Advanced Microeconomics I: Lecture Notes 7

1 Moral hazard models hidden action, where asymmetric information forms after the parties enter into a relationship. A moral hazard is a situation in which a party (agent) is more likely to take risks because the costs that could result will not be borne by the party taking the risk. Moral hazard arises because an individual or institution does not take the full consequences and responsibilities of its actions, and therefore has a tendency to act less carefully than it otherwise would, leaving another party to hold some responsibility for the consequences of those actions.

In a principal-agent problem, one party, called an agent, acts on behalf of another party, called the principal. The agent usually has more information about his or her actions or intentions than the principal does, because the principal usually cannot completely monitor the agent. The agent may have an incentive to act inappropriately (from the viewpoint of the principal) if the interests of the agent and the principal are not aligned.

In particular, consider that a firm (the principal) hires a worker (the agent) to work on a project, which succeeds with probability p if the worker exerts effort. The firm may only observe the outcome of the project but not the agent's effort level. In such a situation, the firm's payment contract can only depend on the outcome, which is an imperfect indicator of the worker's effort level. If the worker is paid fixed wage or if the payment conditional on success is not high enough, since effort is costly, the worker will shirk—moral hazard arises.

1 The basic set-up

2 A principal (employer) hires an agent (employee) for production. The agent can exert a costly effort $e \in \{0, 1\}$. Exerting effort e implies a cost/disutility for the agent that is equal to $g(e)$ with the normalizations $g(0) = 0$ and $g(1) = g > 0$. The agent receives a wage w from the principal.

The agent's utility is assumed to be

$$u(w) - g(e),$$

where u is increasing and concave, and $u(0) = 0$. Denote $h = u^{-1}$, which is increasing and convex. We normalize the agent's reservation utility at zero.

3 Profit is stochastic, and effort affects the profit level as follows: the stochastic profit level π can only take two values $\{\pi_L, \pi_H\}$ with $\pi_H - \pi_L > 0$, and the stochastic influence of effort on profit is characterized by the probabilities

$$\text{Prob}(\pi = \pi_H \mid e = 0) = \lambda_0 \text{ and } \text{Prob}(\pi = \pi_H \mid e = 1) = \lambda_1,$$

with $\lambda_1 - \lambda_0 > 0$.

Effort improves profit in the sense of first-order stochastic dominance.

4 The principal can only offer a contract based on the observable profit level, i.e., $w(\pi)$. Let w_H (resp. w_L) be the wage received by the agent if the profit is π_H (resp. π_L).

5 The risk-neutral principal's expected utility is

$$V_1 = \lambda_1(\pi_H - w_H) + (1 - \lambda_1)(\pi_L - w_L)$$

if the agent makes a positive effort $e = 1$, and

$$V_0 = \lambda_0(\pi_H - w_H) + (1 - \lambda_0)(\pi_L - w_L)$$

if the agent makes no effort $e = 0$.

6 If the agent makes a positive effort $e = 1$, then his expected utility is

$$\lambda_1 u(w_H) + (1 - \lambda_1)u(w_L) - g.$$

If the agent chooses $e = 0$, then his expected utility is

$$\lambda_0 u(w_H) + (1 - \lambda_0)u(w_L).$$

7 The problem of the principal is to decide whether to induce the agent to exert effort or not and, if he chooses to do so, then to decide which contract should be used.

8 The timing is as follows:

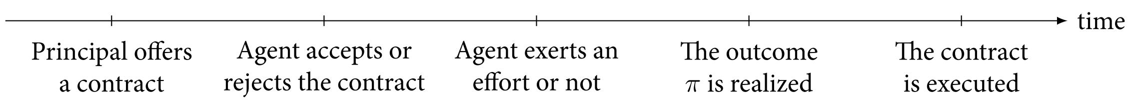


Figure 1

2 Complete information

9 First assume that the principal can observe effort.

10 In this situation, a contract can be regarded as the form (e, w_L, w_H). That is, the agent exerts the effort ($e = 1$) or not ($e = 0$) and he will receive w_L when the profit is low and w_H when the profit is high.

Once accepting the contract (e, w_L, w_H), agent will exert effort e: if the agent were not exerting effort e, his action could be perfectly detected by the principal, and hence the agent could be heavily punished (for example, $-\infty$).

11 It is convenient to think of this problem in two steps:

- For each $e \in \{0, 1\}$ that might be specified in the contract, what is the best (w_L, w_H)?
- What is the best choice of e?

12 To induce the agent to exert effort ($e = 1$), the principal's problem is:

$$\begin{aligned} \underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_1(\pi_H - w_H) + (1 - \lambda_1)(\pi_L - w_L) \\ \text{subject to} \quad & \lambda_1 u(w_H) + (1 - \lambda_1)u(w_L) - g \geq 0. \end{aligned}$$

Indeed, only the agent's individual rationality matters for the principal, because the agent can be forced to exert a positive level of effort.

13 Denoting the multiplier of the individual rationality constraint by μ and optimizing with respect to w_H and w_L yields, respectively, the following first-order conditions:

$$-\lambda_1 + \mu\lambda_1 u'(w_H^*) = 0,$$
$$-(1-\lambda_1) + \mu(1-\lambda_1)u'(w_L^*) = 0,$$

where w_H^* and w_L^* are the first-best wages.

We immediately derive that $\mu = \frac{1}{u'(w_L^*)} = \frac{1}{u'(w_H^*)} > 0$, and finally that $w^* = w_H^* = w_L^*$.

14 Remark:

- The wage w^* the agent receives is the same whatever the state of nature—ex post full insurance.
- Because the IR constraint is binding we also obtain the value of this wage, which is just enough to cover the disutility of effort, namely $w^* = u^{-1}(g)$. It is called the first-best cost C^* of implementing the positive effort level.

