Advanced Microeconomics: Lecture 3

1 Screening: Uninformed parties take step to distinguish/screen the types of informed parties.

- In competitive screening, there are several competing firms.
- In monopolistic screening, there is a single firm screening workers.

1 Competitive screening

2 Literature: Rothschild and Stiglitz (1976) and Wilson (1977).

3 There are two firms.

4 There are two types of workers, θ_H and θ_L, with $\theta_H > \theta_L > 0$ and the fraction of type-θ_H workers is $\lambda \in (0, 1)$.

Workers earn nothing if working at home, i.e., $r(\theta_H) = r(\theta_L) = 0$.

5 Jobs may differ in the "task level" required of the worker.

We assume that the task levels do not affect the output; rather, their only effect is to lower the utility of the worker.

6 The utility of a type-θ worker who faces task level $t \geq 0$ and receives wage w is $w - c(t, \theta)$.

We assume $c(t, \theta)$ is twice continuously differentiable and $c(0, \theta) = 0$, $c_t(t, \theta) > 0$, $c_{tt}(t, \theta) > 0$, $c_\theta(t, \theta) < 0$ for all $t > 0$, and $c_{t\theta}(t, \theta) < 0$.

7 Game:

- Two firms simultaneously announce (finite) sets of contracts. A contract is a pair (w, t).
- Given the offers made by the firms and their types, workers choose whether to accept a contract and, if so, which one.

1.1 Complete information

8 When types are observable, we allow firms to condition their offer on a worker's type, i.e., a firm can offer a contract (w_L, t_L) solely to type-θ_L workers and another contract (w_H, t_H) solely to type-θ_H workers.

9 Proposition: In any SPE of the screening game with observable types, a type-θ_i worker accepts contract $(w_i^*, t_i^*) = (\theta_i, 0)$, and firms earn zero profits.

10 *Proof.* Step 1: Any contract (w_i^*, t_i^*) accepted by type-θ_i workers in SPE will produce zero profits, and $w_i^* = \theta_i$.

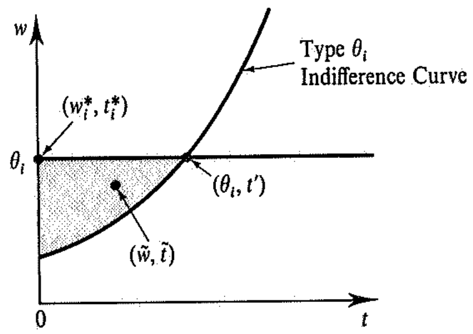
- If $w_i^* > \theta_i$, then the firm who offers (w_i^*, t_i^*) is making a loss and can do better by not offering any contract to type-θ_i workers.

- Assume that $w_i^* < \theta_i$.
  (1) Let $\Pi > 0$ be the aggregate profits earned by two firms on type-θ_i workers.
  (2) There is one firm earning no more than $\frac{\Pi}{2}$, say firm j.
  (3) Firm j can deviate by offering a contract $(w_i^* + \varepsilon, t_i^*)$ for sufficiently small $\varepsilon > 0$.
  (4) Then all type-θ_i workers will accept this contract.
  (5) Thus, the profit of firm j is close to Π. That is, the deviation increases its profit.
- Therefore, $w_i^* = \theta_i$.

Step 2: The SPE task level of type-θ_i workers is 0.

(1) Suppose that $(w_i^*, t_i^*) = (\theta_i, t')$ for some $t' > 0$.

(2) Then either firm could deviate to offer contract $(\tilde{w}, \tilde{t})$ (for type-θ_i-workers):



- Firm: the wage $\tilde{w}$ is lower than $w_i^* = \theta_i$.
- Type-θ_i worker: the utility $\tilde{w} - c(\tilde{t}, \theta_i)$ is larger than $\theta_i - c(t', \theta_i)$.

Contradiction.

(3) The only contract at which there are no profitable deviations is $(\theta_i, 0)$.

□

1.2 Incomplete information

11 The workers' types are not observable. So each contract can be accepted by workers of either type.

12 The outcome in the complete information case $(\theta_H, 0)$ and $(\theta_L, 0)$ cannot arise when types are unobservable: the type-θ_L worker prefers the high-ability contract $(\theta_H, 0)$ to contract $(\theta_L, 0)$.

13 Lemma: In any (separating or pooling) SPE, both firms earn zero profits.

Proof. (1) Let (w_L, t_L) and (w_H, t_H) are the contracts (could be the same) signed by low- and high-ability workers in a SPE, and suppose that the two firms' aggregate profits are $\Pi > 0$.

(2) Then $[w_L - c(t_L, \theta_L)] - [w_H - c(t_H, \theta_L)] \geq 0$ and $[w_H - c(t_H, \theta_H)] - [w_L - c(t_L, \theta_H)] \geq 0$.

(3) The one firm must make no more then $\frac{\Pi}{2}$.

(4) This firm will deviate to offer contracts $(w_L + \varepsilon, t_L)$ and $(w_H + \varepsilon, t_H)$ for sufficiently small $\varepsilon > 0$.

(5) Contract $(w_L+\varepsilon, t_L)$ will attract all type-θ_L workers, and contract $(w_H+\varepsilon, t_H)$ will attract all type-θ_H workers:

- Type-θ_L workers: $[w_L+\varepsilon-c(t_L,\theta_L)]-[w_H+\varepsilon-c(t_H,\theta_L)] = [w_L-c(t_L,\theta_L)]-[w_H-c(t_H,\theta_L)] \geq 0$.
- Type-θ_H workers: $[w_H+\varepsilon-c(t_H,\theta_H)]-[w_L+\varepsilon-c(t_L,\theta_H)] = [w_H-c(t_H,\theta_H)]-[w_L-c(t_L,\theta_H)] \geq 0$.

(6) Such a deviation will make this firm have profit close to Π. It is profitable. Contradiction.

(7) Thus, $\Pi \leq 0$, and hence $\Pi = 0$.

