

# ADVANCED MICROECONOMICS: LECTURE 2

## 1 Signaling for job market

1 The key to resolve the adverse selection: some “mechanisms/procedures” to help distinguish among workers.

2 Signaling is one of such mechanisms, which was first investigated by Spence (1973, 1974).

Basic idea: The high-ability workers may have (costly or costless) actions to distinguish themselves from low-ability workers.

3 The ideal case: Workers can take a costless test that reveals their types.

Then in any SPE, all workers with ability greater than  $\underline{\theta}$  will take the test and the market will achieve the full information outcome.

4 In general, no procedure exists that directly reveals a worker's type.

5 There are two types of workers with productivities  $\theta_L$  and  $\theta_H$ , where  $0 < \theta_L < \theta_H$  and  $\lambda = \text{Prob}(\theta = \theta_H) \in (0, 1)$ .

6 Before entering the job market, a worker can get some education, and the amount of education that a worker receives is observable.

The cost of obtaining education level  $e$  for a type- $\theta$  worker is given by  $c(e, \theta)$ . We assume  $c(e, \theta)$  is twice continuously differentiable and  $c(0, \theta) = 0$ ,  $c_e(e, \theta) > 0$ ,  $c_{ee}(e, \theta) > 0$ ,  $c_\theta(e, \theta) < 0$  for all  $e > 0$ , and  $c_{e\theta}(e, \theta) < 0$ .

Assumption: The education does nothing for a worker's productivity.

7 Utility for a type- $\theta$  worker who chooses education level  $e$  and receives wage  $w$  is  $w - c(e, \theta)$ .

A type- $\theta$  worker can earn  $r(\theta)$  by working at home.

8 For simplicity, assume  $r(\theta) = 0$ .

Thus, the unique equilibrium in the absence of the ability to signal:  $w^* = E[\theta]$ .

9 Game

- A random move of nature determines whether a worker is of high or low ability.
- Conditional on her type, the worker chooses how much education level to obtain. After that, the worker enters the market.
- Conditional on the observed education level, two firms simultaneously make wage offers.
- The worker decides whether to work for a firm and, if so, which one.

Remark: Here we model only a single worker of unknown type. The model with many workers can be thought of as simply having many of these single-worker games going on simultaneously, with the fraction of high-ability workers in the market being  $\lambda$ .

## 2 PBE

10 Perfect Bayesian equilibrium: a pair of strategy profiles and a belief function  $\mu(e) \in [0, 1]$  giving the firms' common probability assessment that the worker is of high ability after observing education level  $e$  such that

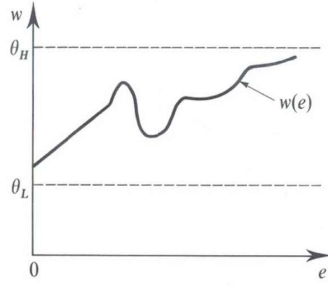
- The worker's strategy  $e^*(\theta)$  is optimal given the firms' strategies  $w_1^*(e)$  and  $w_2^*(e)$ .
- The belief  $\mu^*(e)$  is derived from the workers' strategies  $e^*(\theta)$  via Bayes' rule when possible.
- Following each  $e$  (i.e., given each  $\mu^*(e)$ ), the firms' wage offers  $w_1^*(e)$  and  $w_2^*(e)$  constitute a NE.

11 We focus on pure-strategy PBE.

12 At the end of the game:

- (1) After seeing the education level  $e$ , the firms have belief  $\mu(e)$  that the worker is type  $\theta_H$ .
- (2) The expected productivity is  $\mu(e)\theta_H + (1 - \mu(e))\theta_L$ .
- (3) Like Bertrand pricing game, in any PBE, both firms offer wage  $w(e) = \mu(e)\theta_H + (1 - \mu(e))\theta_L$ .

For any  $e$ ,  $w(e) \in [\theta_L, \theta_H]$ .