15 For the principal, inducing effort yields an expected payoff equal to

$$V_1^* = \lambda_1\pi_H + (1-\lambda_1)\pi_L - u^{-1}(g).$$

16 Had the principal decided to let the agent exert no effort ($e = 0$), his problem is

$$\begin{aligned} \underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_0(\pi_H - w_H) + (1-\lambda_0)(\pi_L - w_L) \\ \text{subject to} \quad & \lambda_0 u(w_H) + (1-\lambda_0)u(w_L) \geq 0. \end{aligned}$$

He would make a zero payment (it is optimal) to the agent whatever the realization of profit. In this scenario, the principal would instead obtain a payoff equal to

$$V_0^* = \lambda_0\pi_H + (1-\lambda_0)\pi_L.$$

17 Inducing effort is optimal from the principal's point of view when $V_1^* \geq V_0^*$, i.e.,

$$(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq u^{-1}(g). \tag{1}$$

18 The left-hand side of Equation (1) captures the gain of increasing effort from $e = 0$ to $e = 1$. This gain comes from the fact that the return π_H, which is greater than π_L, arises more often when a positive effort is exerted.

The right-hand side of Equation (1) is instead the first-best cost of inducing the agent's acceptance when he exerts a positive effort.

19 Summary:

- The first-best outcome (effort level) will be achieved:
  - The first-best outcome calls for $e^* = 1$ if and only if $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq u^{-1}(g)$.
  - When $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq u^{-1}(g)$, to implement the first-best outcome $e^* = 1$, the principal offers a contract $\big(1, u^{-1}(g), u^{-1}(g)\big)$ and the agent will accept.

- When $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) < u^{-1}(g)$, to implement the first-best outcome $e^* = 0$, the principal offers a contract $(0, 0, 0)$ and the agent will accept.

- The agent gets ex post full insurance.

3 Incomplete information with risk-neutral agent

20 In this situation, a contract is of the form (w_L, w_H). That is, the agent will receive w_L when the profit is low and w_H when the profit is high, regardless of his effort level.

21 If the agent is risk-neutral, we can assume that (up to an affine transformation) $u(w) = w$ for all w.

22 We consider this problem in two steps:

- If the principal wants the agent to exert positive effort (or zero effort), what is the best contract (w_L, w_H)?
- What is the best choice for the principal, inducing the agent to exert positive effort or zero effort?

23 To induce the agent to exert effort, the principal's problem is

$$\begin{aligned}
\underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_1(\pi_H - w_H) + (1 - \lambda_1)(\pi_L - w_L) \\
\text{subject to} \quad & \lambda_1 w_H + (1 - \lambda_1) w_L - g \geq \lambda_0 w_H + (1 - \lambda_0) w_L \\
& \lambda_1 w_H + (1 - \lambda_1) w_L - g \geq 0.
\end{aligned}$$

24 The principal's problem is equivalent to

$$\begin{aligned}
\underset{(w_H, w_L)}{\text{minimize}} \quad & \lambda_1 w_H + (1 - \lambda_1) w_L \\
\text{subject to} \quad & \Delta\lambda w_H \geq \Delta\lambda w_L + g \\
& \lambda_1 w_H + (1 - \lambda_1) w_L - g \geq 0.
\end{aligned}$$

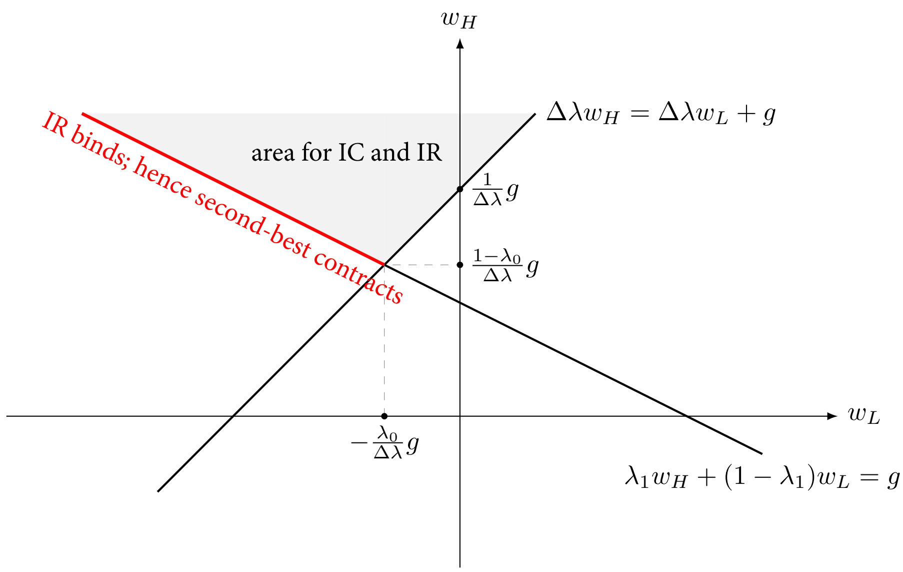
25 IR condition should binds at the optimum; otherwise the principal can decrease w_L without breaking IR condition and IC condition.

26 If the problem has a solution, the expected profit of principal is always

$$V_1^{\text{SB}} = \lambda_1 \pi_H + (1 - \lambda_1)\pi_L - g$$

due to the fact that IR condition binds.

27 Graphic illustration:



28 IC condition does not necessarily bind.

29 To find a solution, we let IC condition be binding. Then we have

$$w_H^{\mathrm{SB}} = g + \frac{1-\lambda_1}{\lambda_1 - \lambda_0} g = \frac{1-\lambda_0}{\Delta\lambda} g \text{ and } w_L^{\mathrm{SB}} = g - \frac{\lambda_1}{\lambda_1 - \lambda_0} g = -\frac{\lambda_0}{\Delta\lambda} g.$$

- The agent is rewarded if profit is high, and his utility is $w_H^{\mathrm{SB}} - g = \frac{1-\lambda_1}{\lambda_1-\lambda_0} g > 0$.
- The agent is punished if profit is low, and his utility is $w_L^{\mathrm{SB}} - g = -\frac{\lambda_1}{\lambda_1-\lambda_0} g < 0$.

