Advanced Microeconomics I: Lecture Notes 6

1 We consider a simple case that the cost function $g(e, \theta) = \theta e$.

- θ_H: high cost (low ability);
- θ_L: low cost (high ability), with probability λ;
- $\theta_H > \theta_L$.
- Single crossing property still holds.

We assume $\bar{u} = 0$ for simplicity.

2 Principal's income function is still $\pi(\cdot)$, with $\pi(0) = 0$, $\pi'(e) > 0$, and $\pi''(e) < 0$ for all $e \in [0, \infty)$.

3 The first-best contracts $\{(e_L^*, w_L^*), (e_H^*, w_H^*)\}$ are

- $\pi'(e_L^*) = \theta_L$;
- $\pi'(e_H^*) = \theta_H$.

$\Rightarrow$ $e_L^* > e_H^*$ since $\pi'' < 0$.

- $w_L^* = \theta_L e_L^*$.
- $w_H^* = \theta_H e_H^*$.

1 Shutdown

4 Proposition: Under asymmetric information, the optimal menu of contracts entails:

- No output distortion for the high-ability agent with respect to the first-best, $e_L^{\text{SB}} = e_L^*$. A downward output distortion for the low-ability agent, $e_H^{\text{SB}} < e_H^*$ with

$$\pi'(e_H^{\text{SB}}) = \theta_H + \frac{\lambda}{1-\lambda}(\theta_H - \theta_L).$$

- The second-best wages are respectively given by

$$w_L^{\text{SB}} = \theta_L e_L^{\text{SB}} + \underbrace{e_H^{\text{SB}}(\theta_H - \theta_L)}_{r_H} > \theta_L e_L^{\text{SB}} = w_L^*,$$
$$w_H^{\text{SB}} = \theta_H e_H^{\text{SB}} < \theta_H e_H^* = w_H^*.$$

Moreover, $w_L^{\text{SB}} = \theta_L e_L^{\text{SB}} + e_H^{\text{SB}}(\theta_H - \theta_L) = e_H^{\text{SB}}\theta_H + \theta_L(e_L^{\text{SB}} - e_H^{\text{SB}}) > w_H^{\text{SB}}$.

- Only the high-ability agent gets a positive information rent given by

$$r_L^{\text{SB}} = e_H^{\text{SB}}(\theta_H - \theta_L).$$

5 Graphical illustration:

Starting from the complete information optimal contract (A^*, B^*) that is not incentive compatible, we can construct an incentive compatible contract (B^*, C) with the same effort levels by giving a higher wage to the agent producing q_L^* (Figure 1).