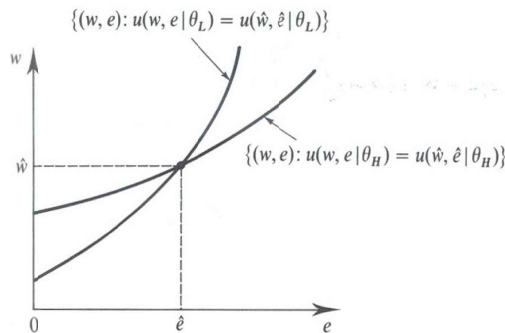


13 Single-crossing property: Due to the assumptions on  $c(e, \theta)$ , an indifference curve of type- $\theta_H$  worker and an indifference curve of type- $\theta_L$  worker cross only once.

At any  $(w, e)$ , the marginal rate of substitution between wages and education is

$$\frac{dw}{de} = c_e(e, \theta),$$

which is decreasing in  $\theta$  since  $c_{e\theta}(e, \theta) < 0$ .



14 Preview of the result: The unique outcome is the best separating PBE outcome:

- High-ability worker:  $(\theta_H, \bar{e})$ .
- Low-ability worker:  $(\theta_L, 0)$ .

### 3 Separating PBE

15 In a separating PBE, two types of workers choose different education levels.

16 Lemma: In any separating PBE,  $w^*(e^*(\theta_H)) = \theta_H$  and  $w^*(e^*(\theta_L)) = \theta_L$ .

*Proof.* (1) Bayes' rule: After seeing  $e^*(\theta_H)$ , the firms should believe that the worker is of high ability  $\theta_H$ ; otherwise, the firms should believe that the worker is of low ability  $\theta_L$ .

(2) The resulting wages are  $\theta_H$  and  $\theta_L$ , respectively.

□

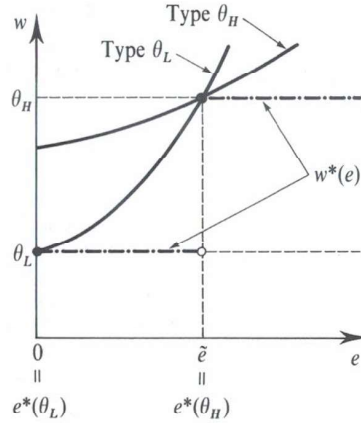
17 Lemma: In any separating PBE,  $e^*(\theta_L) = 0$ .

*Proof.* (1) The type- $\theta_L$  worker always receive wage  $\theta_L$ .

(2) Thus, choosing  $e = 0$  will save her cost of education, and is optimal.

□

18 Let  $(\tilde{e}, \theta_H)$  be the intersection point of the curve  $\theta_L = w - c(e, \theta_L)$  and the curve  $w = \theta_H$ .



Lemma: In any separating PBE,  $e^*(\theta_H) \geq \tilde{e}$ .

*Proof.* (1) Suppose  $e^*(\theta_H) < \tilde{e}$ .

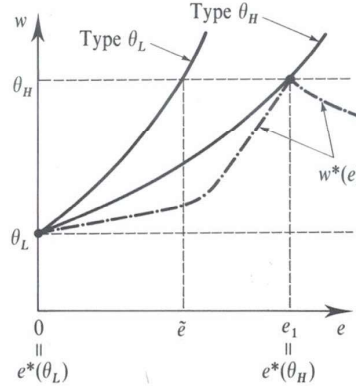
(2) Then the type- $\theta_L$  worker will mimic the type- $\theta_H$  worker by choosing  $e^*(\theta_H)$ :

$$\theta_L = \theta_H - c(\tilde{e}, \theta_L) < \theta_H - c(e^*(\theta_H), \theta_L).$$

(3) It is not an equilibrium. Contradiction.

□

19 Let  $(e_1, \theta_H)$  be the intersection point of the curve  $\theta_L = w - c(e, \theta_H)$  and the curve  $w = \theta_H$ .