The principal makes an expected payment

$$\lambda_1 w_H^{\mathrm{SB}} + (1-\lambda_1) w_L^{\mathrm{SB}} = g,$$

which is equal to the disutility of effort he would incur if he could control the effort level perfectly or if he was carrying the agent's task himself.

30 The wages $(w_H^{\mathrm{SB}}, w_L^{\mathrm{SB}})$ yield one possible implementation of the first-best outcome, where IC binds.

Let us consider another pair of wages

$$w_H^{\mathrm{SB}'} = g + 2\frac{1-\lambda_1}{\lambda_1 - \lambda_0} g \text{ and } w_L^{\mathrm{SB}'} = g - 2\frac{\lambda_1}{\lambda_1 - \lambda_0} g.$$

Clearly, IR binds and IC is strictly satisfied.

Indeed, there are infinitely many solutions.

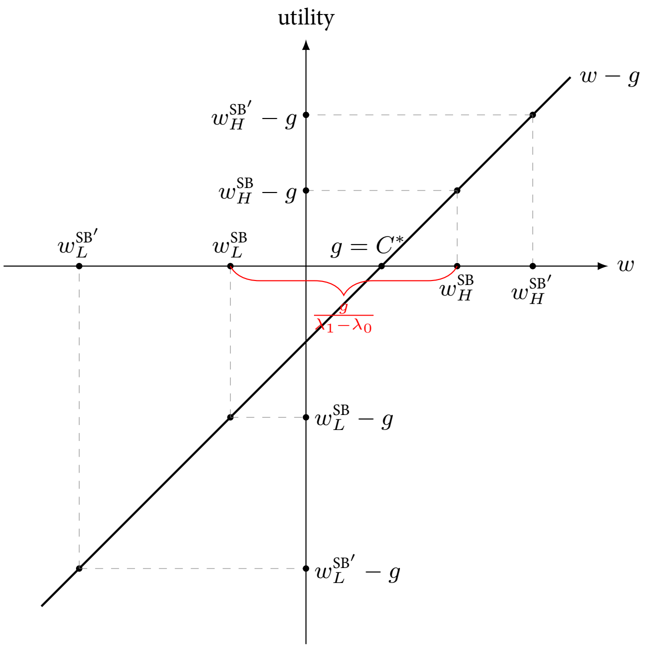
31 Graphic illustration:

(1) $w - g$ is the agent's utility function when he exerts effort.

(2) In the complete information case, the agent's utility is zero, and the wage is always g.

(3) Since IR binds, the contract $(w_H^{\mathrm{SB}}, w_L^{\mathrm{SB}})$ makes the agent's expected utility be zero, shown as in the graph. That is, $\lambda_1 w_H^{\mathrm{SB}} + (1-\lambda_1) w_L^{\mathrm{SB}} - g = 0$.

(4) The expected wage should be $\lambda_1 w_H^{\mathrm{SB}} + (1-\lambda_1) w_L^{\mathrm{SB}} = g$.



(5) To induce the agent to exert effort, the principal needs to set w_H and w_L to satisfy $(\lambda_1 - \lambda_0)(w_H - w_L) \geq g$. That is, $w_H - w_L$ should be at least $\frac{g}{\lambda_1 - \lambda_0}$.
为了能够激励代理人付出努力，需要将 w_H 和 w_L 的差距拉大。

(6) IC could not bind: the principal can increase w_H^{SB} to $w_H^{SB'}$ and decrease w_L^{SB} to $w_L^{SB'}$ such that the expected wage $\lambda_1 w_H^{SB'} + (1 - \lambda_1) w_L^{SB'} = g$.

32 Had the principal decided to let the agent exert no effort ($e = 0$), the principal's problem is

$$
\begin{aligned}
\underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_0(\pi_H - w_H) + (1 - \lambda_0)(\pi_L - w_L) \\
\text{subject to} \quad & \lambda_0 w_H + (1 - \lambda_0) w_L \geq \lambda_1 w_H + (1 - \lambda_1) w_L - g \\
& \lambda_0 w_H + (1 - \lambda_0) w_L \geq 0.
\end{aligned}
$$

Thus, principal would make the following payment:

- $w_H^{SB} = g + \frac{1-\lambda_1}{\lambda_1 - \lambda_0} g - \epsilon_1$ and $w_L^{SB} = g - \frac{\lambda_1}{\lambda_1 - \lambda_0} g + \epsilon_2$, or
- zero payment to the agent whatever the realization of profit.

The expected profit is

$$V_0 = \lambda_0 \pi_H + (1 - \lambda_0) \pi_L.$$

33 The optimal outcome calls for $e^* = 1$ if and only if $V_1^{SB} \geq V_0$, i.e.,

$$(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq g = u^{-1}(g).$$

Therefore, we have shown: Moral hazard is not an issue with a risk-neutral agent despite the nonobservability of effort. The first-best level of effort is still implemented.

34 The principal can costlessly structure the agent's payment so that the agent has the right incentives to exert effort. Indeed, by increasing effort from $e = 0$ to $e = 1$, the agent receives the wage w_H^{SB} more often than the wage w_L^{SB}.

His expected gain from exerting effort is thus $(\lambda_1 - \lambda_0)(w_H^{\text{SB}} - w_L^{\text{SB}}) = g$, i.e., it exactly compensates the agent for the extra disutility of effort that he incurs when increasing his effort from $e = 0$ to $e = 1$.

35 Suppose that $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq g$. Then the optimal outcome is $e^* = 1$.

Let us consider a pair of wages

$$w_H^{\text{SB}''} = \pi_H - T_1 \text{ and } w_L^{\text{SB}''} = \pi_L - T_1,$$

where T_1 is an up-front payment made by the agent before output realizes.

