Advanced Microeconomics I: Lecture 10

1 Externalities

1 An externality is present whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy.

In particular, an externality is a cost or a benefit imposed upon someone by actions taken by others.

2 When we say "directly," we mean to exclude any effects that are mediated by prices.

3 An externally imposed benefit is a positive externality:

- a well-maintained garden next door,
- the pleasant scent of the perfume wore by a lady seated next to you,
- etc.

4 An externally imposed cost is a negative externality:

- gas emission,
- water pollution,
- second-hand cigarette smoke,
- etc.

5 Goal: We explore the implications of external effects for competitive equilibria and public policy.

6 There are two consumers (1 and 2) and L goods. Consumer i's initial wealth is w_i.

We suppose that the actions of these consumers do not affect the prices $p \in \mathbb{R}^L$.

7 We assume that each consumer has preference not only over her consumption $(x_{1i}, x_{2i}, \ldots, x_{Li})$ but also over some action $h \in \mathbb{R}_+$ taken by consumer 1.

Consumer i's utility function takes the form $u_i(x_{1i}, x_{2i}, \ldots, x_{Li}, h)$. We assume that $\frac{\partial u_2}{\partial h} \neq 0$. Consumer 1's choice of h affects consumer 2's well-being, then it generates an externality.

8 It is convenient to define a derived utility function:

$$\begin{aligned} v_i(p, w_i, h) = \underset{x_i}{\text{maximize}} \quad & u_i(x_i, h) \\ \text{subject to} \quad & p \cdot x_i \leq w_i. \end{aligned}$$

9 We also assume that consumers' utility functions take a quasilinear form. Thus, v_i has the form of $\phi_i(p, h) + w_i$.

Since prices are assumed to be unaffected, $\phi_i(p, h)$ can be simply rewritten as $\phi_i(h)$.

We assume that ϕ_i is twice differentiable with $\phi_i'' < 0$.

10 Suppose that there is a competitive equilibrium with prices p. Therefore, consumer 1 will choose h to maximize $\phi_1(h)$.

Since $\phi_1'' < 0$, the equilibrium level h^* satisfies the necessary and sufficient first-order condition

$$\phi_1'(h^*) \leq 0 \text{ with equality if } h^* > 0.$$

11 In any Pareto optimal allocation, the optimal level h^o maximizes the joint surplus of the two consumers, and os solves:

$$\max_{h \geq 0} \phi_1(h) + \phi_2(h).$$

h^o satisfies the necessary and sufficient first-order condition

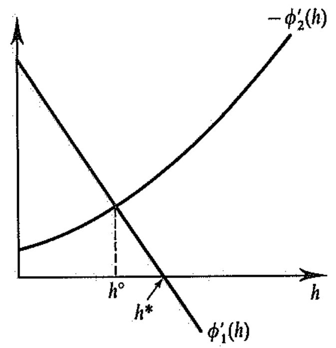
$$\phi_1'(h^o) \leq -\phi_2'(h^o) \text{ with equality if } h^o > 0.$$

12 When external effects are present, so that $\phi_2'(h) \neq 0$ at all h, the equilibrium level is not optimal unless $h^o = h^* = 0$.

13 Suppose we have interior solutions.

Case 1: If $\phi_2' < 0$, so h generates a negative externality. The we have $\phi_1'(h^o) = -\phi_2'(h^o) > 0$. Since ϕ_1' is decreasing and $\phi_1'(h^*) = 0$, we have $h^* > h^o$.

Case 2: If $\phi_2' > 0$, so h generates a positive externality. The we have $\phi_1'(h^o) = -\phi_2'(h^o) < 0$. Since ϕ_1' is decreasing and $\phi_1'(h^*) = 0$, we have $h^* < h^o$.



14 Quotas.

Suppose that h generates a negative externality, so $h^o < h^*$. The most direct sort of government intervention to achieve efficiency is the direct control of the externality-generating activity itself.

The government can simply mandate that h be no larger than h^o. With this constraint, consumer 1 will indeed fix the level at h^o.

15 Pigouvian taxation.

Suppose that consumer 1 is made to pay a tax of t_h per unit of h.

Then consumer 1 will choose the level that solves

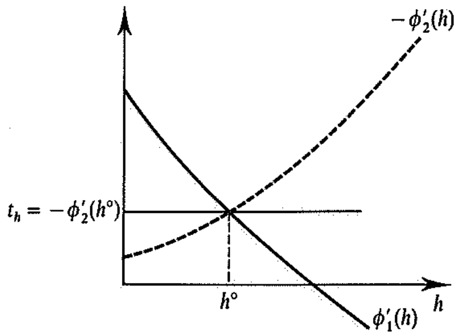
$$\max_h \phi_1(h) - t_h h.$$

The necessary and sufficient first-order condition is

$$\phi_1'(h) \le t_h \text{ with equality if } h > 0.$$

Since $t_h = -\phi_2'(h^o)$, h^o satisfies the above condition.

When faced with this tax, consumer 1 carries out an individual cost-benefit computation that internalizes the externality that she imposes on consumer 2.



16 Property rights.

We establish enforceable property rights with regard to the externality-generating activity.

For example, we assign the right to an externality-free environment to consumer 2. In this case, consumer 1 is unable to engage in the externality-producing activity without consumer 2's permission.

We assume that consumer 2 makes consumer 1 a take-it-or-leave-it offer, demanding a payment T in return for permission to generate externality level h.

Consumer 1 will agree to this demand T if and only if she will be at least as well off as she would be by rejecting it, i.e., $\phi_1(h) - T \ge \phi_1(0)$.