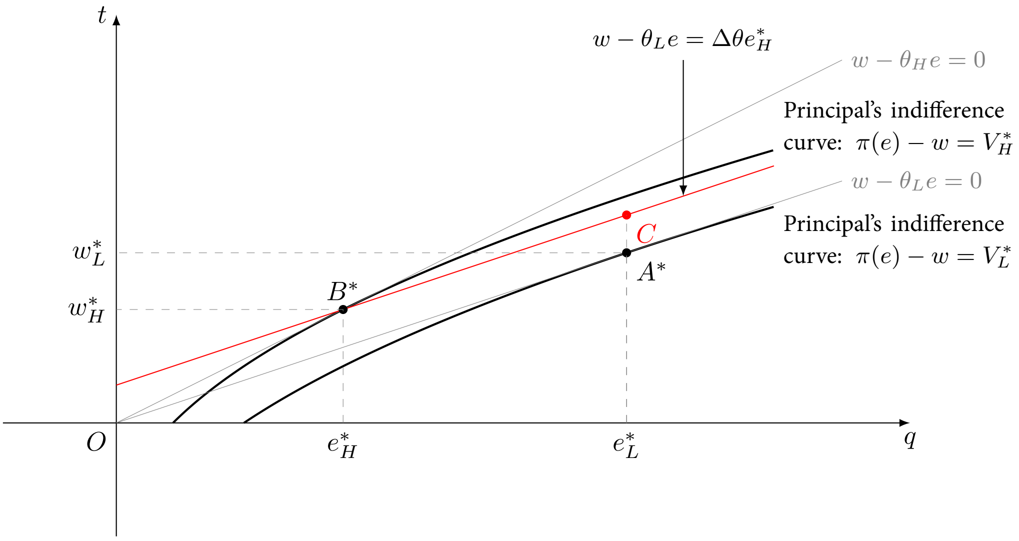

Figure 1: Rent needed to implement the first-best outputs

The contract C is on the θ_L-agent's indifference curve passing through B^*. Hence, the θ_L-agent is now indifferent between B^* and C. (B^*, C) becomes an incentive-compatible menu of contracts. The rent that is given up to the θ_L-agent is now $\Delta\theta e_H^*$.

Rather than insisting on the first-best production level e_H^* for an inefficient type, the principal prefers to slightly decrease e_H by an amount $\mathrm{d}e$.

- By doing so, expected efficiency is just diminished by a second-order term $\frac{1}{2}|\pi''(e_H^*)|(\mathrm{d}e)^2$ since e_H^* is the first-best output that maximizes efficiency when the agent is inefficient.
- Instead, the information rent left to the efficient type diminishes to the first-order $\Delta\theta\,\mathrm{d}e$.

Of course, the principal stops reducing the inefficient type's output when a further decrease would have a greater efficiency cost than the gain in reducing the information rent it would bring about. The optimal trade-off finally occurs at $(A^{\text{SB}}, B^{\text{SB}})$ as shown in Figure 2.

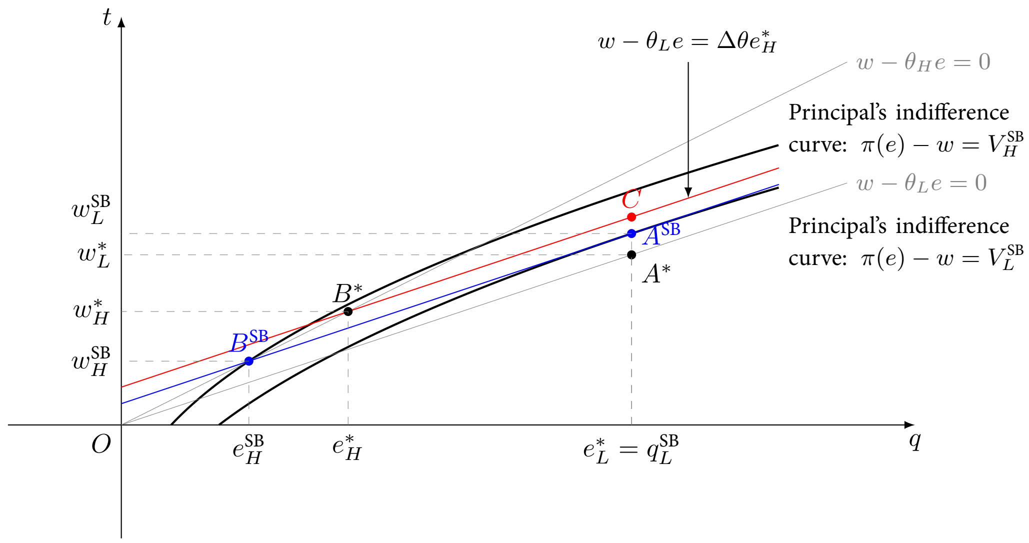

Figure 2: Second-best contracts

6 信息租金取决于 e_H^{SB} 和 $\theta_H - \theta_L$.

- 之所以降低 θ_H 的努力程度，是为了尽可能减少支付给 θ_L 的信息租金，从而有一个更好的利润。
- Principal 扭曲的 θ_H 的努力程度，依赖于两种 agent 之间的差异。
  - 当 $\theta_H - \theta_L \to 0$ 时，θ_L 的信息租金趋于零，此时 θ_H 会趋于有效的努力程度。
  - 而当 $\theta_H - \theta_L \to \infty$ 时，θ_L 的信息租金趋于无穷大，此时 principal 会采取将 θ_H 停工的排斥性合约，以避免支付高额的信息租金。

7 The above proposition holds when $\pi'(e_H^{\mathrm{SB}}) = \theta_H + \frac{\lambda}{1-\lambda}(\theta_H - \theta_L)$ has a positive solution.