□

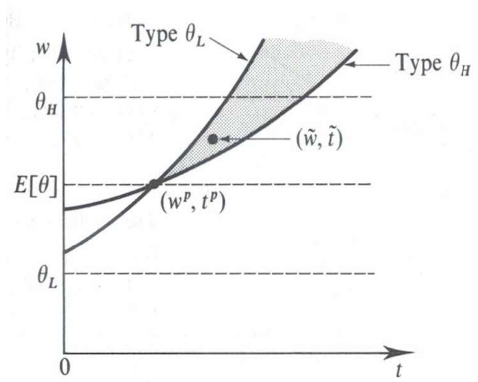
14 Lemma: No pooling SPE exists.

Proof. (1) Suppose that there is a pooling SPE contract (w^p, t^p); firm j offers this contract, and both type-θ_L and type-θ_H workers accept it.

(2) Thus, the expected productivity is $\mathbf{E}[\theta]$.

(3) Since the firms have zero profit in SPE, $w^p = \mathbf{E}[\theta]$.

(4) Firm k can deviate to offer a single contract $(\tilde{w}, \tilde{t})$.



(5) This contract will attract all the type-θ_H workers and none of the type-θ_L workers (they prefer contract (w^p, t^p)).

(6) Since $\tilde{w} < \theta_H$, firm k makes strictly positive profit $\theta_H - \tilde{w}$.

(7) Contradiction.

□

15 Lemma: If (w_L, t_L) and (w_H, t_H) are the contracts signed by low- and high-ability workers in a separating SPE, then both contracts yield zero profits, i.e., $w_L = \theta_L$ and $w_H = \theta_H$.

Proof. Step 1: $w_L \geq \theta_L$.

(1) Suppose that $w_L < \theta_L$ and firm j offers contract (w_L, t_L).

(2) Then firm k can deviate by only offering contract $(\tilde{w}_L, t_L)$, where $\theta_L > \tilde{w}_L > w_L$.

(3) The deviating firm will earn strictly positive profit.

- All low-ability workers will accept this contract ⇒ positive profit.

- If high-ability workers do not accept this contract $\Rightarrow$ zero profit.
- If high-ability workers accept this contract $\Rightarrow$ positive profit.

(4) Contradiction. Thus, $w_L \geq \theta_L$.

Step 2: $w_H \geq \theta_H$.

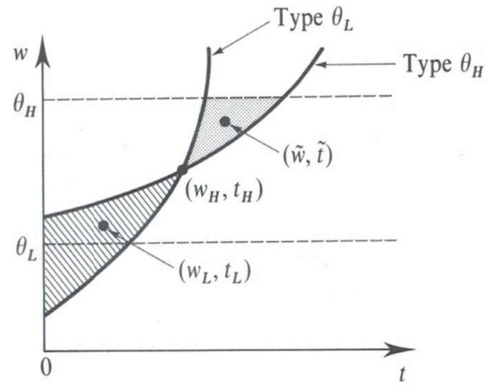
(1) Suppose that $w_H < \theta_H$.

(2) Then the low-ability contract (w_L, t_L) must lie in the hatched region:

- High-ability workers will choose $(w_H, t_H) \Rightarrow (w_L, t_L)$ is below the θ_H-indifference curve through (w_H, t_H).
- Low-ability workers will choose $(w_L, t_L) \Rightarrow (w_L, t_L)$ is above the θ_L-indifference curve through (w_H, t_H).
- Since firms earn strictly positive profits on high-ability workers, $w_L > \theta_L$.

(3) Suppose that firm j is offering the low-ability contract (w_L, t_L).

(4) Then firm $k \neq j$ can deviate by only offering a contract $(\tilde{w}, \tilde{t})$ lying in the shaded region.



(5) This contract will be accepted by all the θ_H workers and none of θ_L workers. θ_L workers will accept the contract (w_L, t_L) offered by firm j.

(6) This deviation leads to a strictly positive profit for firm k, since $\tilde{w} < \theta_H$. Contradiction.

(7) Thus, $w_H \geq \theta_H$.

Step 3: each firm earns zero profit, so $w_L = \theta_L$ and $w_H = \theta_H$. □

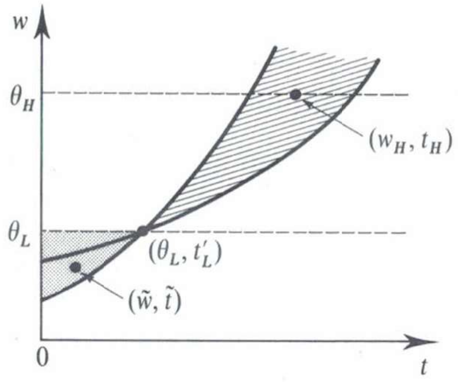
16 Lemma: In any separating SPE, the low-ability workers accept contract $(\theta_i, 0)$; that is, they receive the same contract as when no informational asymmetry is present.

Proof. (1) In any separating SPE, $w_L^* = \theta_L$.

(2) Suppose that the low-ability contract is (θ_L, t'_L) with $t'_L > 0$.

(3) Suppose that firm j is offering the high-ability contract (w_H, t_H), which lies on the segment of the line $w = \theta_H$ lying in the hatched region.

(4) Then firm k can deviate by only offering a contract $(\tilde{w}, \tilde{t})$ lying in the shaded region.



(5) This contract will be accepted by all the θ_L workers and none of θ_H workers. θ_H workers will accept the contract (w_H, t_H) offered by firm j.

(6) This deviation leads to a strictly positive profit for firm k, since $\tilde{w} < \theta_L$. Contradiction.