Lemma: In any separating PBE,  $e^*(\theta_H) \leq e_1$ .

*Proof.* (1) Suppose  $e^*(\theta_H) > e_1$ .

(2) Then the type- $\theta_H$  worker will mimic the type- $\theta_L$  worker by choosing 0:

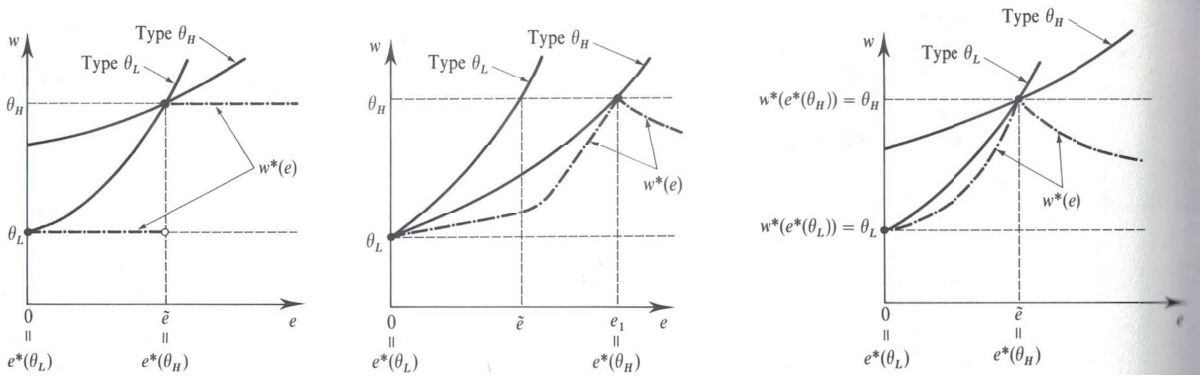
$$\theta_L = \theta_H - c(e_1, \theta_H) > \theta_H - c(e^*(\theta_H), \theta_H).$$

(3) It is not an equilibrium. Contradiction.

□

20 Proposition: For each  $e_0 \in [\tilde{e}, e_1]$ , there is a separating PBE:

$$e^*(\theta_H) = e_0, \quad e^*(\theta_L) = 0, \quad \mu^*(e) = \begin{cases} 0, & \text{if } e < e_0, \\ 1, & \text{if } e \geq e_0. \end{cases}, \quad w^*(e) = \begin{cases} \theta_L, & \text{if } e < e_0, \\ \theta_H, & \text{if } e \geq e_0. \end{cases}.$$



*Proof.* • Type- $\theta_L$  worker:

- Deviation  $e \in (0, e_0)$ : worse off since  $\theta_L - c(e, \theta_L) < \theta_L$ .
- Deviation  $e \geq e_0$ : not better off since  $\theta_H - c(e, \theta_L) \leq \theta_H - c(\tilde{e}, \theta_L) = \theta_L$ .

• Type- $\theta_H$  worker:

- Deviation  $e < e_0$ : not better off since  $\theta_L = \theta_H - c(e_1, \theta_H) \leq \theta_H - c(e_0, \theta_H)$ .
- Deviation  $e > e_0$ : worse off since  $\theta_H - c(e, \theta_H) < \theta_H - c(e_0, \theta_H)$ .

• Belief:  $\mu^*(0) = 0$  and  $\mu^*(e_0) = 1$ . For  $e \neq e_0$ , set  $\mu^*(e_0)$  as in the statement.

- Wage: Given the belief, it is optimal.

□

Notice:

- The Bayes' rule only requires that  $\mu^*(0) = 0$  and  $\mu^*(e_0) = 1$ .
- However, after seeing  $e \notin \{0, e_0\}$ , the belief  $\mu^*(e)$  could be arbitrary. It leads to multiple equilibria.

21 Key: The useless education can serve as a signal because the marginal cost of education is higher for a low-ability worker.