If $\pi'(e_H^{\mathrm{SB}}) = \theta_H + \frac{\lambda}{1-\lambda}(\theta_H - \theta_L)$ has no positive solution (for example, when λ is close to 1, or when $\theta_H - \theta_L$ is sufficiently large), e_H^{SB} should be set at zero, and w_H^{SB} will thus be set at zero as well—it is the special case of a contract with shutdown.

8 When the shutdown of θ_H agents occurs, the contract offered to θ_L agents is

$$e_L^{\mathrm{SB}} = e_L^* \text{ and } w_L^{\mathrm{SB}} = w_L^*.$$

The information rent for θ_L agents is zero.

9 直觉：

- 如果 θ_L 的比例很大（λ 接近于 1），导致方程没有正数解：若给 θ_H 提供非零合约，或者说提高 θ_H 的配置效率，则甄别中需要支付给 θ_L 过多的信息租金，对于 principal 并不划算。
- 如果两种 agent 的差异较大（$\theta_H - \theta_L$ 很大），导致方程没有正数解：若给 θ_H 提供非零合约，则甄别中需要支付给 θ_L 过多的信息租金，principal 也会选择不给 θ_H 提供合约。

10 With such a policy, a significant inefficiency emerges because the inefficient type θ_H does not make effort. The benefit of such a policy is that no rent is given up to the efficient type θ_L.

11 To guarantee the contracts without shutdown, we assume that

- $\pi'(0) = \infty$ (Inada condition).

- $\lim_{e\to\infty} \pi'(e)e = 0$.
- Since $\pi'(0) = \infty$, $\pi'(e_H^{\mathrm{SB}}) = \theta_H + \frac{\lambda}{1-\lambda}(\theta_H - \theta_L)$ has a positive solution.
- Besides, principal is not optimal to offer contracts with shutdown:
  (1) The profit of principal for optimal contracts without shutdown is
$$\lambda(\pi(e_L^{\mathrm{SB}}) - \theta_L e_L^{\mathrm{SB}} - \Delta\theta e_H^{\mathrm{SB}}) + (1-\lambda)(\pi(e_H^{\mathrm{SB}}) - \theta_H e_H^{\mathrm{SB}}).$$
  (2) The profit of principal for optimal contracts with shutdown is
$$\lambda(\pi(e_L^*) - \theta_L e_L^*).$$
  (3) Since $e_L^* = e_L^{\mathrm{SB}}$, the difference is
$$(1-\lambda)(\pi(e_H^{\mathrm{SB}}) - \theta_H e_H^{\mathrm{SB}}) - \lambda\Delta\theta e_H^{\mathrm{SB}} = (1-\lambda)\Big[\pi(e_H^{\mathrm{SB}}) - \underbrace{\Big(\theta_H + \frac{\lambda}{1-\lambda}\Delta\theta\Big)}_{\pi'(e_H^{\mathrm{SB}})} e_H^{\mathrm{SB}}\Big].$$
  (4) We can rewrite $\pi(e_H^{\mathrm{SB}}) - \Big(\theta_H + \frac{\lambda}{1-\lambda}\Delta\theta\Big)e_H^{\mathrm{SB}}$ as
$$\pi(e_H^{\mathrm{SB}}) - \pi'(e_H^{\mathrm{SB}})e_H^{\mathrm{SB}},$$
  which is strictly positive since $\pi(e) - \pi'(e)e$ is strictly increasing with e and is equal to zero for $e = 0$.
  (5) Hence, $\pi(e_H^{\mathrm{SB}}) - \pi'(e_H^{\mathrm{SB}})e_H^{\mathrm{SB}} > 0$, and shutdown of θ_H does not occur.

2 Nonresponsiveness

12 We assume that the principal's return π depends also on θ: $\pi(e, \theta)$.

13 Assumptions:

- $\pi_e(e, \theta) > 0$,
- $\pi_{ee}(e, \theta) < 0$,
- $\pi_{e\theta}(e, \theta) > 1$: the marginal value of the principal increases faster than the type of agent.