These wages satisfy the agent's IC constraint since:

$$(\lambda_1 - \lambda_0)(w_H^{\text{SB}''} - w_L^{\text{SB}''}) = (\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq g.$$

The up-front payment T can be adjusted by the principal to have the agent's IR constraint be binding:

$$T_1 = \lambda_1 \pi_H + (1 - \lambda_1)\pi_L - g.$$

With the wages $w_H^{\text{SB}''}$ and $w_L^{\text{SB}''}$, the agent becomes residual claimant for the profit of the firm. The up-front payment T_1 is precisely equal to this expected profit. The principal chooses this ex ante payment to reap all gains from delegation.

此处相当于委托人将项目出售给代理人，价格是其预期净利润 T_1。代理人自行管理这个项目，其收入就是项目最终的利润。

36 Suppose that $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \leq g$. Then the optimal outcome is $e^* = 0$.

Let us consider a pair of wages

$$w_H^{\text{SB}''} = \pi_H - T_0 \text{ and } w_L^{\text{SB}''} = \pi_L - T_0,$$

where T_0 is an up-front payment made by the agent before output realizes.

These wages satisfy the agent's IC constraint since:

$$(\lambda_1 - \lambda_0)(w_H^{\text{SB}''} - w_L^{\text{SB}''}) = (\lambda_1 - \lambda_0)(\pi_H - \pi_L) \leq g.$$

The up-front payment T_0 can be adjusted by the principal to have the agent's IR constraint be binding:

$$T_0 = \lambda_0 \pi_H + (1 - \lambda_0)\pi_L.$$

With the wages $w_H^{\text{SB}''}$ and $w_L^{\text{SB}''}$, the agent becomes residual claimant for the profit of the firm. The up-front payment T_0 is precisely equal to this expected profit. The principal chooses this ex ante payment to reap all gains from delegation.

4 Incomplete information with limited liability

37 Clearly, in an optimal contract, w_L has a upper bound: $w_L \leq -\frac{\lambda_0}{\Delta\lambda} g$.

In many situation, it also has a lower bound: the responsibility is limited.

38 Let us consider a risk-neutral agent. Let us also assume that the agent's wage must always be greater than some exogenous level $-l$, with $l \geq 0$.

Limited liability in both states are thus written as

$$w_H \geq -l \text{ and } w_L \geq -l.$$

39 The principal's problem is

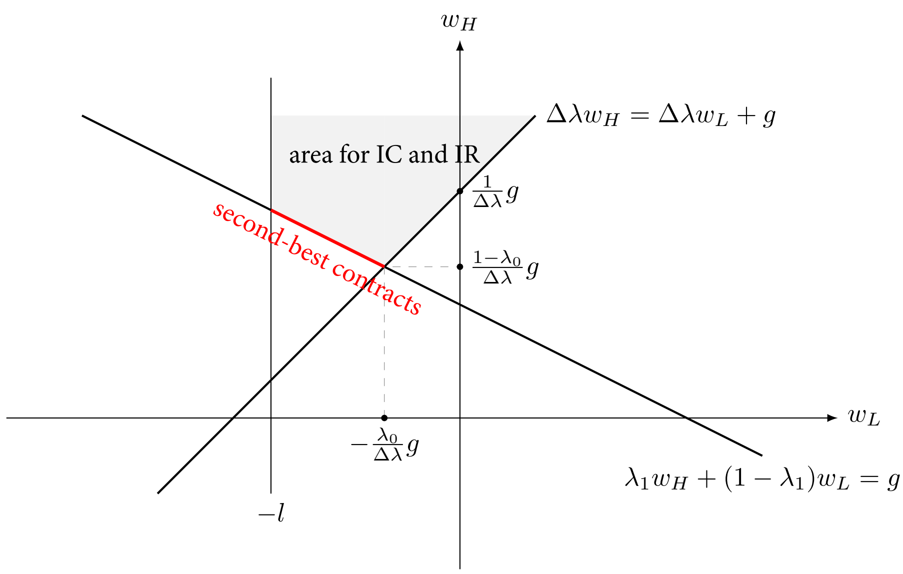
$$\begin{aligned}
\underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_1(\pi_H - w_H) + (1-\lambda_1)(\pi_L - w_L) \\
\text{subject to} \quad & \lambda_1 w_H + (1-\lambda_1) w_L - g \geq \lambda_0 w_H + (1-\lambda_0) w_L \\
& \lambda_1 w_H + (1-\lambda_1) w_L - g \geq 0 \\
& w_H \geq -l \\
& w_L \geq -l
\end{aligned}$$

40 For $l > \frac{\lambda_0}{\Delta\lambda} g$, the first-best outcome can be implemented, and one optimal wages are

$$w_H^{\text{SB}} = g + \frac{1-\lambda_1}{\lambda_1 - \lambda_0} g \text{ and } w_L^{\text{SB}} = g - \frac{\lambda_1}{\lambda_1 - \lambda_0} g.$$

In this case, the agent has no expected limited liability rent.

41 Graphic illustration:



42 For $0 \leq l \leq \frac{\lambda_0}{\Delta\lambda} g$, we conjecture that the IC condition and the limited liability condition for low effort are only relevant constraints.

(1) The limited liability condition for high effort is obviously irrelevant (IC implies $w_H \geq \frac{g}{\Delta\lambda} + w_L$).

(2) The IR condition is also irrelevant:

$$\lambda_1 w_H + (1-\lambda_1) w_L - g \geq \lambda_1\left(-l + \frac{g}{\Delta\lambda}\right) + (1-\lambda_1)(-l) - g = \frac{\lambda_0}{\Delta\lambda} g - l \geq 0.$$

(3) Since the principal is willing to minimize the wages made to the agent, both constraints must be binding.

