

Social and Economic Networks

Games on Networks

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Outline

- 1 Games on networks
- 2 Strategic complements and strategic substitutes
- 3 Properties of equilibrium
 - Strategic complements
 - Strategic substitutes

Reference

- Jackson, Chapter 9

- 1 Games on networks
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Game theoretic reasoning

- To understand how the structure of social networks influences behavior, beyond diffusion and learning.

Depending in more complicated ways on what neighbors are doing.

- An individual only wants to buy a product or make an investment when his or her neighbors do not.
 - If an individual is choosing a piece of software or some other product and wants it to be compatible with a majority of neighbors.
- This interactive considerations require **game theoretic reasoning**, adapted and extended to a network setting.

Canonical setting

- Each player i chooses action x_i in $\{0, 1\}$.
 - 1: buy a book/invest a new technology/learn a language.
 - 0: does not buy a book/invest technology/learn a language.
- Payoff will depend on
 - how many neighbors choose each action.
 - how many neighbors a player has.
- Consider the case

$$u_{d_i}(x_i, m_{N_i}),$$

depending only on d_i and m_{N_i} (the number of neighbors of i choosing action 1).

Canonical setting: Limitations

- Only two actions.
- Only care about the number of friends taking the action, not the identities of them.
- Treat friends equally in terms of who is taking the action.
- It only matters how many friends they have.

Example 1: Simple complement

- Agent i is willing to choose 1 if and only if **at least t neighbors** do.
- Payoff of action 0:

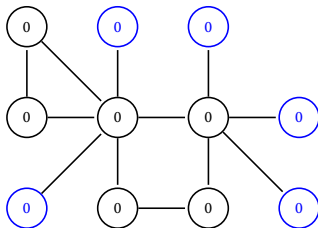
$$u_{d_i}(0, m_{N_i}) = 0.$$

- Payoff of action 1:

$$u_{d_i}(1, m_{N_i}) = -t + m_{N_i},$$

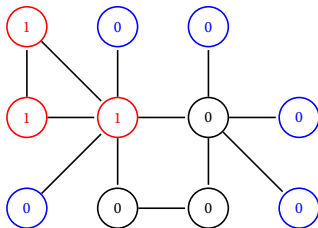
where m_{N_i} is the number of neighbors of i choosing action 1.

Example 1: Simple complement (Cont.)



An agent is willing to take action 1 if and only if **at least 2 neighbors** do.

Example 1: Simple complement (Cont.)



An agent is willing to take action 1 if and only if **at least 2 neighbors** do.

Example 2: Best shot

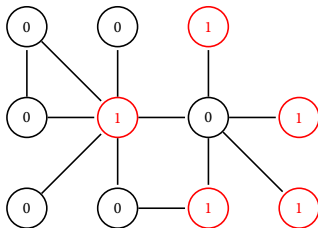
- Agent i is willing to choose 1 if and only if **no neighbors** do.
- For example, one of my friends buys the book, I do not buy the book because now I can borrow it from them.
- Payoff of action 0:

$$u_{d_i}(0, m_{N_i}) = \begin{cases} 1, & \text{if } m_{N_i} > 0, \\ 0, & \text{if } m_{N_i} = 0. \end{cases}$$

- Payoff of action 1:

$$u_{d_i}(1, m_{N_i}) = 1 - c.$$

Example 2: Best shot (Cont.)



An agent is willing to take action 1 if and only if **at least 2 neighbors** do.

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Strategic complements/substitutes

- Strategic complements:
as more of my friends take the action, it is a more attractive action to me.
- Strategic substitutes:
as more of my friends take the action, it is a less attractive action for me to take.

Strategic complements

- Strategic complements: for all $d, m \geq m'$, differences are increasing:

$$u_d(1, m) - u_d(0, m) \geq u_d(1, m') - u_d(0, m').$$

- * As the number of friends who take the action increases, the payoff to taking the action compared to not taking the action has gone up.
- * So the difference between taking the action and not taking it, it is more attractive than it was before.
- Positive relationship.

Strategic substitutes

- Strategic substitutes: for all $d, m \geq m'$, differences are decreasing:

$$u_d(1, m) - u_d(0, m) \leq u_d(1, m') - u_d(0, m').$$

- As more people take the action, as we move from m' to m , it becomes less attractive to take the action.
- Negative relationship.

Examples

- Strategic complements
 - education decisions
 - smoking and other behavior among peers
 - technology adoption
 - learn a language
 - cheating
- Strategic substitutes
 - information gathering
 - local public goods (shareable products)
 - competing firms (oligopoly with local markets)

Strategic complements/substitutes

- Complements:

There is a threshold $t(d)$, such that i prefers 1 if $m_{N_i} > t(d)$ and 0 if $m_{N_i} < t(d)$.