□

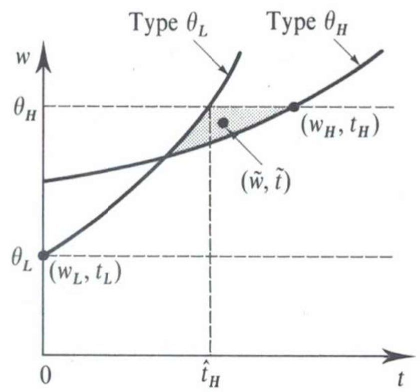
17 Lemma: In any separating SPE, the high-ability workers accept contract $(\theta_H, \hat{t}_H)$, where $\hat{t}_H$ satisfies $\theta_H - c(\hat{t}_H, \theta_L) = \theta_L - c(0, \theta_L)$.

Proof. (1) In any separating SPE, $(\theta_L, 0)$ is the contract for θ_L workers and (θ_H, t_H) is the contract for θ_H workers. In the following, we shall determine t_H.

(2) For θ_L workers, $t_H \geq \hat{t}_H$; otherwise, θ_L workers will choose the contract (θ_H, t_H).

(3) Suppose that $t_H > \hat{t}_H$.

(4) Then either firm can deviate by offering, in addition to its current contracts, a contract $(\tilde{w}, \tilde{t})$ lying in the shaded region.



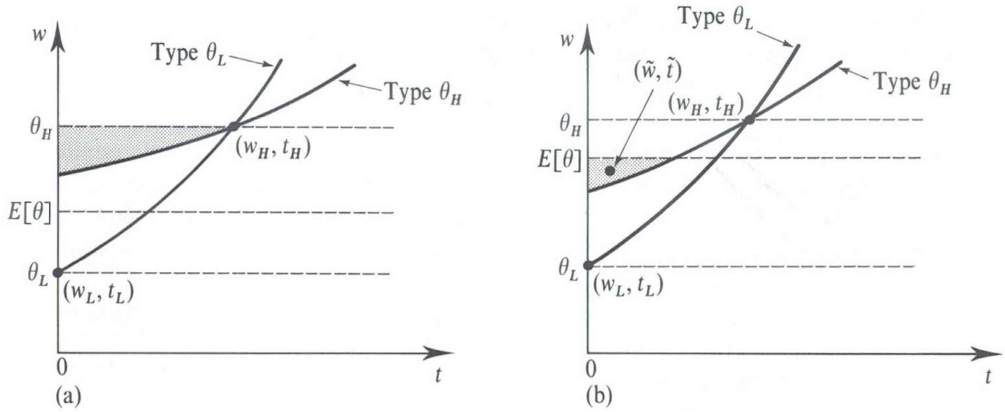
(5) This contract attracts all the θ_H workers and does not change the choice of θ_L workers.

(6) This deviation leads to a strictly positive profit, since $\tilde{w} < \theta_H$. Contradiction.

□

18 The existence of separating SPE: We just know what any equilibrium must look like, but we do not know whether one exists.

19 Example 1 on nonexistence.



(1) Assume both firms offer contracts $(\theta_i, 0)$ and (θ_H, t_H) as in Lemmas.

(2) Either firm can deviate to offer a single contract $(\tilde{w}, \tilde{t})$ (right figure).

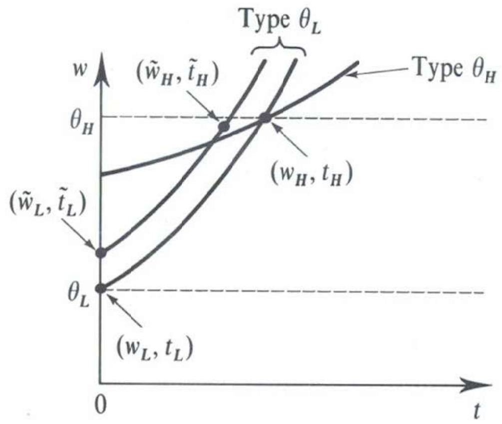
(3) This contract attracts all the workers.

(4) On the other hand, the deviating firm earns strictly positive profit: $\mathbf{E}[\theta] > \tilde{w}$.

Note that the single contract attracts all the workers if and only if the contract lies in the shaded region. If the line $w = \mathbf{E}[\theta]$ is below the shaded region, then the single contract does not give a strictly positive profit for the deviating firms. (left figure)

Note that no firm can earn strictly positive profits by deviating in a manner that attracts either only high-ability workers or only low-ability workers.

20 Example 2 on nonexistence.



(1) Assume both firms offer contracts $(\theta_i, 0)$ and (θ_H, t_H) as in Lemmas.

(2) Either firm can deviate to offer $(\tilde{w}_L, \tilde{t}_L)$ and $(\tilde{w}_H, \tilde{t}_H)$.

(3) θ_L workers will choose $(\tilde{w}_L, \tilde{t}_L)$ and θ_H workers will choose $(\tilde{w}_H, \tilde{t}_H)$.

(4) If the profit is strictly positive, then this deviation breaks the separating contracts $(\theta_i, 0)$ and (θ_H, t_H).

21 Welfare: We focus on the case when a SPE exists.

- Asymmetric information leads to Pareto inefficient outcomes: high-ability workers end up signing contracts that make them engage in useless tasks merely to distinguish themselves from low-ability workers.

- The low-ability workers are worse off when screening is possible than when it is not.
- Since a SPE exists, the high-ability workers are better off when screening is possible.
- The SPE outcome is constrained Pareto optimal.

2 Monopolistic screening

22 An owner (principal) wishes to hire a manager (agent) to run a one-time project.

If the agent's effort level is $e \in [0, \infty)$, then principal's income is $\pi(e)$, with $\pi(0) = 0$, $\pi'(e) > 0$, and $\pi''(e) < 0$ for all e.

If the principal pays wage w to the agent, the agent's utility/profit is $\pi(e) - w$.

