort from $e = 0$ to $e = 1$, the agent receives the wage w_H^* more often than the wage w_L^*. His expected gain from exerting effort is thus $(\lambda_1 - \lambda_0)(w_H^* - w_L^*) = g$, i.e., it exactly compensates the agent for the extra disutility of effort that he incurs when increasing his effort from $e = 0$ to $e = 1$.

30 Suppose that $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq g$. Then the optimal outcome is $e^* = 1$.

Let us consider a pair of wages

$$w_H^{\mathrm{SB}''} = \pi_H - T_1 \text{ and } w_L^{\mathrm{SB}''} = \pi_L - T_1,$$

where T_1 is an up-front payment made by the agent before output realizes.

These wages satisfy the agent's IC constraint since:

$$(\lambda_1 - \lambda_0)(w_H^{\mathrm{SB}''} - w_L^{\mathrm{SB}''}) = (\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq g.$$

The up-front payment T can be adjusted by the principal to have the agent's IR constraint be binding:

$$T_1 = \lambda_1 \pi_H + (1 - \lambda_1)\pi_L - g.$$

With the wages $w_H^{\mathrm{SB}''}$ and $w_L^{\mathrm{SB}''}$, the agent becomes residual claimant for the profit of the firm. The up-front payment T_1 is precisely equal to this expected profit. The principal chooses this *ex ante* payment to reap all gains from delegation.

31 Suppose that $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \leq g$. Then the optimal outcome is $e^* = 0$.

Let us consider a pair of wages

$$w_H^{\mathrm{SB}''} = \pi_H - T_0 \text{ and } w_L^{\mathrm{SB}''} = \pi_L - T_0,$$

where T_0 is an up-front payment made by the agent before output realizes.

These wages satisfy the agent's IC constraint since:

$$(\lambda_1 - \lambda_0)(w_H^{\mathrm{SB}''} - w_L^{\mathrm{SB}''}) = (\lambda_1 - \lambda_0)(\pi_H - \pi_L) \leq g.$$

The up-front payment T_0 can be adjusted by the principal to have the agent's IR constraint be binding:

$$T_0 = \lambda_0 \pi_H + (1 - \lambda_0)\pi_L.$$

With the wages $w_H^{\mathrm{SB}''}$ and $w_L^{\mathrm{SB}''}$, the agent becomes residual claimant for the profit of the firm. The up-front payment T_0 is precisely equal to this expected profit. The principal chooses this *ex ante* payment to reap all gains from delegation.

4 Incomplete information with risk-averse agent

32 Assume that the agent is risk-averse.

33 To induce the agent to exert effort, the principal's program is written as:

$$\begin{aligned}
\underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_1(\pi_H - w_H) + (1 - \lambda_1)(\pi_L - w_L) \\
\text{subject to} \quad & \lambda_1 u(w_H) + (1 - \lambda_1)u(w_L) - g \geq \lambda_0 u(w_H) + (1 - \lambda_0)u(w_L) \\
& \lambda_1 u(w_H) + (1 - \lambda_1)u(w_L) - g \geq 0.
\end{aligned}$$

34 Let $u_H = u(w_H)$ and $u_L = u(w_L)$. Then the principal's program can be written as:

$$\begin{aligned}
\underset{(u_H, u_L)}{\text{maximize}} \quad & \lambda_1(\pi_H - h(u_H)) + (1 - \lambda_1)(\pi_L - h(u_L)) \\
\text{subject to} \quad & \lambda_1 u_H + (1 - \lambda_1)u_L - g \geq \lambda_0 u_H + (1 - \lambda_0)u_L \\
& \lambda_1 u_H + (1 - \lambda_1)u_L - g \geq 0.
\end{aligned}$$

Note that the principal's objective function is now strictly concave in (u_H, u_L) because h is strictly convex. The constraints are now linear and the interior of the constrained set is obviously nonempty, and therefore it is a concave problem, with the Kuhn and Tucker conditions being sufficient and necessary for characterizing optimality.

35 Letting γ and μ be the non-negative multipliers associated respectively with the constraints, the first-order conditions of this program can be expressed as

$$-\lambda_1 h'(u_H^{\mathrm{SB}}) + \gamma(\lambda_1 - \lambda_0) + \mu\lambda_1 = -\frac{\lambda_1}{u'(w_H^{\mathrm{SB}})} + \gamma(\lambda_1 - \lambda_0) + \mu\lambda_1 = 0$$
$$-(1-\lambda_1) h'(u_L^{\mathrm{SB}}) - \gamma(\lambda_1 - \lambda_0) + \mu(1-\lambda_1) = -\frac{1-\lambda_1}{u'(w_L^{\mathrm{SB}})} - \gamma(\lambda_1 - \lambda_0) + \mu(1-\lambda_1) = 0,$$

where w_H^{SB} and w_L^{SB} are the second-best optimal wages.

