Advanced Microeconomics III: Lecture Notes 2

1 Introduction

1 We look at problems of adverse selection (逆向选择) where one party to a transaction knows things pertaining to the transaction that are relevant to but unknown by the second party.

Adverse selection models hidden characteristics/information, where asymmetric information exists before the parties enter into a relationship. It refers to a market process in which undesired results occur when buyers and sellers have asymmetric information (access to different information); the "bad" products or services are more likely to be selected.

当交易中某一方掌握了信息，而另一方缺少信息时，掌握信息的一方可以利用对方的“无知”来侵害对方的利益，同时谋求自己的利益。反过来，处于信息劣势的一方也不一定会坐以待毙，他知道对方在乘机牟利，因此对任何交易都持怀疑态度。这样会使得本来对双方都有利的交易无法达成，或者即使达成，效率也不高。

2 One example is the market of used cars. In the market, buyers often do not observe the quality of the cars, which is private information of the sellers.

Due to the common existence of low-quality used cars (the "lemons"—没有价值的东西, 次品), buyers will be reluctant to pay high price for a high-quality car (the "peach"—(同类事物中) 极好的 [极吸引人的] 事物), since they cannot tell its quality.

As a consequence of low market prices, high-quality sellers are driven out of the market (they lose if they sell), and whoever sells on the market is more likely to be selling a low-quality car—adverse selection arises.

As a result, buyer's willingness to pay decreases further, and eventually, the market of high-quality cars disappears.

3 Simple example: There are two types of used cars: peaches and lemons. A peach, if it is known to be a peach, is worth \$3,000 to a buyer and \$2,500 to a seller. A lemon, on the other hand, is worth \$2,000 to a buyer and \$1,000 to a seller. There are twice as many lemons as peaches.

- Case 1 (complete information): If buyers and sellers both had the ability to look at a car and see whether it was a peach or a lemon, there would be no problem: Peaches would sell for \$3,000 and lemons for \$2,000.
- Case 2 (incomplete but symmetric information): If neither buyer nor seller knew whether a particular car was a peach or a lemon, we would have no problem (at least, assuming risk neutrality, which we will to avoid complications): A seller, thinking she has a peach with probability $\frac{1}{3}$ and a lemon with probability $\frac{2}{3}$, has a car that (in expectation) is worth \$1,500. A buyer, thinking that the car might be a peach with probability $\frac{1}{3}$ and a lemon with $\frac{2}{3}$, thinks that the car is worth on average \$2,333.33. Assuming an inelastic supply of cars and perfectly elastic demand, the market clears at \$2,333.33.

- Case 3 (asymmetric information): The seller, having lived with the car for quite a while, knows whether it is a peach or a lemon. Buyers typically can not tell. If we make the extreme assumption that the buyers can not tell at all, then the peach market breaks down.

  Therefore, the expected value of the car to buyers is $2,333.33, and that would be the maximal amount she is willing to pay for the car. Given this, only sellers of lemons sell, because a peach values $2,500 to sellers. So the market attracts only sellers of lemons and the way it selects sellers is a version of adverse selection.

  Moreover, if only lemons are put on the market, buyer's beliefs update: they understand the logic behind adverse selection (sellers of peaches are not willing to sell), the actually probability that they are facing a peach is zero. As a result, we get as equilibrium: Only lemons are put on the market, at a price of $2,000.

  This example says that owners of good cars will not place their cars on the used car market. This is sometimes summarized as "the bad driving out the good" in the market.

4 Another example is in life/health insurance.

If premiums are set at actuarially fair rates for the population as a whole, insurance may be a bad deal for healthy people, who then will refuse to buy. Only the sick and dying will sign up.

Premium rates then must be set to reflect this.

2 Labor market model

5 There are many identical potential firms that can hire workers. Each products the same output using an identical constant returns to scale technology in which labor is the only input.

The firms are risk neutral, seek to maximize their expected profits, and act as price takers. For simplicity, we take the price p of the firms' output to equal 1.

6 Workers differ in the productivity, denoted by θ. Let $[\underline{\theta}, \bar{\theta}] \subseteq \mathbb{R}$ denote the set of possible worker productivity levels, where $0 \leq \underline{\theta} < \bar{\theta} < \infty$.

The proportion of workers with productivity of θ or less is given by the distribution function $F(\theta)$.

We assume that F is nondegenerate. The total measure/number of workers is N.