- a type- $\theta_H$  worker may find it worthwhile to get some positive level of education to raise her wage by some amount,
- a type- $\theta_L$  worker may be unwilling to get this same level of education in return for the same wage increase.

22 Pareto efficiency among all the separating PBEs:

- Firms earn zero profits.
- A type- $\theta_L$  worker's utility is  $\theta_L$ .
- A type- $\theta_H$  worker does strictly better in separating PBE where she gets a lower level of education.

Thus, the separating PBE in which the high-ability worker gets  $\tilde{e}$  Pareto dominate all the others.

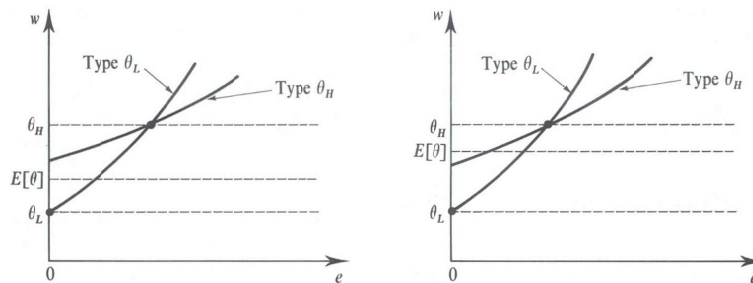
On the other hand, the Pareto dominated separating PBE are sustained because of the high-ability worker's fear: if she chooses a lower level of education than equilibrium education, firms will believe that she is not a high-ability worker. These beliefs can be maintained because in PBE they are never disconfirmed (off-equilibrium path).

23 Welfare for type- $\theta_L$  workers: they are strictly worse off when signaling is possible, i.e.,  $E[\theta] > \theta_L$ .

24 Welfare for type- $\theta_H$  workers: they may be either better or worse off when signaling is possible.

- If  $E[\theta] < \theta_H - c(\tilde{e}, \theta_H)$ , then the high-ability workers are better off because of the increase in their wages arising through signaling.
- If  $E[\theta] > \theta_H - c(\tilde{e}, \theta_H)$ , then the high-ability workers are worse off than when signaling is impossible.

In a separating PBE, the outcome  $(E[\theta], 0)$  from no-signaling situation is no longer available to the high-ability workers.



Summary:

- The set of separating PBE is completely unaffected by the fraction  $\lambda$ .

- As  $\lambda$  grows, it becomes more likely that the high-ability workers are worse off by the possibility of signaling.

25 Refinement:

- (1) For any  $e_0 > \tilde{e}$ , a type- $\theta_L$  worker will never be better off by choosing  $e_0$  than 0 regardless of what firms believe about her as a result.
- (2) Upon seeing  $e_0 > \tilde{e}$ , any belief other than  $\mu(e_0) = 1$  seems unreasonable.
- (3) Thus,  $w^*(e_0) = \theta_H$ .
- (4) As a consequence, the type- $\theta_H$  worker will deviate from  $e_0$  to  $\tilde{e}$ .

By this logic, the only reasonable separating SPE outcome is  $(\theta_L, 0)$  for  $\theta_L$  workers and  $(\theta_H, \tilde{e})$  for  $\theta_H$  workers.

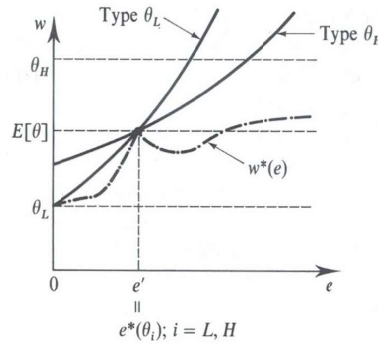
## 4 Pooling PBE

26 In a pooling PBE, the two types of workers choose the same level of education,  $e^*(\theta_L) = e^*(\theta_H) = e^*$ .