23 The agent is an expected utility maximizer with utility $v\big(w - g(e, \theta)\big)$.

- $\theta \in \{\theta_L, \theta_H\}$ represents agent's ability. Here, $\theta_H > \theta_L$ and $\text{Prob}(\theta_H) = \lambda \in (0, 1)$.
- $g(e, \theta)$ measures the disutility of effort.
- $g(0, \theta) = 0$, $g_e(e, \theta) \begin{cases} > 0, & \text{if } e > 0 \\ = 0, & \text{if } e = 0 \end{cases}$, $g_{ee} > 0$, $g_\theta < 0$, $g_{e\theta}(e, \theta) \begin{cases} < 0, & \text{if } e > 0 \\ = 0, & \text{if } e = 0 \end{cases}$.

$\Rightarrow$ The agent's indifference curves have single-crossing property.

- The agent is risk averse: $v' > 0$ and $v'' < 0$.[1]
- The agent has a reservation utility $\bar{u}$.

24 The economic variables are effort level e and the wage w. These variables are both observable and verifiable by a third party such as a benevolent court of law.

A contract is a pair (e, w). Let $\mathcal{A}$ be the set of all feasible contracts, that is, $\mathcal{A} = \{(e, w) \mid e \in \mathbb{R}_+, w \in \mathbb{R}\}$.

25 The sequence of play is as follows:

Figure 1

2.1 Complete information

26 First suppose that there is no asymmetry of information between the principal and the agent, i.e., θ is observable.

27 The principal will try to maximize her utility subject to inducing the agent to accept the proposed contract. Clearly, the agent obtains $\bar{u}$ if he does not take the principal's contract. So the principal will solve the following problem:

$$\begin{aligned} \underset{(e_i, w_i) \in \mathcal{A}}{\text{maximize}} \quad & \pi(e_i) - w_i \\ \text{subject to} \quad & v\big(w_i - g(e_i, \theta_i)\big) \geq \bar{u}. \end{aligned}$$

[1] Question: How about when the manager is risk neutral?

28 In any solution, the IR constraint must bind; otherwise, the principal could lower the wage offered and still have the agent accept the contract. Thus, the maximization problem becomes:

$$\max_{(e_i,w_i)\in\mathcal{A}} \pi(e_i) - v^{-1}(\bar{u}) - g(e_i,\theta_i).$$

Then the solution (e_i^*, w_i^*) must satisfy the first-order condition:

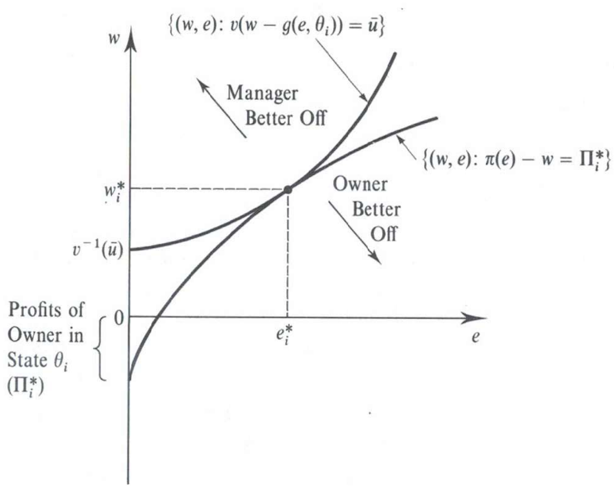
$$\pi'(e_i^*)\begin{cases}\leq g_e(e_i^*,\theta_i), & \\ = g_e(e_i^*,\theta_i), & \text{if } e_i^* > 0.\end{cases}$$

Since $\pi'(0) > 0$ and $g_e(0,\theta_i) = 0$, $e_i^* > 0$. Thus,

$$\pi'(e_i^*) = g_e(e_i^*,\theta_i).$$

Interpretation: The optimal level of effort e_i^* (for θ_i agent) equals the principal's marginal value and the agent's marginal cost.

29 Graphic illustration



- Agent's reservation utility is $\bar{u}$, which is equivalent to the contract $\big(0, v^{-1}(\bar{u})\big)$.
- Principal seeks to find the most profitable point on the indifference curve with utility $\bar{u}$, i.e., through the point $\big(0, v^{-1}(\bar{u})\big)$.
- For a θ_i agent, principal pays the wage w_i^* such that $w_i^* - g(e_i^*,\theta_i) = v^{-1}(\bar{u})$.
- For a θ_i agent, principal's profit is $\Pi_i^* = \pi(e_i^*) - v^{-1}(\bar{u}) - g(e_i^*,\theta_i)$.

  This profit is exactly equal to the distance from the origin to the intersection point between the indifference curve through (e_i^*, w_i^*) and the vertical axis: letting $e = 0$ in the indifference curve $\pi(e) - w = \Pi_i^*$, we have $-w = \Pi_i^*$.
- If $\bar{u}$ is small (especially, $\bar{u} = 0$), then this profit could be strictly positive. If $\bar{u}$ is very large, this profit could be negative; in this case, the principal will not provide such a contract.

  Interpretation: If agent's reservation utility is low, principal can attract him to accept some contract; otherwise, agent will not accept any contract that is acceptable for principal.

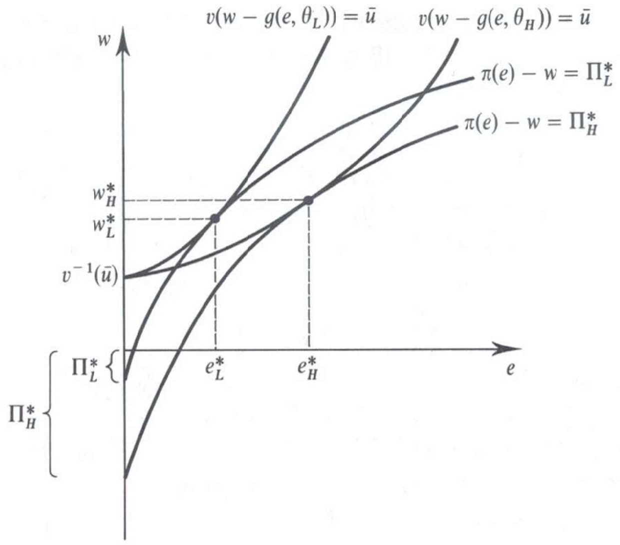
30 Since $\theta_H > \theta_L$, $\pi'' < 0$, $g_{e\theta} < 0$, $g_{ee} > 0$, $\pi'(e_i^*) = g_e(e_i^*, \theta_i)$ for $i \in \{H, L\}$, we have $e_H^* > e_L^*$:

- It is impossible that $e_H^* = e_L^*$.
- If $e_H^* < e_L^*$, then we have

$$\pi'(e_H^*) > \pi'(e_L^*) \text{ and } g_e(e_H^*, \theta_H) < g_e(e_L^*, \theta_H) < g_e(e_L^*, \theta_L).$$

Contradiction.