Hence consumer 2 will choose her offer (h, T) to solve

$$\begin{aligned} \underset{(h,T)}{\text{maximize}} \quad & \phi_2(h) + T \\ \text{subject to} \quad & \phi_1(h) - T \ge \phi(0). \end{aligned}$$

Since the constraint is binding, $T = \phi_1(h) - \phi_1(0)$. Therefore, consumer 2's optimal offer involves h that solves

$$\max_h \phi_2(h) + \phi_1(h) - \phi_1(0).$$

Thus, the solution is precisely h^0.

Coase Theorem: If trade of the externality can occur, then bargaining will lead to an efficient outcome no matter how property rights are allocated.

2 Public goods

17 A public good is a commodity for which use of a unit of the good by one agent does not preclude its use by other agents.

18 Example:

- Knowledge,
- Broadcast radio and TV programs,
- Public highways,
- Clean air,
- National parks,
- etc.

19 There are I consumers, 1 public good, and L usual goods.

20 We assume that each consumer's utility function is quasilinear. Therefore we can define, for each consumer, a derived utility function over the level of the public good.

Let x denote the quantity of the public good, we denote consumer i's utility from the public good by $\phi_i(x)$. We also assume that it is twice differentiable and $\phi_i''(x) < 0$ for all $x \geq 0$.

21 The cost of supplying q units of the public good is $c(q)$. We assume that $c(\cdot)$ is twice differentiable with $c''(q) > 0$ at all $q \geq 0$.

- The production of a desirable public good is costly: $\phi_i'(\cdot) > 0$ for all i and $c'(\cdot) > 0$.
- The reduction of a public bad is costly: $\phi_i'(\cdot) < 0$ for all i and $c'(\cdot) < 0$.

22 Any Pareto optimal allocation maximize aggregate surplus, and involve a level of the public good that solves:

$$\max_q \sum_{i=1}^{I} \phi_i(q) - c(q).$$

The necessary and sufficient first-order condition for q^o is

$$\sum_{i=1}^{I} \phi_i'(q^o) \leq c'(q^o) \text{ with equality if } q^o > 0.$$

23 Consider the case in which the public good is provided by means of private purchases by consumers. There is a market for the public good and each consumer i chooses how much of the public good to buy (x_i).

The supply side is a single profit-maximizing firm with cost function $c(\cdot)$.

At a competitive equilibrium with price p^*, each consumer i's purchase x_i^* must maximize her utility and solve

$$\max_{x_i} \phi_i(x_i + \sum_{k \neq i} x_k^*) - p^* x_i.$$

Thus, x_i^* satisfy the necessary and sufficient condition

$$\phi_i'(x^*) \leq p^* \text{ with the equality if } x_i^* > 0,$$

where $x^* = \sum x_i^*$.

The firm's supply q^* solve

$$\max_q p^* q - c(q),$$

and satisfy the necessary and sufficient condition

$$p^* \leq c'(q^*) \text{ with equality if } q^* > 0.$$

At a competitive equilibrium, we have $q^* = x^*$. Then we have

$$\sum_i I_{x_i^*>0}[\phi_i'(q^*) - c'(q^*)] = 0.$$

Whenever $I > 1$ and $q^* > 0$ (so that $x_i^* > 0$ for some i), we have

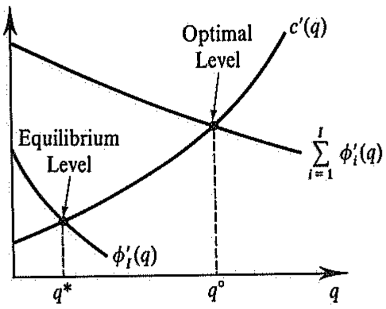
$$\sum_i \phi_i'(q^*) > \phi_i'(q^*) = c'(q^*).$$

Thus, $q^* < q^o$.

24 Inefficiency: each consumer's purchase provides a direct benefit not only to the consumer herself but also to every other consumer. Hence, private provision creates a situation in which externalities are present.

The failure of each consumer to consider the benefits for others of her public good provision is referred to as the free-rider problem: Each consumer has an incentive to enjoy the benefits of the public good provided by others while providing it insufficiently herself.

25 Graph



26 Quantity-based intervention (direct governmental provision) or price-based intervention in the form of taxes or subsidies.

27 Lindahl equilibria: Suppose that each consumer's consumption of the public good is a distinct commodity with its own market. We denote the price of this personalized good by p_i.

Given the equilibrium price p_i^{**}, each consumer i sees herself as deciding the total amount of the public good she will consume, x_i, so as to solve

$$\max_{x_i} \phi_i(x_i) - p_i^{**} x_i.$$

Her equilibrium consumption level x_i^{**} satisfies the necessary and sufficient condition

$$\phi_i'(x_i^{**}) \leq p_i^{**} \text{ with equality } x_i^{**} > 0.$$

The firm is viewed as producing a bundle of I goods with a fixed-proportions technology (i.e., the level of production of each personalized good is necessarily the same). Thus, the firm sloves

$$\max_q \sum_i p_i^{**} q - c(q).$$

The firm's equilibrium level of q^{**} satisfies the necessary and sufficient condition

$$\sum_i p_i^{**} \leq c'(q^{**}) \text{ with equality if } q^{**} > 0.$$

The market-clearing condition implies that $x_i^{**} = q^{**}$ for all i. And hence,

$$\sum_i \phi_i'(q^{**}) \leq c'(q^{**}) \text{ with equality if } q^{**} > 0.$$

Thus, $q^{**} = q^o$.

28 Problem of Lindahl equilibria: it is impossible to exclude a consumer from use of the public good from others' purchase.

3 Homework

- Reading: Chapter 11.2–11.3 in MWG.
- Homework: 11.B.3, 11.C.1