27 After seeing  $e^*$  (on the equilibrium path), the firms should believe the worker is of high ability with probability  $\lambda$ .

Thus, the wage  $w^*(e^*) = \lambda\theta_H + (1 - \lambda)\theta_L = E[\theta]$ .

28 Let  $(e', E[\theta])$  be the intersection point between the curve  $\theta_L = w - c(e, \theta_L)$  and the curve  $w = E[\theta]$ .



Lemma: In a pooling PBE,  $e^* \leq e'$ .

*Proof.* (1) Suppose  $e^* > e'$ .

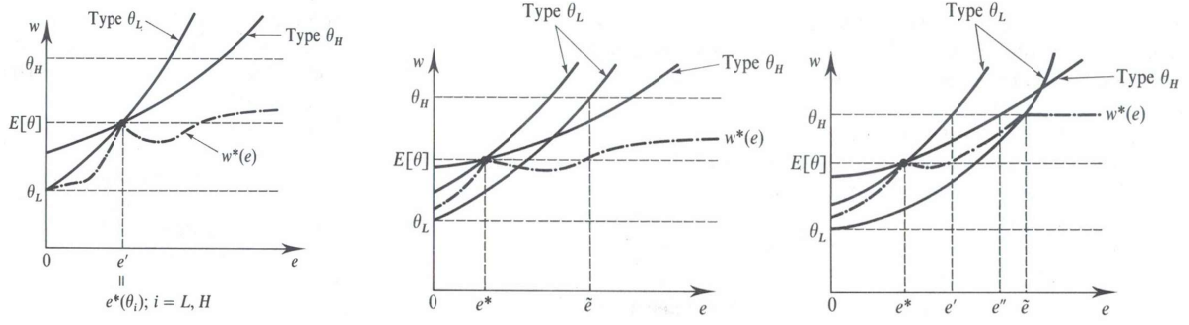
(2) Then the type- $\theta_L$  worker will deviate to 0:  $\theta_L = E[\theta] - c(e', \theta_L) > E[\theta] - c(e^*, \theta_L)$ .

(3) Thus, it is not an equilibrium. Contradiction.

□

29 Proposition: For any  $e_0 \in [0, e']$ , there is a pooling PBE:

$$e^*(\theta_L) = e^*(\theta_H) = e_0, \mu^*(e) = \begin{cases} 0, & \text{if } e < e_0, \\ \lambda, & \text{if } e \geq e_0. \end{cases}, w^*(e) = \begin{cases} \theta_L, & \text{if } e < e_0, \\ E[\theta], & \text{if } e \geq e_0. \end{cases}$$



*Proof.* • For type- $\theta_L$  worker:

- Deviation  $e < e_0$ : not better off since  $\theta_L = E[\theta] - c(e', \theta_L) \leq E[\theta] - c(e_0, \theta_L)$ .
- Deviation  $e > e_0$ : worse off since  $E[\theta] - c(e, \theta_L) < E[\theta] - c(e_0, \theta_L)$ .

• For type- $\theta_H$  worker:

- Deviation  $e < e_0$ : worse off since  $\theta_L = E[\theta] - c(e', \theta_L) < E[\theta] - c(e_0, \theta_H)$ .
- Deviation  $e > e_0$ : worse off since  $E[\theta] - c(e, \theta_H) < E[\theta] - c(e_0, \theta_H)$ .

- Belief:  $\mu^*(e_0) = \lambda$ . For  $e \neq e_0$ ,  $\mu^*(e)$  could be arbitrary. We set  $\mu^*(e)$  as in the statement.
- Wage: Given the belief, it is optimal.

□

30 Remark:  $e' < \tilde{e} < e_1$ .

31 Pareto efficiency:

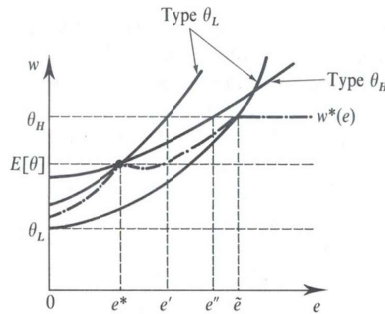
A pooling PBE in which both types of worker get no education Pareto dominates any pooling PBE with a positive education level.