Interpretation: the optimal effort level of a high-ability agent is greater than that of a low-ability agent.



31 In the figure, the wage w_H^* is greater than w_L^*, but we note that w_H^* can be greater or smaller than w_L^* depending on the curvature of the functions π, g, and v, as it can be easily seen graphically.

32 The principal's profit:

$$\Pi_H^* = \overbrace{\pi(e_H^*) - g(e_H^*, \theta_H) - v^{-1}(\bar{u}) \geq \pi(e_L^*) - g(e_L^*, \theta_H) - v^{-1}(\bar{u})}^{e_H^* \text{ maximizes } \pi(e) - v^{-1}(\bar{u}) - g(e, \theta_H)} \underbrace{\geq \pi(e_L^*) - g(e_L^*, \theta_L) - v^{-1}(\bar{u})}_{\theta_L < \theta_H} = \Pi_L^*.$$

33 For contract to be always carried out, it is thus enough that profit is positive for a θ_L agent, *i.e.*, the following condition must be satisfied

$$\Pi_L^* = \pi(e_L^*) - g(e_L^*, \theta_L) - v^{-1}(\bar{u}) \geq 0,$$

i.e., $\bar{u} \leq v\big(\pi(e_L^*) - g(e_L^*, \theta_L)\big)$. We will maintain this hypothesis hereafter.

34 First-best contract menu $\{(e_i^*, w_i^*)\}_{i=H,L}$.

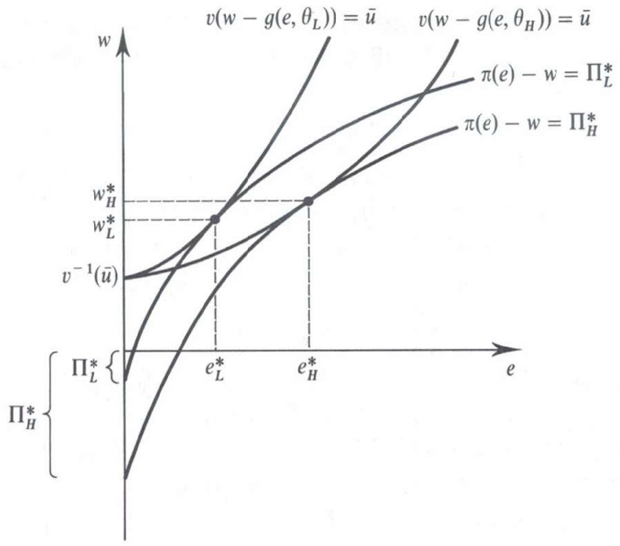
To implement the first-best effort levels e_H^* and e_L^*, the principal can make the following take-it-or-leave-it offers to the agent: If $\theta = \theta_H$ (resp. θ_L), the principal offers the wage w_H^* (resp. w_L^*) for the effort level e_H^* (resp. e_L^*) with $w_i^* - g(e_i^*, \theta_i) = v^{-1}(\bar{u})$.

Whatever his type, agent accepts the offer and makes zero utility. The complete information optimal contracts are thus (e_H^*, w_H^*) if $\theta = \theta_H$ and (e_L^*, w_L^*) if $\theta = \theta_L$.

2.2 Incomplete information

35 Suppose that θ is the agent's private information.

36 Consider the case where the principal offers the menu of contracts $\{(e_H^*, w_H^*), (e_L^*, w_L^*)\}$ hoping that an agent with type θ_L will select (e_L^*, w_L^*) and an agent with type θ_H will select instead (e_H^*, w_H^*).



We see that (e_L^*, w_L^*) is preferred to (e_H^*, w_H^*) by both types of agents:

- The θ_H-agent's isoutility curve that passes through (e_L^*, w_L^*) corresponds to a utility level higher than $\bar{u}$ at (e_H^*, w_H^*).
- The θ_L-agent's isoutility curve that passes through (e_H^*, w_H^*) corresponds to a utility level lower than $\bar{u}$ at (e_L^*, w_L^*).

Offering the menu of contracts $\{(e_H^*, w_H^*), (e_L^*, w_L^*)\}$ fails to have the agents self-selecting properly within this menu. The high-ability agent mimics the low-ability one and selects also contract (e_L^*, w_L^*). The complete information optimal contracts can no longer be implemented under asymmetric information.

37 Definition: A menu of contracts $\{(e_L, w_L), (e_H, w_H)\}$ is incentive compatible when (e_L, w_L) is weakly preferred to (e_H, w_H) by the type-θ_L agent and (e_H, w_H) is weakly preferred to (e_L, w_L) by the type-θ_H agent.

Mathematically,

$$w_L - g(e_L, \theta_L) \geq w_H - g(e_H, \theta_L), \tag{IC$_L$}$$

$$w_H - g(e_H, \theta_H) \geq w_L - g(e_L, \theta_H). \tag{IC$_H$}$$

38 If a menu of contracts $\{(e_L, w_L), (e_H, w_H)\}$ is incentive compatible, then $e_H \geq e_L$, which is called the monotonicity constraint. Indeed,

$$\int_{e_L}^{e_H} g_e(e, \theta_L)\,\mathrm{d}e = \overbrace{g(e_H, \theta_L) - g(e_L, \theta_L) \geq w_H - w_L}^{\text{By Equation (IC}_L\text{)}} \underbrace{\geq g(e_H, \theta_H) - g(e_L, \theta_H)}_{\text{By Equation (IC}_H\text{)}} = \int_{e_L}^{e_H} g_e(e, \theta_H)\,\mathrm{d}e,$$

and hence $e_H \geq e_L$.