(4) Therefore,

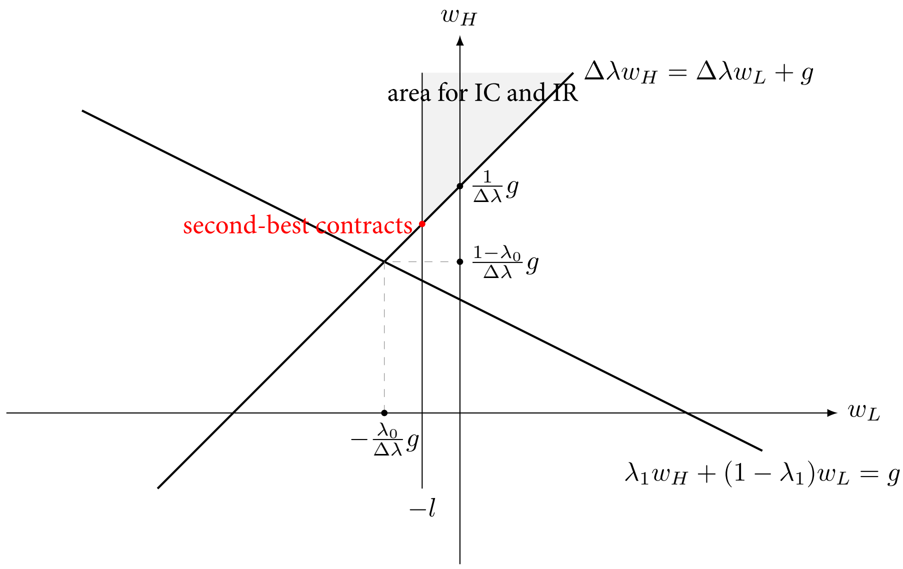
$$w_H^{\mathrm{SB}} = -l + \frac{g}{\Delta\lambda} \text{ and } w_L^{\mathrm{SB}} = -l.$$

In this case, the agent's expected limited liability rent is non-negative:

$$\lambda_1 w_H^{\mathrm{SB}} + (1-\lambda_1) w_L^{\mathrm{SB}} - g = -l + \frac{\lambda_0}{\Delta\lambda} g > 0.$$

这个租金源于道德风险和有限责任的共同作用所导致的委托人对代理人的额外支付。

43 Graphic illustration:



44 Remark:

- Only the limited liability constraint for the bad state may be binding.
- When the limited liability constraint for the bad state is binding, the principal is limited in his punishments to induce effort.

  The principal has to increases awards when high production is realized to induce high effort.

  As a result, the agent receives a non-negative ex ante limited liability rent. Compared with the case without limited liability, this rent is actually the additional payment that the principal must incur because of the conjunction of moral hazard and limited liability.
- As the agent is endowed with more assets, i.e., as l gets larger, the conflict between moral hazard and limited liability diminishes and then disappears whenever l is large enough. In this case, the agent avoids bankruptcy even when he has to pay the optimal penalty to the principal in the bad state of nature.

45 For the sake of simplicity, we assume $l = 0$.

When the principal induces positive effort from the agent, the optimal contract is

$$w_H^{\mathrm{SB}} = \frac{g}{\Delta\lambda} \text{ and } w_L^{\mathrm{SB}} = 0,$$

and his expected utility is

$$V_1^{\mathrm{SB}} = \lambda_1 \pi_H + (1-\lambda_1)\pi_L - \frac{\lambda_1}{\Delta\lambda} g.$$

When the principal gives up the goal of inducing effort from the agent, he can choose $w_H = w_L = 0$ and instead obtain the expected utility level

$$V_0 = \lambda_0 \pi_H + (1 - \lambda_0)\pi_L.$$

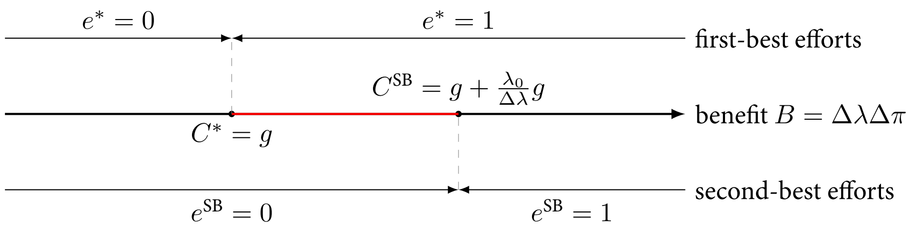
It is worth inducing effort if $V_1^{\text{SB}} \geq V_0$, i.e., when

$$\Delta\lambda\Delta\pi \geq \frac{\lambda_1}{\Delta\lambda} g = g + \frac{\lambda_0}{\Delta\lambda} g.$$

The left-hand side is the gain of inducing effort, i.e., the gain of increasing the probability of a high production level. The right-hand side is instead the second-best cost C^{SB} of inducing effort, which is the disutility of effort g plus the limited liability rent $\frac{\lambda_0}{\Delta\lambda} g$. This second-best cost of implementing effort obviously exceeds the first-best cost. It is clear that the limited liability and moral hazard together make it more costly to induce effort.

46 Summary ($l = 0$ and $\Delta\lambda\Delta\pi \geq C^{\text{SB}} = g + \frac{\lambda_0}{\Delta\lambda} g$):

- There is conflict between moral hazard and limited liability.
- IR does not bind. IC binds and limited liability for bad state binds.
- The agent has a positive expected utility $\frac{\lambda_0}{\Delta\lambda} g$.
- Efficiency loses since $C^{\text{SB}} = g + \frac{\lambda_0}{\Delta\lambda} > g = C^*$. The loss part $\frac{\lambda_0}{\Delta\lambda}$ is the limited liability rent for the agent, which is paid by the principal.