36 Rearranging terms, we get

$$\frac{1}{u'(w_H^{\mathrm{SB}})} = \mu + \gamma\frac{\lambda_1 - \lambda_0}{\lambda_1} \text{ and } \frac{1}{u'(w_L^{\mathrm{SB}})} = \mu - \gamma\frac{\lambda_1 - \lambda_0}{1-\lambda_1}.$$

Multiplying the left equation by λ_1 and the right equation by $1-\lambda_1$, and then adding those two modified equations, we obtain

$$\mu = \frac{\lambda_1}{u'(w_H^{\mathrm{SB}})} + \frac{1-\lambda_1}{u'(w_L^{\mathrm{SB}})} > 0.$$

Hence, the IR condition is binding.

37 The IC condition implies

$$u_H^{\mathrm{SB}} - u_L^{\mathrm{SB}} \geq \frac{g}{\lambda_1 - \lambda_0} > 0,$$

and thus $w_H^{\mathrm{SB}} > w_L^{\mathrm{SB}}$.

Therefore,

$$\gamma = \frac{\lambda_1(1-\lambda_1)}{\lambda_1 - \lambda_0}\left(\frac{1}{u'(w_H^{\mathrm{SB}})} - \frac{1}{u'(w_L^{\mathrm{SB}})}\right) > 0,$$

and hence the IC condition is also binding.

38 Since the IR and IC conditions are binding, we have

$$u_H^{\mathrm{SB}} = g + \frac{1-\lambda_1}{\lambda_1 - \lambda_0}g \text{ and } u_L^{\mathrm{SB}} = g - \frac{\lambda_1}{\lambda_1 - \lambda_0}g,$$

and hence

$$w_H^{\mathrm{SB}} = h\left(g + \frac{1-\lambda_1}{\lambda_1 - \lambda_0}g\right) \text{ and } w_L^{\mathrm{SB}} = h\left(g - \frac{\lambda_1}{\lambda_1 - \lambda_0}g\right).$$

39 The agent receives more than the complete information wage when a high output is realized, $w_H^{\mathrm{SB}} > h(g)$. When a low output is realized, the agent instead receives less than the complete information wage, $w_L^{\mathrm{SB}} < h(g)$.

A risk premium must be paid to the risk-averse agent to induce his participation since he now incurs a risk by the fact that $w_L^{\mathrm{SB}} < w_H^{\mathrm{SB}}$. Indeed, we have

$$g = \lambda_1 u(w_H^{\mathrm{SB}}) + (1-\lambda_1)u(w_L^{\mathrm{SB}}) < u\left(\lambda_1 w_H^{\mathrm{SB}} + (1-\lambda_1) w_L^{\mathrm{SB}}\right),$$

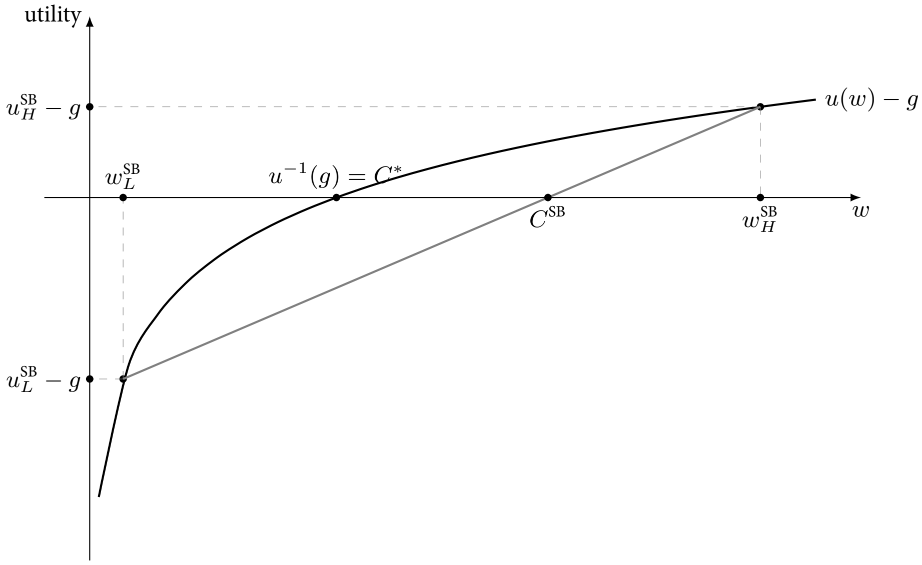
where the inequality follows from Jensen's inequality. That is, the expected payment $\lambda_1 w_H^{\mathrm{SB}} + (1-\lambda_1) w_L^{\mathrm{SB}}$ given by the principal is thus larger than the first-best cost $h(g)$, which is incurred by the principal when effort is observable.