The Pareto-dominated pooling PBE are sustained by the worker's fear: A deviation will lead firms to have an unfavorable impression of her ability.

32 For any pooling PBE  $(e^*, \mu^*, w^*)$  where  $e^* \in [0, e']$ ,

- let  $(e_\ell, \theta_H)$  be the intersection point between the curve  $E[\theta] - c(e^*, \theta_L) = w - c(e, \theta_L)$  and the curve  $w = \theta_H$ ,
- let  $(e_h, \theta_H)$  be the intersection point between the curve  $E[\theta] - c(e^*, \theta_H) = w - c(e, \theta_H)$  and the curve  $w = \theta_H$ .

33 Refinement (intuition criterion):



- (1) To support the education choice  $e^*$  as a pooling PBE outcome, we must have  $\mu(e) < 1$  after seeing  $e \in (e_\ell, e_h)$ :
- If  $\mu(e) = 1$  for some  $e \in (e_\ell, e_h)$ , then the wage should be  $\theta_H$ , and the type- $\theta_H$  worker will be better off by deviating to  $e$ :

$$\theta_H - c(e, \theta_H) > \theta_H - c(e_h, \theta_H) = E[\theta] - c(e^*, \theta_H) \geq E[\theta].$$

- (2) Consider the off-equilibrium path: Suppose that a firm is confronted with a deviation to some education level  $e \in (e_\ell, e_h)$  when it was expecting the equilibrium level of education  $e^*$  to be chosen.
- (3) The firm will reason as follows:

- a type- $\theta_L$  worker would be worse off deviating to  $e$  regardless of what beliefs firms have after that:

$$E[\theta] - c(e^*, \theta_L) = \theta_H - c(e_\ell, \theta_L) > \theta_H - c(e, \theta_L).$$

- a type- $\theta_H$  worker might be better off by doing this:

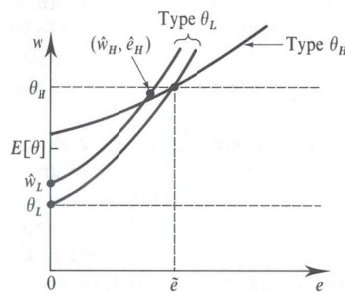
$$E[\theta] - c(e^*, \theta_H) = \theta_H - c(e_h, \theta_H) < \theta_H - c(e, \theta_H).$$

Thus, this must not be a low-ability worker.

- (4) Thus,  $e^*$  cannot be a pooling PBE education level. No pooling PBE survives.

## 5 Second-best intervention

- 34 In the presence of signaling, although the central planner cannot observe workers' types, it may be able to achieve a Pareto improvement relative to the market outcome.
- 35 Case 1: When the best separating PBE is Pareto dominated by the no-signaling outcome, a Pareto improvement can be achieved simply by banning the signaling activity.
- 36 Case 2: When the no-signaling outcome does not Pareto dominate the best separating PBE, a Pareto improvement can be achieved by "cross-subsidization":



The outcomes  $(\hat{w}_L, 0)$  and  $(\hat{w}_H, \hat{e}_H)$  can be achieved by mandating

- workers with education levels below  $\hat{e}_H$  receive wage  $\hat{w}_L$ ,
- workers with education levels of at least  $\hat{e}_H$  receive wage  $\hat{w}_H$ .

Thus, low-ability workers will choose  $e = 0$  and high-ability workers will choose  $e = \hat{e}_H$ .



## 6 Homework

- Key: The economic intuition behind separating and pooling SPE
- Reading: 13.C
- Homework: 13.C.2
- Recommendation: How about the signaling game where there is only a single firm?