5 Incomplete information with risk-averse agent

47 Assume that the agent is risk-averse.

48 We also consider this problem in two steps:

- If the principal wants the agent to exert positive effort (or zero effort), what is the best contract (w_L, w_H)?
- What is the best choice for the principal, inducing the agent to exert positive effort or zero effort?

49 To induce the agent to exert effort, the principal's program is written as:

$$\begin{aligned} \underset{(w_H, w_L)}{\text{maximize}} \quad & \lambda_1(\pi_H - w_H) + (1 - \lambda_1)(\pi_L - w_L) \\ \text{subject to} \quad & \lambda_1 u(w_H) + (1 - \lambda_1)u(w_L) - g \geq \lambda_0 u(w_H) + (1 - \lambda_0)u(w_L) \\ & \lambda_1 u(w_H) + (1 - \lambda_1)u(w_L) - g \geq 0. \end{aligned}$$

50 Let $u_H = u(w_H)$ and $u_L = u(w_L)$. Then the principal's program can be written as:

$$\begin{aligned} \underset{(u_H, u_L)}{\text{maximize}} \quad & \lambda_1(\pi_H - h(u_H)) + (1-\lambda_1)(\pi_L - h(u_L)) \\ \text{subject to} \quad & \lambda_1 u_H + (1-\lambda_1)u_L - g \geq \lambda_0 u_H + (1-\lambda_0)u_L \\ & \lambda_1 u_H + (1-\lambda_1)u_L - g \geq 0. \end{aligned}$$

Note that the principal's objective function is now strictly concave in (u_H, u_L) because h is strictly convex. The constraints are now linear and the interior of the constrained set is obviously nonempty, and therefore it is a concave problem, with the Kuhn and Tucker conditions being sufficient and necessary for characterizing optimality.

51 Letting γ and μ be the non-negative multipliers associated respectively with the constraints, the first-order conditions of this program can be expressed as

$$\begin{aligned} -\lambda_1 h'(u_H^{\text{SB}}) + \gamma(\lambda_1 - \lambda_0) + \mu\lambda_1 &= -\frac{\lambda_1}{u'(w_H^{\text{SB}})} + \gamma(\lambda_1 - \lambda_0) + \mu\lambda_1 = 0 \\ -(1-\lambda_1)h'(u_L^{\text{SB}}) - \gamma(\lambda_1 - \lambda_0) + \mu(1-\lambda_1) &= -\frac{1-\lambda_1}{u'(w_L^{\text{SB}})} - \gamma(\lambda_1 - \lambda_0) + \mu(1-\lambda_1) = 0, \end{aligned}$$

where w_H^{SB} and w_L^{SB} are the second-best optimal wages.

52 Rearranging terms, we get

$$\frac{1}{u'(w_H^{\text{SB}})} = \mu + \gamma\frac{\lambda_1 - \lambda_0}{\lambda_1} \text{ and } \frac{1}{u'(w_L^{\text{SB}})} = \mu - \gamma\frac{\lambda_1 - \lambda_0}{1-\lambda_1}.$$

Multiplying the left equation by λ_1 and the right equation by $1-\lambda_1$, and then adding those two modified equations, we obtain

$$\mu = \frac{\lambda_1}{u'(w_H^{\text{SB}})} + \frac{1-\lambda_1}{u'(w_L^{\text{SB}})} > 0.$$

Hence, the IR condition is binding.

53 The IC condition implies

$$u_H^{\text{SB}} - u_L^{\text{SB}} \geq \frac{g}{\lambda_1 - \lambda_0} > 0,$$

and thus $w_H^{\text{SB}} > w_L^{\text{SB}}$.

Therefore,

$$\gamma = \frac{\lambda_1(1-\lambda_1)}{\lambda_1 - \lambda_0}\left(\frac{1}{u'(w_H^{\text{SB}})} - \frac{1}{u'(w_L^{\text{SB}})}\right) > 0,$$

and hence the IC condition is also binding.

54 Since the IR and IC conditions are binding, we have

$$u_H^{\text{SB}} = g + \frac{1-\lambda_1}{\lambda_1 - \lambda_0}g \text{ and } u_L^{\text{SB}} = g - \frac{\lambda_1}{\lambda_1 - \lambda_0}g,$$

and hence

$$w_H^{\text{SB}} = h\left(g + \frac{1-\lambda_1}{\lambda_1 - \lambda_0}g\right) \text{ and } w_L^{\text{SB}} = h\left(g - \frac{\lambda_1}{\lambda_1 - \lambda_0}g\right).$$

55 The agent receives more than the complete information wage when a high output is realized, $w_H^{\text{SB}} > h(g)$. When a low output is realized, the agent instead receives less than the complete information wage, $w_L^{\text{SB}} < h(g)$.

A risk premium must be paid to the risk-averse agent to induce his participation since he now incurs a risk by the fact that $w_L^{\text{SB}} < w_H^{\text{SB}}$. Indeed, we have

$$g = \lambda_1 u(w_H^{\text{SB}}) + (1-\lambda_1)u(w_L^{\text{SB}}) < u\left(\lambda_1 w_H^{\text{SB}} + (1-\lambda_1)w_L^{\text{SB}}\right),$$