14 The first-best efforts θ_L^* and θ_H^* are now given by
$$\pi_e(e_L^*, \theta_L) = \theta_L \text{ and } \pi_e(e_H^*, \theta_H) = \theta_H.$$

15 Consider the first order condition $\pi_e(e^*(\theta), \theta) = \theta$. We have
$$\pi_{ee}(e^*(\theta), \theta)\frac{\mathrm{d}e^*(\theta)}{\mathrm{d}\theta} + \pi_{e\theta}(e^*(\theta), \theta) = 1.$$
It leads to
$$\frac{\mathrm{d}e^*(\theta)}{\mathrm{d}\theta} = \frac{1 - \pi_{e\theta}(e^*(\theta), \theta)}{\pi_{ee}(e^*(\theta), \theta)} = \frac{-}{-} > 0.$$
Thus, $e_H^* > e_L^*$—it does not satisfy the monotonicity condition for IC contracts.

16 Conflict:

- For efficiency, principal want θ_H agents to produce more;
- For incentive compatibility, θ_L agents has to produce (weakly) more (monotonicity constraint).

It is called a phenomenon of nonresponsiveness.

17 This phenomenon makes screening of types quite difficult.

Let $e_L^{\mathrm{SB}} = e_L^*$ and e_H^{SB} be defined by

$$\pi_e(e_H^{\mathrm{SB}}, \theta_H) = \theta_H + \frac{\lambda}{1-\lambda}(\theta_H - \theta_L).$$

By incentive compatibility, screening only possible when $e_L^{\mathrm{SB}} > e_H^{\mathrm{SB}}$.

18 If λ is very small, e_H^{SB} is very close to e_H^*. We thus have $e_H^{\mathrm{SB}} \sim e_H^* > e_L^* = e_L^{\mathrm{SB}}$.

It means that the screening is impossible. It forces the principal to use a pooling contract.

19 The principal's problem is to solve

$$\begin{aligned} \underset{(e^p, w^p)}{\text{maximize}} \quad & \lambda\big(\pi(e^p, \theta_L) - w^p\big) + (1-\lambda)\big(\pi(e^p, \theta_H) - w^p\big) \\ \text{subject to} \quad & w^p - \theta_L e^p \geq 0 \text{ and } w^p - \theta_H e^p \geq 0. \end{aligned}$$

20 Clearly, if $w^p - \theta_H e^p \geq 0$, then $w^p - \theta_L e^p \geq 0$.

Moreover, $w^p - \theta_H e^p \geq 0$ should be binding at the optimum.

21 The reduced problem is

$$\max_{e^p} \lambda \pi(e^p, \theta_L) + (1-\lambda)\pi(e^p, \theta_H) - \theta_H e^p.$$

Then e^p is characterized by

$$\lambda \pi_e(e^p, \theta_L) + (1-\lambda)\pi_e(e^p, \theta_H) = \theta_H.$$

22 Since $\pi_{e\theta} > 0$, we have that

$$\begin{aligned} \lambda \pi_e(e^p, \theta_L) + (1-\lambda)\pi_e(e^p, \theta_H) = \theta_H &= \pi_e(e_H^*, \theta_H) \\ &> \lambda \pi_e(e_H^*, \theta_L) + (1-\lambda)\pi_e(e_H^*, \theta_H). \end{aligned}$$

Since $\pi_{ee} < 0$, we have that $e^p < e_H^*$.

23 In summary, when nonresponsiveness occurs, the sharp conflict between the principal's preferences and the incentive constraints (which reflect the agent's preferences) makes it impossible to use any information transmitted by the agent about his type.

3 Three-type model

24 There are three types $\{\theta_L, \theta_M, \theta_H\}$ with $\theta_H - \theta_M = \theta_M - \theta_L = \Delta\theta$.

The respective probabilities are λ_L, λ_M, and λ_H with $\lambda_L + \lambda_M + \lambda_H = 1$.