39 Definition: A menu of contracts $\{(e_L, w_L), (e_H, w_H)\}$ is individually rational if

$$w_L - g(e_L, \theta_L) \geq v^{-1}(\bar{u}), \tag{IR$_L$}$$
$$w_H - g(e_H, \theta_H) \geq v^{-1}(\bar{u}). \tag{IR$_H$}$$

40 Information rent: Under complete information, the principal is able to maintain all types of agents at their reservation utility. Their respective utility levels at the first-best contracts satisfy

$$w_H^* - g(e_H^*, \theta_H) = v^{-1}(\bar{u}) \text{ and } w_L^* - g(e_L^*, \theta_L) = v^{-1}(\bar{u}).$$

Generally this will not be possible anymore under incomplete information, at least when the principal wants both types of agents to be active.

Let $r_H = w_H - g(e_H, \theta_H) - v^{-1}(\bar{u})$ and $r_L = w_L - g(e_L, \theta_L) - v^{-1}(\bar{u})$ denote the respective information rent (the utility in excess of the reservation utility) of each type.

41 The principal's problem is to solve

$$\begin{array}{cl} \underset{(e_L,w_L),(e_H,w_H)}{\text{maximize}} & \lambda\big(\pi(e_H) - w_H\big) + (1-\lambda)\big(\pi(e_L) - w_L\big) \\ \text{subject to} & \text{Equations (IC}_L\text{)–(IR}_H\text{)}. \end{array}$$

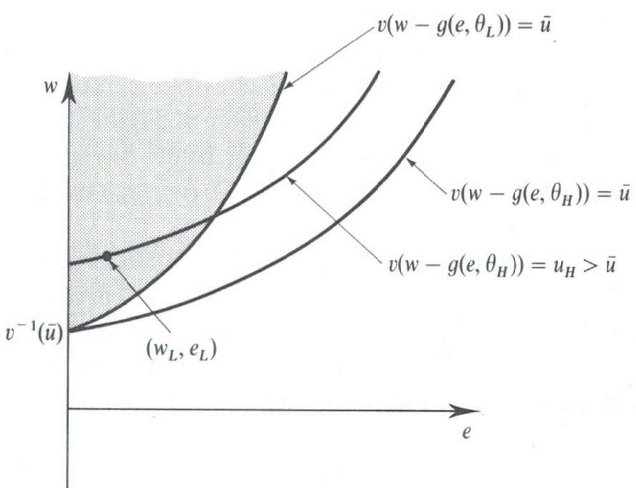
We can rewrite as $\lambda\big(\pi(e_H) - g(e_H, \theta_H) - v^{-1}(\bar{u})\big) + (1-\lambda)\big(\pi(e_L) - g(e_L, \theta_L) - v^{-1}(\bar{u})\big) - [\underbrace{\lambda r_H + (1-\lambda) r_L}_{\text{expected information rent}}]$.

42 Lemma: The constraint (IR$_H$) is always satisfied due to constraints (IC$_H$) and (IR$_L$).

Proof.

$$w_H - g(e_H, \theta_H) \geq w_L - g(e_L, \theta_H) \geq w_L - g(e_L, \theta_L) \geq \bar{u}.$$

□



43 Lemma: The constraint (IR$_L$) is binding at the optimal.

Proof. Suppose that $w_L - g(e_L, \theta_L) - v^{-1}(\bar{u}) = \varepsilon > 0$. Then the principal can decease w_L by ε and consequently also w_H by ε and gain ε. □

44 Lemma: The constraint (IC_H) is binding at the optimal.

Proof. Suppose that $[w_H - g(e_H, \theta_H)] - [w_L - g(e_L, \theta_H)] = \varepsilon > 0$. Then the principal can decrease w_H by ε and gain $\lambda\varepsilon$. □

45 We obtain a reduced program

$$\max_{e_L, e_H} \lambda\big(\pi(e_H) - g(e_H, \theta_H) - v^{-1}(\bar{u}) + g(e_L, \theta_H) - g(e_L, \theta_L)\big) + (1-\lambda)\big(\pi(e_L) - g(e_L, \theta_L) - v^{-1}(\bar{u})\big).$$

Compared with the full information setting, asymmetric information alters the principal's optimization simply by the subtraction of the expected rent that has to be given up to the efficient type. The inefficient type gets no rent, but the efficient type θ_H gets the information rent that he could obtain by mimicking the inefficient type θ_L. This rent depends only on the level of production requested from this inefficient type.

46 The first order condition on e_H implies

$$\pi'(e_H^{\text{SB}}) = g_e(e_H^{\text{SB}}, \theta_H), \text{ that is, } e_H^{\text{SB}} = e_H^*.$$

Hence, there is no distortion away from the first-best for the efficient type's output.

Notice that: $\pi'(0) > 0$, $\pi'' < 0$, $g_e(0, \theta_H) = 0$, and $g_{ee} > 0$, such a $e_H^{\text{SB}} > 0$ exists.

47 The first order condition on e_L implies

$$(1-\lambda) \cdot \big(\pi'(e_L^{\text{SB}}) - g_e(e_L^{\text{SB}}, \theta_L)\big) = \lambda \cdot \big(g_e(e_L^{\text{SB}}, \theta_L) - g_e(e_L^{\text{SB}}, \theta_H)\big).$$

This equation expresses the important trade-off between efficiency and rent extraction which arises under asymmetric information. The expected marginal efficiency gain (resp. cost) and the expected marginal cost (resp. gain) of the rent brought about by an infinitesimal increase (resp. decrease) of θ_L agent's output are equated. Thus, the principal is neither willing to increase nor to decrease θ_L agent's effort.