- Substitutes:

There is a threshold $t(d)$, such that i prefers 1 if $m_{N_i} < t(d)$ and 0 if $m_{N_i} > t(d)$.

Externalities

- Others' behaviors affect my utility/welfare.
- Others' behaviors affect the relative payoffs to my behaviors.
- Complements: Choice to take an action by my friends increases my relative payoff to taking that action.
- Substitutes: Choice to take an action by my friends decreases my relative payoff to taking that action.

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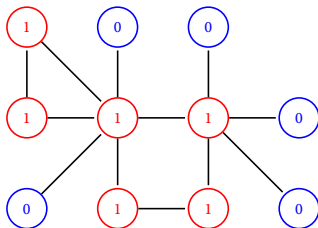
Equilibrium

- Nash equilibrium:

Every player's action is optimal for that player given the actions of others.

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Example 1: Simple complement

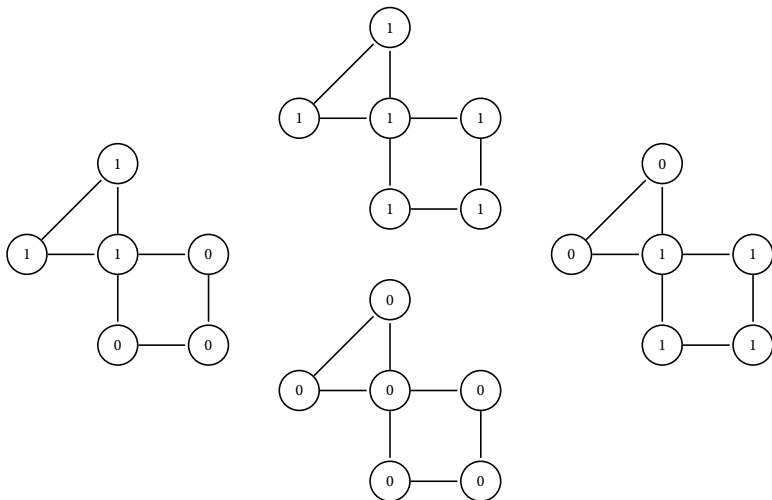


- An agent is willing to take action 1 if and only if **at least 2 neighbors** do.
- Each player takes the maximal action that she can in any equilibrium.

Complete lattice

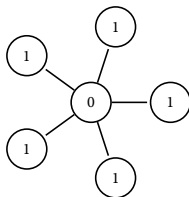
- In a game of strategic complements, the set of pure strategy Nash equilibria has a nice structure: complete lattice.
- Complete lattice: for every subset of equilibria X ,
 - there exists an equilibrium x' such that $x' \geq x$ for all $x \in X$,
 - there exists an equilibrium x'' such that $x'' \leq x$ for all $x \in X$.

Complete lattice (Cont.)

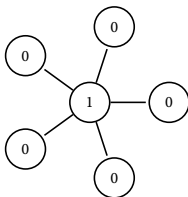


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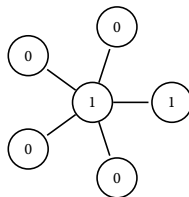
Example 2: Best shot



Equilibrium



Equilibrium



Not an equilibrium

Maximal independent set: each 1 has no 1's in its neighborhood, each 0 has at least one 1 in its neighborhood.

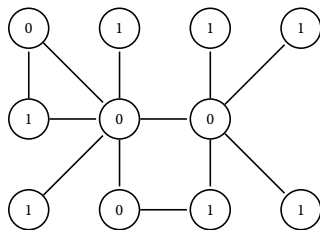
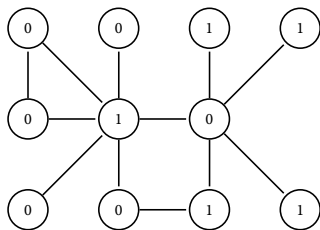
Maximal independent set

- Independent set:
A set S of nodes such that no two nodes in S are linked.
- Maximal:
Every node in N is either in S or linked to a node in S .

Equilibrium: Strategic substitutes

- Best shot game: pure strategy equilibria exist and are related to maximal independent sets.
- Others: pure strategy may not exist, but mixed will (with finite action spaces).
- Equilibria usually do not form a lattice.

Example: Best shot



- Invest if and only if no neighbors do (threshold is 1).
- Multiple equilibria.
- No lattice structure.

Find maximal independent sets

- Pick one person to be 1, and fill in all the 0's for her neighbors
- A bunch of people who are still left. Pick any one of those people who are still left. They are not one of the neighbors of the first person, so they still do not have any neighbors who have taken any action, put them as a 1.
- Now all of their neighbors would have to be 0's
- So forth.
- This algorithm will give us one of the maximum independent sets.
- Every possible order for picking nodes \Rightarrow all the maximum independent sets.