25 As a benchmark, the first-best effort levels are respectively given by

$$\pi'(e_L^*) = \theta_L,\ \pi'(e_M^*) = \theta_M, \pi'(e_H^*) = \theta_H.$$

26 Principal would like to offer a menu of contracts $\{(e_L, w_L), (e_M, w_M), (e_H, w_H)\}$ hoping that θ_L agents will select (e_L, w_L), θ_M agents will select (e_M, w_M), and θ_H agents will select (e_H, w_H).

27 IC constraints for $\{(e_L, w_L), (e_M, w_M), (e_H, w_H)\}$:

$$w_L - \theta_L e_L \geq w_M - \theta_L e_M, \tag{IC$_{LM}$}$$
$$w_L - \theta_L e_L \geq w_H - \theta_L e_H, \tag{IC$_{LH}$}$$
$$w_M - \theta_M e_M \geq w_H - \theta_M e_H, \tag{IC$_{MH}$}$$
$$w_M - \theta_M e_M \geq w_L - \theta_M e_L, \tag{IC$_{ML}$}$$
$$w_H - \theta_H e_H \geq w_M - \theta_H e_M, \tag{IC$_{HM}$}$$
$$w_H - \theta_H e_H \geq w_L - \theta_H e_L. \tag{IC$_{HL}$}$$

- 4 local incentive constraints: involving adjacent types.
- 2 global incentive constraints: involving nonadjacent types.

28 Monotonicity condition (or implementability condition): Constraints (IC$_{LM}$) and (IC$_{ML}$) imply that $e_L \geq e_M$. Constraints (IC$_{MH}$) and (IC$_{HM}$) imply that $e_M \geq e_H$.

$$e_L \geq e_M \geq e_H. \tag{Monotonicity condition}$$

29 Two local incentive constraints (IC$_{LM}$) and (IC$_{MH}$) lead to the global one (IC$_{LH}$) under $e_M \geq e_H$.

Similarly, two local incentive constraints (IC$_{ML}$) and (IC$_{HM}$) lead to the global one (IC$_{HL}$) under $e_L \geq e_M$.

30 Intuitively, more efficient types tend to claim to be less efficient. Momentarily, we ignore the incentive constraints (IC$_{ML}$), (IC$_{HL}$) and (IC$_{HM}$).

31 So we consider only (IC$_{LM}$), (IC$_{MH}$) and (Monotonicity condition).

32 IR constraints for $\{(e_L, w_L), (e_M, w_M), (e_H, w_H)\}$:

$$w_L - \theta_L e_L \geq 0, \tag{IR$_L$}$$
$$w_M - \theta_M e_M \geq 0, \tag{IR$_M$}$$
$$w_H - \theta_H e_H \geq 0. \tag{IR$_H$}$$

33 Clearly, (IR$_H$) and (IC$_{MH}$) imply (IR$_M$). Similarly, (IR$_H$) and (IC$_{LH}$) imply (IR$_L$).

That is, given that IC constraints hold, IR constraints of all 3 types are satisfied as long as (IR$_H$) holds.

34 The principal's problem is to solve

$$\begin{aligned} \underset{(e_L,w_L),(e_M,w_M),(e_H,w_H)}{\text{maximize}} \quad & \lambda_L\big(\pi(e_L) - w_L\big) + \lambda_M\big(\pi(e_M) - w_M\big) + \lambda_H\big(\pi(e_H) - w_H\big) \\ \text{subject to} \quad & \text{Constraints (IC}_{LM}\text{), (IC}_{MH}\text{), (Monotonicity condition) and (IR}_H\text{).} \end{aligned}$$

35 As usual, constraints (IC_{LM}), (IC_{MH}) and (IR_H) should be binding at the optimum:

$$w_L - \theta_L e_L = w_M - \theta_L e_M, \; w_M - \theta_M e_M = w_H - \theta_M e_H, \; w_H - \theta_H e_H = 0.$$

That is,

$$\begin{aligned}
w_H &= \theta_H e_H, \\
w_M &= w_H + \theta_M e_M - \theta_M e_H = \theta_H e_H + \theta_M e_M - \theta_M e_H, \\
w_L &= w_M + \theta_L e_L - \theta_L e_M = \theta_H e_H + \theta_M e_M - \theta_M e_H + \theta_L e_L - \theta_L e_M.
\end{aligned}$$