Notice: Such a $e_L^{\text{SB}} > 0$ exists.

48 Since $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$ and $\pi'(e_L^{\text{SB}}) = g_e(e_L^{\text{SB}}, \theta_L) + \frac{\lambda}{1-\lambda}[g_e(e_L^{\text{SB}}, \theta_L) - g_e(e_L^{\text{SB}}, \theta_H)]$, we have the following inequality

$$e_H^{\text{SB}} = e_H^* > \underbrace{e_L^* > e_L^{\text{SB}}}_{\pi'' < 0},$$

and hence

$$\begin{aligned} w_L^{\text{SB}} - g(e_L^{\text{SB}}, \theta_L) - w_H^{\text{SB}} + g(e_H^{\text{SB}}, \theta_L) &= g(e_L^{\text{SB}}, \theta_H) - g(e_H^{\text{SB}}, \theta_H) - g(e_L^{\text{SB}}, \theta_L) + g(e_H^{\text{SB}}, \theta_L) \\ &= \int_{\theta_L}^{\theta_H} g_\theta(e_L^{\text{SB}}, \theta) - g_\theta(e_H^{\text{SB}}, \theta)\, \mathrm{d}\theta \geq 0. \end{aligned}$$

That is, the constraint (IC_L) is strictly satisfied.

49 Proposition: Under asymmetric information, the optimal menu of contracts entails:

- No output distortion for the high-ability agent with respect to the first-best, $e_H^{\text{SB}} = e_H^*$. A downward output distortion for the low-ability agent, $e_L^{\text{SB}} < e_L^*$ with

$$\pi'(e_L^{\text{SB}}) = g_e(e_L^{\text{SB}}, \theta_L) + \frac{\lambda}{1-\lambda}[g_e(e_L^{\text{SB}}, \theta_L) - g_e(e_L^{\text{SB}}, \theta_H)].$$

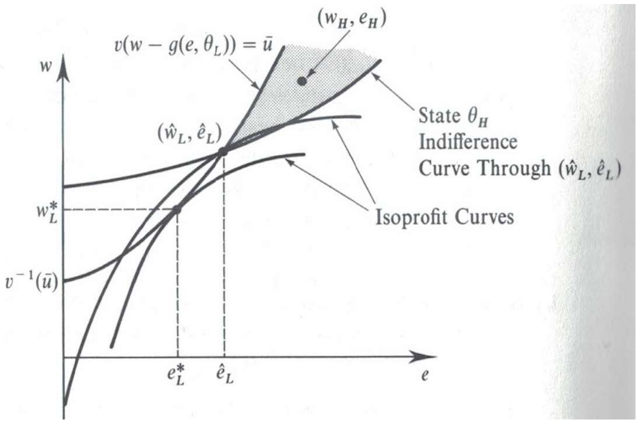
- The second-best wages are respectively given by

$$w_H^{\mathrm{SB}} = g(e_H^{\mathrm{SB}}, \theta_H) + v^{-1}(\bar{u}) + g(e_L^{\mathrm{SB}}, \theta_L) - g(e_L^{\mathrm{SB}}, \theta_H) > w_H^*,$$
$$w_L^{\mathrm{SB}} = g(e_L^{\mathrm{SB}}, \theta_L) + v^{-1}(\bar{u}) < w_L^*.$$

- Only the high-ability agent gets a positive information rent given by

$$r_H^{\mathrm{SB}} = g(e_L^{\mathrm{SB}}, \theta_L) - g(e_L^{\mathrm{SB}}, \theta_H).$$

50 Graphic illustration for $e_L^{\mathrm{SB}} \leq e_L^*$.



(1) Suppose that $e_L^{\mathrm{SB}} > e_L^*$.

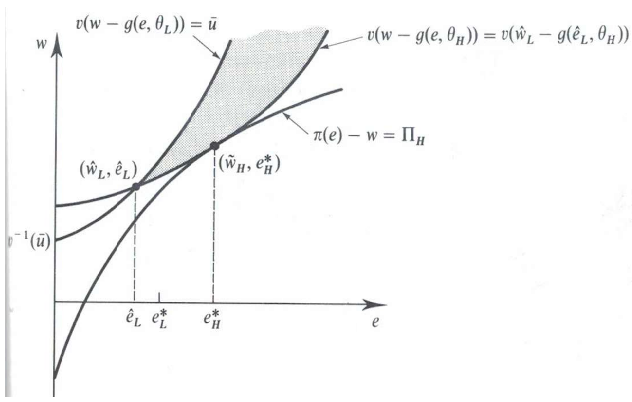
(2) Since θ_L-IR binds, $(e_L^{\mathrm{SB}}, w_L^{\mathrm{SB}})$ lies on the indifference curve through $v^{-1}(\bar{u})$.

(3) To make θ_L-IC and θ_H-IC hold, $(e_H^{\mathrm{SB}}, w_H^{\mathrm{SB}})$ lies in the shade region.

(4) Principal can raise her profit by moving $(e_L^{\mathrm{SB}}, w_L^{\mathrm{SB}})$ to (e_L^*, w_L^*): θ_L-IC and θ_H-IC still hold.

(5) Thus, $e_L^{\mathrm{SB}} > e_L^*$ cannot be optimal.

51 Graphic illustration for $e_H^{\mathrm{SB}} = e_H^*$.



(1) Suppose that $e_L^{\mathrm{SB}} \leq e_L^*$.