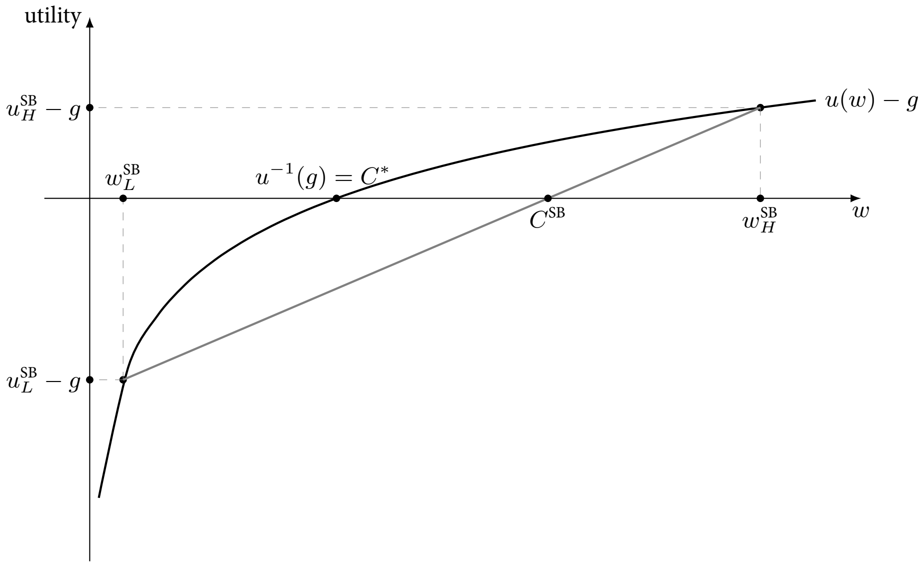
where the inequality follows from Jensen's inequality. That is, the expected payment $\lambda_1 w_H^{\text{SB}} + (1-\lambda_1)w_L^{\text{SB}}$ given by the principal is thus larger than the first-best cost $h(g)$, which is incurred by the principal when effort is observable.
为了保证代理人获得保留效用，委托人就需要花费更多的奖励。

56 The second-best cost of inducing effort under moral hazard is the expected payment made to the agent

$$C^{\text{SB}} = \lambda_1 w_H^{\text{SB}} + (1-\lambda_1)w_L^{\text{SB}} = \lambda_1 h\left(g + \frac{1-\lambda_1}{\lambda_1-\lambda_0}g\right) + (1-\lambda_1)h\left(g - \frac{\lambda_1}{\lambda_1-\lambda_0}g\right) > h(g) = C^*,$$

where the inequality follows from Jensen's inequality (h is strictly convex).

57 Graphic illustration:



(1) $u(w) - g$ is the agent's utility function when he exerts effort.

(2) In the complete information case, the agent's utility is zero, and the wage is always $u^{-1}(g)$.

(3) Since IR binds, the contract $(w_H^{\text{SB}}, w_L^{\text{SB}})$ makes the agent's expected utility be zero, shown as in the graph. That is, $\lambda_1 u(w_H^{\text{SB}}) + (1-\lambda_1)u(w_L^{\text{SB}}) - g = 0$.

(4) The expected wage should be $\lambda_1 w_H^{\text{SB}} + (1-\lambda_1)w_L^{\text{SB}} = C^{\text{SB}}$.

(5) Since u is concave, $C^{\text{SB}} > C^*$.

(6) To induce the agent to exert effort, the principal needs to set w_H and w_L to satisfy $(\lambda_1-\lambda_0)\big(u(w_H) - u(w_L)\big) \geq g$. That is, $w_H - w_L$ should be sufficiently large.
为了能够激励代理人付出努力，需要将 w_H 和 w_L 的差距拉大。

(7) IC should be binding; otherwise, the principal can decrease w_H and increase w_L, so that the expected wage $\lambda_1 w_H + (1-\lambda_1)w_L$ decreases.
委托人只会将 w_H 和 w_L 的差距拉大到恰好能够激励代理人付出努力的程度。

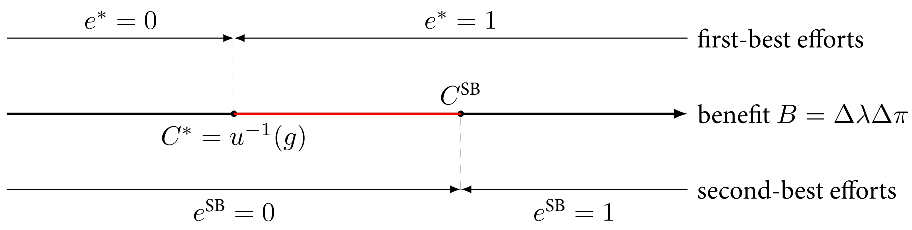
58 Had the principal decided to let the agent exert no effort, $e = 0$, he would (optimally) make a zero payment to the agent whatever the realization of profit. The profit is $\lambda_0 \pi_H + (1 - \lambda_0)\pi_L$.

59 The benefit of inducing effort is still $(\lambda_1 - \lambda_0)(\pi_H - \pi_L)$, and a positive effort $e^* = 1$ is the optimal choice of the principal whenever

$$(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq C^{\text{SB}} > C^*.$$

60 Summary (given $(\lambda_1 - \lambda_0)(\pi_H - \pi_L) \geq C^{\text{SB}}$):

- The agent's utility is always zero.
- The principal sets $w_H^{\text{SB}} > w_L^{\text{SB}}$ to induce the agent to exert effort.
- Efficiency loses since $C^{\text{SB}} > C^*$, which is paid by the principal (“蒸发”掉了).



6 A continuum of profits

61 We assume that profit π is drawn from a distribution $F(\cdot \mid e)$ on the support $[\underline{\pi}, \bar{\pi}]$.

This distribution is conditional on the agent's effort $e \in \{0, 1\}$. We denote by $f(\cdot \mid e)$ the density corresponding to the distribution $F(\cdot \mid e)$.

62 Complete information:

To induce $e = 1$, the principal's problem is

$$\begin{aligned} \underset{w(\pi)}{\text{maximize}} \quad & \int [\pi - w(\pi)] f(\pi \mid 1) \, \mathrm{d}\pi \\ \text{subject to} \quad & \int u(w(\pi)) f(\pi \mid 1) \, \mathrm{d}\pi - g \geq 0 \end{aligned}$$

Denoting the multipliers by γ. Optimizing pointwise with respect to w yields

$$-f(\pi \mid 1) + \gamma u'(w(\pi)) f(\pi \mid 1) = 0.$$

Thus, $\gamma = \frac{1}{u'(w(\pi))} > 0$ and the wage is constant. It implies that $w^* = u^{-1}(g)$, which is the same as the two-profit case. The profit is

$$\int \pi f(\pi \mid 1) \, \mathrm{d}\pi - u^{-1}(g).$$

Had the principal decided to let the agent exert no effort, $e = 0$, he would (optimally) make a zero payment to the agent whatever the realization of profit. The payoff is $\int \pi f(\pi \mid 0) \, \mathrm{d}\pi$.