Hence, the information rents are

$$\begin{aligned}
r_H &= w_H - \theta_H e_H = 0, \\
r_M &= w_M - \theta_M e_M = \theta_H e_H - \theta_M e_H = \Delta\theta e_H, \\
r_L &= w_L - \theta_L e_L = \Delta\theta e_H + \Delta\theta e_M.
\end{aligned}$$

36 The principal's problem is rewritten as:

$$\begin{aligned}
\underset{e_L, e_M, e_H}{\text{maximize}} \quad & \lambda_L\big(\pi(e_L) - \theta_H e_H - \theta_M e_M + \theta_M e_H - \theta_L e_L + \theta_L e_M\big) \\
& + \lambda_M\big(\pi(e_M) - \theta_H e_H - \theta_M e_M + \theta_M e_H\big) + \lambda_H\big(\pi(e_H) - \theta_H e_H\big) \\
\text{subject to} \quad & \text{Constraint (Monotonicity condition).}
\end{aligned}$$

37 Ignore constraint (Monotonicity condition) first.

First order condition for e_L:

$$\pi'(e_L^{\mathrm{SB}}) = \theta_L.$$

First order condition for e_M:

$$\pi'(e_M^{\mathrm{SB}}) = \theta_M + \frac{\lambda_L}{\lambda_M}(\theta_M - \theta_L) = \theta_M + \frac{\lambda_L}{\lambda_M}\Delta\theta.$$

First order condition for e_H:

$$\pi'(e_H^{\mathrm{SB}}) = \theta_H + \frac{\lambda_M}{\lambda_H}(\theta_H - \theta_M) + \frac{\lambda_L}{\lambda_H}(\theta_H - \theta_M) = \theta_H + \frac{\lambda_M + \lambda_L}{\lambda_H}\Delta\theta.$$

38 Then check constraint (Monotonicity condition):

- Clearly, $e_L^{\mathrm{SB}} > e_M^{\mathrm{SB}}$ automatically.
- $e_M^{\mathrm{SB}} > e_H^{\mathrm{SB}}$ iff $\pi'(e_M^{\mathrm{SB}}) < \pi'(e_H^{\mathrm{SB}})$ iff

$$\theta_M + \frac{\lambda_L}{\lambda_M}\Delta\theta < \theta_H + \frac{\lambda_M + \lambda_L}{\lambda_H}\Delta\theta,$$

  which is equivalent to

$$\lambda_M > \lambda_L \lambda_H.$$

In this case, the information rents are

$$\begin{aligned} r_H &= w_H - \theta_H e_H = 0, \\ r_M &= w_M - \theta_M e_M = \theta_H e_H - \theta_M e_H = \Delta\theta e_H, \\ r_L &= w_L - \theta_L e_L = \Delta\theta e_H + \Delta\theta e_M. \end{aligned}$$

39 On the other hand (if $\lambda_M \leq \lambda_L \lambda_H$), bunching (集束) result occurs:

For a given λ_H, if λ_L is rather big and λ_M is small, then the information rent of θ_M agents is not too costly but that of θ_L is much more. Therefore, reducing rents calls for strongly reducing e_M, but a reduction in e_H is less necessary. However, due to the implementability condition, e_M cannot be reduced to be lower than e_H. We thus have $e_M = e_H$ at the optimum.

In this case, principal's problem is rewritten as:

$$\max_{e_L, e^p} \lambda_L \big(\pi(e_L) - \theta_H e^p - \theta_L e_L + \theta_L e^p\big) + \lambda_M \big(\pi(e^p) - \theta_H e^p\big) + \lambda_H \big(\pi(e^p) - \theta_H e^p\big).$$

First order condition for e^p:

$$(\lambda_M + \lambda_H)\pi'(e^p) = \lambda_M \theta_H + \lambda_H \theta_H + \lambda_L(\theta_H - \theta_L).$$

That is,

$$\pi'(e^p) = \theta_H + \frac{\lambda_L}{\lambda_M + \lambda_H} 2\Delta\theta.$$