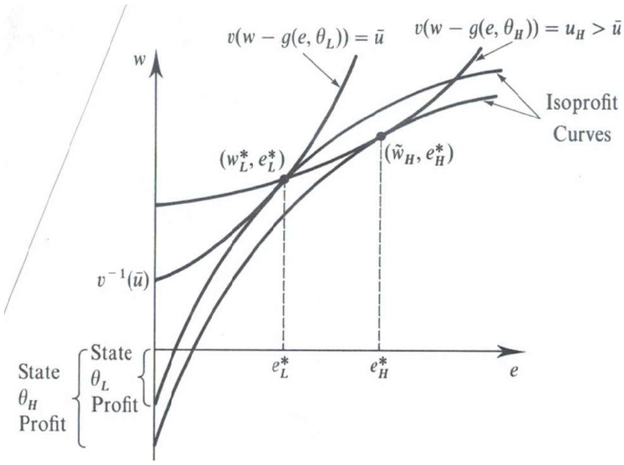
(2) To make θ_L-IC and θ_H-IC hold, $(e_H^{\mathrm{SB}}, w_H^{\mathrm{SB}})$ lies in the shade region.

(3) Principal's problem is to find the allocation of $(e_H^{\mathrm{SB}}, w_H^{\mathrm{SB}})$ that maximizes her profit.

(4) The optimal solution occurs at a point of tangency between the indifference curve of θ_H agent through $(e_H^{\text{SB}}, w_H^{\text{SB}})$ and an isoprofit curve for principal.

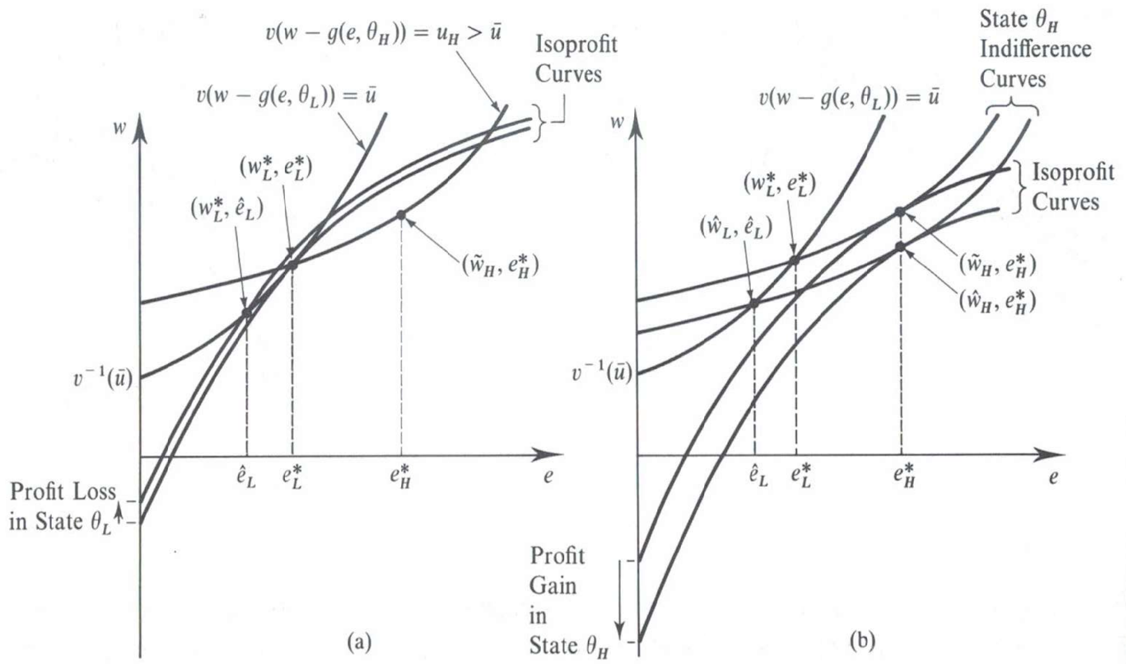
(5) All points of tangency between indifference curves of θ_H agent and isoprofit curves of principal occur at e_H^*.

52 Graphic illustration for the optimal contracts.



(1) We start with (e_L^*, w_L^*).

(2) Since θ_H-IC binds, $(e_H^*, \tilde{w}_H)$ lies on θ_H agent's indifference curve through (e_L^*, w_L^*).



(1) Principal firstly moves (e_L^*, w_L^*) to $(e_L^{\text{SB}}, w_L^{\text{SB}})$, where $e_L^* > e_L^{\text{SB}}$. Note that (e_L^*, w_L^*) and $(e_L^{\text{SB}}, w_L^{\text{SB}})$ lie on θ_L agent's indifference curve through $v^{-1}(\bar{u})$.

(2) This change lowers the profit that principal earns from θ_L agent.

(3) On the other hand, it relaxes θ_H-IC.

(4) Principal then moves $(e_H^*, \tilde{w}_H)$ to $(e^*, \hat{w}_H)$.

(5) This change increases the profit that principal earns from θ_H agent.

(6) Comparison:

- The derivative of principal's profit from θ_L agent with respect to e_L at e_L^* is zero:

$$\frac{\mathrm{d}}{\mathrm{d}e_L}[\pi(e_L) - g(e_L, \theta_L) - v^{-1}(\bar{u})]\Big|_{e_L = e_L^*} = 0.$$

- The derivative of principal's profit from θ_H agent with respect to e_L at e_L^* is strictly negative:

$$\frac{\mathrm{d}}{\mathrm{d}e_L}[\pi(e_H^*) - g(e_H^*, \theta_H) - v^{-1}(\bar{u}) + g(e_L, \theta_H) - g(e_L, \theta_L)]\Big|_{e_L = e_L^*} < 0.$$

(7) How far should principal go in lowering e_L—When the marginal loss from θ_L agent equals the marginal gain from θ_H agent, i.e.,

$$(1 - \lambda) \cdot [\pi'(e_L^{\mathrm{SB}}) - g_e(e_L^{\mathrm{SB}}, \theta_L)] = \lambda \cdot [g_e(e_L^{\mathrm{SB}}, \theta_L) - g_e(e_L^{\mathrm{SB}}, \theta_H)].$$

3 Homework

- Key: The SPE contracts in competitive screening, the optimal contracts in monopolistic screening, and their relationship.
- Reading: 13.D, 14.C, 2.1–2.9 in *The Theory of Incentives*
- Homework: 13.D.1, 14.C.6