40 The second-best cost of inducing effort under moral hazard is the expected payment made to the agent

$$C^{\text{SB}} = \lambda_1 w_H^{\text{SB}} + (1-\lambda_1) w_L^{\text{SB}} = \lambda_1 h\left(g + \frac{1-\lambda_1}{\lambda_1 - \lambda_0} g\right) + (1-\lambda_1) h\left(g - \frac{\lambda_1}{\lambda_1 - \lambda_0} g\right) > h(g) = C^*,$$

where the inequality follows from Jensen's inequality (h is strictly convex).

41 Graphic illustration:



(1) $u(w) - g$ is the agent's utility function when he exerts effort.

(2) In the complete information case, the agent's utility is zero, and the wage is always $u^{-1}(g)$.

(3) Since IR binds, the contract $(w_H^{\text{SB}}, w_L^{\text{SB}})$ makes the agent's expected utility be zero, shown as in the graph. That is, $\lambda_1 u(w_H^{\text{SB}}) + (1-\lambda_1) u(w_L^{\text{SB}}) - g = 0$.

(4) The expected wage should be $\lambda_1 w_H^{\text{SB}} + (1-\lambda_1) w_L^{\text{SB}} = C^{\text{SB}}$.

(5) Since u is concave, $C^{\text{SB}} > C^*$.

(6) To induce the agent to exert effort, the principal needs to set w_H and w_L to satisfy $(\lambda_1 - \lambda_0)\big(u(w_H) - u(w_L)\big) \geq g$. That is, $w_H - w_L$ should be sufficiently large.

(7) IC should be binding; otherwise, the principal can decrease w_H and increase w_L, so that the expected wage $\lambda_1 w_H + (1-\lambda_1) w_L$ decreases.

42 Had the principal decided to let the agent exert no effort, $e = 0$, he would (optimally) make a zero payment to the agent whatever the realization of profit. The profit is $\lambda_0 \pi_H + (1-\lambda_0)\pi_L$.

43 The benefit of inducing effort is still $(\lambda_1 - \lambda_0)(\pi_H - \pi_L)$, and a positive effort $e^* = 1$ is the optimal choice of the principal whenever

$$(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq C^{\text{SB}} > C^*.$$

44 Summary (given $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq C^{\text{SB}} > C^*$):

- The agent's utility is always zero.
- The principal sets $w_H^{\text{SB}} > w_L^{\text{SB}}$ to induce the agent to exert effort.
- Efficiency loses since $C^{\text{SB}} > C^*$, which is paid by the principal.

5 A continuum of profits

45 We assume that profit π is drawn from a distribution $F(\cdot \mid e)$ on the support $[\underline{\pi}, \bar{\pi}]$.

This distribution is conditional on the agent's effort $e \in \{0,1\}$. We denote by $f(\cdot \mid e)$ the density corresponding to the distribution $F(\cdot \mid e)$.

46 Complete information:

To induce $e = 1$, the principal's problem is

$$\begin{aligned} \underset{w(\pi)}{\text{maximize}} \quad & \int [\pi - w(\pi)] f(\pi \mid 1)\,\mathrm{d}\pi \\ \text{subject to} \quad & \int u(w(\pi)) f(\pi \mid 1)\,\mathrm{d}\pi - g \geq 0 \end{aligned}$$

Denoting the multipliers by γ. Optimizing pointwise with respect to w yields

$$-f(\pi \mid 1) + \gamma u'(w(\pi)) f(\pi \mid 1) = 0.$$

Thus, $\gamma = \frac{1}{u'(w(\pi))} > 0$ and the wage is constant. It implies that $w^* = u^{-1}(g)$, which is the same as the two-profit case. The profit is

$$\int \pi f(\pi \mid 1)\,\mathrm{d}\pi - u^{-1}(g).$$

Had the principal decided to let the agent exert no effort, $e = 0$, he would (optimally) make a zero payment to the agent whatever the realization of profit. The payoff is $\int \pi f(\pi \mid 0)\,\mathrm{d}\pi$.