40 Theorem:

- Constraints (IC_{LM}), (IC_{MH}) and (IR_H) are all binding.
- When $\lambda_M > \lambda_H \lambda_L$, Constraint (Monotonicity condition) is strictly satisfied. Optimal outputs are given by $e_L^{\text{SB}} = e_L^*$, $e_M^{\text{SB}} < e_M^*$ and $e_L^{\text{SB}} < e_L^*$ with

$$\begin{aligned} \pi'(e_M^{\text{SB}}) &= \theta_M + \frac{\lambda_L}{\lambda_M} \Delta\theta, \\ \pi'(e_H^{\text{SB}}) &= \theta_H + \frac{\lambda_M + \lambda_L}{\lambda_H} \Delta\theta. \end{aligned}$$

- When $\lambda_M \leq \lambda_H \lambda_L$, some bunching emerges. We still have $e_L^{\text{SB}} = e_L^*$, but now $e_M^{\text{SB}} = e_H^{\text{SB}} = e^p < e_L^{\text{SB}}$, with

$$\pi'(e^p) = \theta_H + \frac{\lambda_L}{\lambda_M + \lambda_H} 2\Delta\theta.$$

41 To avoid bunching, modelers often chose to impose a sufficient condition on the distribution of types, the monotonicity of the hazard rate.

Definition: A distribution of types satisfies the monotone hazard rate property if and only if

$$\frac{\text{Prob}(\theta < \theta_M)}{\text{Prob}(\theta = \theta_M)} = \frac{\lambda_L}{\lambda_M} < \frac{\text{Prob}(\theta < \theta_H)}{\text{Prob}(\theta = \theta_H)} = \frac{\lambda_L + \theta_M}{\lambda_H}.$$

Indeed, the hazard rate is defined as $\frac{f(\theta)}{1-F(\theta)}$, where $F(\theta) = \text{Prob}(\hat{\theta} \leq \theta)$ is a cumulative distribution function of random variable $\hat{\theta}$ and $f(\theta)$ is F's density. That is, the hazard rate is the probability of observing a type within a neighborhood of θ (for example, $[\theta, \theta + \mathrm{d}\theta]$), conditional on the type being no less than θ.

42 The virtual costs of the different types, namely θ_L, $\theta_M + \frac{\lambda_L}{\lambda_M}\Delta\theta$ and $\theta_H + \frac{\lambda_M+\lambda_L}{\lambda_H}\Delta\theta$, are ranked exactly as the true physical costs.

The virtual surplus is maximized by a decreasing schedule of outputs ($e_L^{\text{SB}} > e_M^{\text{SB}} > e_H^{\text{SB}}$). Asymmetric information does not perturb the ranking of types.

4 Summary

43 When it comes to solving the screening problem, it is useful to start from the benchmark problem without adverse selection, which involves maximizing the payoff of the principal subject to IR constraints. At the optimum, allocative efficiency is then achieved, because the principal can treat each type of agent separately and offer a type-specific package.

44 In the presence of adverse selection, however, the principal has to offer all types of agents the same menu of options. He has to anticipate that each type of agent will choose her favorite opinion. Without loss of generality, he can restrict the menu to the set of opinions actually chosen by at least one type of agent. It reduces the program of the principal to the maximization of his expected payoff subject to IC and IR constraints.

45 One can disregard the IC for low-ability agent and IR for high-ability agent. Contract then trades off optimally the allocative inefficiency of the low-ability agent with the information rent conceded to the high-ability agent.

In contrast, there is no allocative inefficiency for the high-ability agent and no rent for the low-ability agent.

46 For generalizations to more than two types, IC constraints can often be replaced by fewer local IC constraints and monotonicity condition. We have full separation under natural restrictions (monotone hazard rate).

47 In some cases, the distribution of types does not lead to full separation—for example, when there are intermediate types that the principal considers to be of low probability. There would then be an incentive for the principal to have severe allocative inefficiency for these types, in order to reduce the rents of adjacent types. But this incentive conflicts with the monotonicity condition. In this case, a procedure of "bunching and ironing" has been outlined to solved for the optimal contract. The monotonicity condition then binds for some types where bunching occurs.