$e^* = 1$ is the optimal choice of principal if and only if

$$\int \pi f(\pi \mid 1) \, \mathrm{d}\pi - u^{-1}(g) \geq \int \pi f(\pi \mid 0) \, \mathrm{d}\pi.$$

63 In an environment with incomplete information, a contract $w(\pi)$ inducing a positive effort must satisfy the IC constraint

$$\int u(w(\pi))f(\pi \mid 1)\,\mathrm{d}\pi - g \geq \int u(w(\pi))f(\pi \mid 0)\,\mathrm{d}\pi,$$

and the IR constraint

$$\int u(w(\pi))f(\pi \mid 1)\,\mathrm{d}\pi - g \geq 0.$$

64 Incomplete information with a risk-neutral agent.

(1) To induce $e = 1$, the principal's problem is

$$\begin{aligned} \underset{w(\pi)}{\text{maximize}} \quad & \int [\pi - w(\pi)]f(\pi \mid 1)\,\mathrm{d}\pi \\ \text{subject to} \quad & \int w(\pi)f(\pi \mid 1)\,\mathrm{d}\pi - g \geq 0 \\ & \int w(\pi)f(\pi \mid 1)\,\mathrm{d}\pi - g \geq \int w(\pi)f(\pi \mid 0)\,\mathrm{d}\pi \end{aligned}$$

Principal can set $w(\pi) = \pi - \int \pi f(\pi \mid 1)\,\mathrm{d}\pi + g$. The expected payoff is $\int \pi f(\pi \mid 1)\,\mathrm{d}\pi - g$.

(2) To induce $e = 0$, the principal's problem is

$$\begin{aligned} \underset{w(\pi)}{\text{maximize}} \quad & \int [\pi - w(\pi)]f(\pi \mid 0)\,\mathrm{d}\pi \\ \text{subject to} \quad & \int w(\pi)f(\pi \mid 0)\,\mathrm{d}\pi \geq 0 \\ & \int w(\pi)f(\pi \mid 0)\,\mathrm{d}\pi \geq \int w(\pi)f(\pi \mid 1)\,\mathrm{d}\pi - g \end{aligned}$$

Principal can set $w(\pi) = 0$ or $w(\pi) = \pi - \int \pi f(\pi \mid 0)\,\mathrm{d}\pi$. The expected payoff is $\int \pi f(\pi \mid 0)\,\mathrm{d}\pi$.

(3) $e = 1$ is the optimal of principal if and only if

$$\int \pi f(\pi \mid 1)\,\mathrm{d}\pi - g \geq \int \pi f(\pi \mid 0)\,\mathrm{d}\pi.$$

65 Incomplete information with a risk-averse agent.

(1) To induce $e = 1$, the principal's problem is

$$\begin{aligned} \underset{w(\pi)}{\text{maximize}} \quad & \int [\pi - w(\pi)]f(\pi \mid 1)\,\mathrm{d}\pi \\ \text{subject to} \quad & \int u(w(\pi))f(\pi \mid 1)\,\mathrm{d}\pi - g \geq 0 \\ & \int u(w(\pi))f(\pi \mid 1)\,\mathrm{d}\pi - g \geq \int u(w(\pi))f(\pi \mid 0)\,\mathrm{d}\pi \end{aligned}$$

Denoting the multipliers by γ and μ, respectively, the Lagrangian writes as

$$[\pi - w(\pi)]f(\pi \mid 1) + \gamma[u(w)[f(\pi \mid 1) - f(\pi \mid 0)] - g] + \mu[u(w)f(\pi \mid 1) - g].$$

Optimizing pointwise with respect to w yields

$$\frac{1}{u'(w^{\mathrm{SB}}(\pi))} = \mu + \gamma\left[1 - \frac{f(\pi \mid 1)}{f(\pi \mid 0)}\right].$$

We can verify that $\gamma > 0$ and $\mu > 0$. Then

$$u\left(\int w^{\text{SB}}(\pi) f(\pi \mid 1)\,\mathrm{d}\pi\right) > \int u(w^{\text{SB}}(\pi)) f(\pi \mid 1)\,\mathrm{d}\pi = g.$$

That is, the expected wage $C^{\text{SB}} = \int w^{\text{SB}}(\pi) f(\pi \mid 1)\,\mathrm{d}\pi$ is larger than $u^{-1}(g) = C^*$.

(2) To induce $e = 0$, the principal's problem is

$$\begin{aligned} \underset{w(\pi)}{\text{maximize}} \quad & \int [\pi - w(\pi)] f(\pi \mid 0)\,\mathrm{d}\pi \\ \text{subject to} \quad & \int u(w(\pi)) f(\pi \mid 0)\,\mathrm{d}\pi \geq 0 \\ & \int u(w(\pi)) f(\pi \mid 0)\,\mathrm{d}\pi \geq \int u(w(\pi)) f(\pi \mid 1)\,\mathrm{d}\pi - g \end{aligned}$$

Principal can set $w(\pi) = 0$. The expected payoff is $\int \pi f(\pi \mid 0)\,\mathrm{d}\pi$.

(3) $e = 1$ is optimal if and only if

$$\int \pi f(\pi \mid 1)\,\mathrm{d}\pi - C^{\text{SB}} \geq \int \pi f(\pi \mid 0)\,\mathrm{d}\pi.$$

7 Homework

- Key: Optimal contract.
- Reading: 14.B
- Homework: 14.B.4