$e^* = 1$ is the optimal choice of principal if and only if

$$\int \pi f(\pi \mid 1)\,\mathrm{d}\pi - u^{-1}(g) \geq \int \pi f(\pi \mid 0)\,\mathrm{d}\pi.$$

47 In an environment with incomplete information, a contract $w(\pi)$ inducing a positive effort must satisfy the IC constraint

$$\int u(w(\pi)) f(\pi \mid 1)\,\mathrm{d}\pi - g \geq \int u(w(\pi)) f(\pi \mid 0)\,\mathrm{d}\pi,$$

and the IR constraint

$$\int u(w(\pi)) f(\pi \mid 1)\,\mathrm{d}\pi - g \geq 0.$$

48 Incomplete information with a risk-neutral agent.

(1) To induce $e = 1$, the principal's problem is

$$\begin{aligned} \underset{w(\pi)}{\text{maximize}} \quad & \int [\pi - w(\pi)] f(\pi \mid 1)\,\mathrm{d}\pi \\ \text{subject to} \quad & \int w(\pi) f(\pi \mid 1)\,\mathrm{d}\pi - g \geq 0 \\ & \int w(\pi) f(\pi \mid 1)\,\mathrm{d}\pi - g \geq \int w(\pi) f(\pi \mid 0)\,\mathrm{d}\pi \end{aligned}$$

Principal can set $w(\pi) = \pi - \int \pi f(\pi \mid 1)\,\mathrm{d}\pi + g$. The expected payoff is $\int \pi f(\pi \mid 1)\,\mathrm{d}\pi - g$.

(2) To induce $e = 0$, the principal's problem is

$$\begin{aligned} \underset{w(\pi)}{\text{maximize}} \quad & \int [\pi - w(\pi)] f(\pi \mid 0) \, \mathrm{d}\pi \\ \text{subject to} \quad & \int w(\pi) f(\pi \mid 0) \, \mathrm{d}\pi \geq 0 \\ & \int w(\pi) f(\pi \mid 0) \, \mathrm{d}\pi \geq \int w(\pi) f(\pi \mid 1) \, \mathrm{d}\pi - g \end{aligned}$$

Principal can set $w(\pi) = 0$ or $w(\pi) = \pi - \int \pi f(\pi \mid 0) \, \mathrm{d}\pi$. The expected payoff is $\int \pi f(\pi \mid 0) \, \mathrm{d}\pi$.

(3) $e = 1$ is the optimal of principal if and only if

$$\int \pi f(\pi \mid 1) \, \mathrm{d}\pi - g \geq \int \pi f(\pi \mid 0) \, \mathrm{d}\pi.$$

49 Incomplete information with a risk-averse agent.

(1) To induce $e = 1$, the principal's problem is

$$\begin{aligned} \underset{w(\pi)}{\text{maximize}} \quad & \int [\pi - w(\pi)] f(\pi \mid 1) \, \mathrm{d}\pi \\ \text{subject to} \quad & \int u(w(\pi)) f(\pi \mid 1) \, \mathrm{d}\pi - g \geq 0 \\ & \int u(w(\pi)) f(\pi \mid 1) \, \mathrm{d}\pi - g \geq \int u(w(\pi)) f(\pi \mid 0) \, \mathrm{d}\pi \end{aligned}$$

Denoting the multipliers by γ and μ, respectively, the Lagrangian writes as

$$[\pi - w(\pi)] f(\pi \mid 1) + \gamma [u(w)[f(\pi \mid 1) - f(\pi \mid 0)] - g] + \mu [u(w) f(\pi \mid 1) - g].$$

Optimizing pointwise with respect to w yields

$$\frac{1}{u'(w^{\mathrm{SB}}(\pi))} = \mu + \gamma \left[1 - \frac{f(\pi \mid 1)}{f(\pi \mid 0)} \right].$$

We can verify that $\gamma > 0$ and $\mu > 0$. Then

$$u \left(\int w^{\mathrm{SB}}(\pi) f(\pi \mid 1) \, \mathrm{d}\pi \right) > \int u(w^{\mathrm{SB}}(\pi)) f(\pi \mid 1) \, \mathrm{d}\pi = g.$$

That is, the expected wage $C^{\mathrm{SB}} = \int w^{\mathrm{SB}}(\pi) f(\pi \mid 1) \, \mathrm{d}\pi$ is larger than $u^{-1}(g) = C^*$.

(2) To induce $e = 0$, the principal's problem is

$$\begin{aligned} \underset{w(\pi)}{\text{maximize}} \quad & \int [\pi - w(\pi)] f(\pi \mid 0) \, \mathrm{d}\pi \\ \text{subject to} \quad & \int u(w(\pi)) f(\pi \mid 0) \, \mathrm{d}\pi \geq 0 \\ & \int u(w(\pi)) f(\pi \mid 0) \, \mathrm{d}\pi \geq \int u(w(\pi)) f(\pi \mid 1) \, \mathrm{d}\pi - g \end{aligned}$$

Principal can set $w(\pi) = 0$. The expected payoff is $\int \pi f(\pi \mid 0) \, \mathrm{d}\pi$.

(3) $e = 1$ is optimal if and only if

$$\int \pi f(\pi \mid 1)\, \mathrm{d}\pi - C^{\mathrm{SB}} \geq \int \pi f(\pi \mid 0)\, \mathrm{d}\pi.$$

6 Homework

- Key: Optimal contract.
- Reading: 14.B
- Homework: 14.B.4