7 Workers seek to maximize the amount that they earn from their labor.

A worker can choose to work either at a firm or at home. We suppose that a worker of type θ can earn $r(\theta)$ from working at home.

Thus, $r(\theta)$ is the opportunity cost to a type-θ worker of accepting employment; she will accept employment at a firm iff the wage is at least $r(\theta)$ (For simplicity, we assume that she accepts if she is indifferent).

8 Complete information: workers' productivity levels are publicly observable.

Because the labor of each different type of worker is a distinct good, there is a distinct equilibrium wage $w^*(\theta)$ for each type θ.

Given competitive, constant returns natures of firms, in a competitive equilibrium we have $w^*(\theta) = \theta$ for all θ, and the set of workers accepting employment in a firm is $\{\theta \mid r(\theta) \leq \theta\}$.

It is Pareto optimal:

(1) Let $I(\theta)$ be a binary variable that equals 1 if the type-θ worker works for a firm and 0 otherwise. The aggregate surplus (the total revenue generated by the workers' labor) is

$$N \int_{\underline{\theta}}^{\bar{\theta}} [I(\theta)\theta + (1 - I(\theta))r(\theta)] \, \mathrm{d}F(\theta).$$

(2) The aggregate surplus is maximized by setting $I(\theta) = 1$ for those θ with $r(\theta) \leq \theta$ and $I(\theta) = 0$ otherwise.

(3) For a type-θ worker, $r(\theta) \leq \theta$ iff he produces at a firm is no less than at home. Thus, the set of employed workers must be $\{\theta \mid r(\theta) \leq \theta\}$.

9 Asymmetric information: workers' productivity levels are unobservable by the firms.

Definition: In the competitive labor market with unobservable worker productivity levels, a competitive equilibrium is a wage w^* and a set Θ^* of worker types who accept employment such that

$$\Theta^* = \{\theta \mid r(\theta) \leq w^*\} \text{ and } w^* = \underbrace{\mathbf{E}[\theta \mid \theta \in \Theta^*]}_{\text{rational expectation}}.$$

(1) Since workers' types are unobservable, the wage must be independent of a worker's type. So we have a single wage w for all workers.

(2) Supply: A type-θ worker is willing to work for a firm iff $r(\theta) \leq w$. Hence, the supply of labor at wage w (the set of worker types who are willing to accept employment at wage w) is

$$\Theta(w) = \{\theta \mid r(\theta) \leq w\}.$$

(3) Demand: If a firm believes that the average productivity of workers who accept employment is μ, then its demand for labor is given by

$$z(w) = \begin{cases} 0, & \text{if } \mu < w, \\ [0, \infty], & \text{if } \mu = w, \\ \infty, & \text{if } \mu > w. \end{cases}$$

If worker types in Θ^* are accepting employment offers in a competitive equilibrium, and if firms' beliefs about the productivity of potential employees correctly reflect the actual average productivity of the workers hired, then we must have

$$\mu = \mathbf{E}[\theta \mid \theta \in \Theta^*].$$

The demand can equal the supply in an equilibrium with a positive level of employment iff

$$w = \mu = \mathbf{E}[\theta \mid \theta \in \Theta^*].$$

In an equilibrium, we have

$$w^* = \mathbf{E}[\theta \mid r(\theta) \leq w^*].$$

10 Pareto inefficiency: A competitive equilibrium may fail to be Pareto optimal.

Consider a simple case where $r(\theta) = r$ for all θ and suppose that $F(r) \in (0, 1)$, so that there are some workers with $\theta > r$ and some with $\theta < r$.

(1) The set of workers who are willing to accept employment at wage w is

$$\Theta(w) = \begin{cases} [\underline{\theta}, \bar{\theta}], & \text{if } w \geq r, \\ \emptyset, & \text{if } w < r. \end{cases}$$

(2) $\mathbf{E}[\theta \mid \theta \in \Theta(w)] = \mathbf{E}[\theta]$ for all w. Thus, the equilibrium wage $w^* = \mathbf{E}[\theta]$.

(3) If $w^* = \mathbf{E}[\theta] \geq r$, then all workers accept employment at a firm; Otherwise, none do.

(4) In a Pareto optimal allocation, workers with $\theta \geq r$ will accept employment at a firm and those with $\theta < r$ will not.

(5) If there is a high fraction of high-productivity workers, then $w^* = \mathbf{E}[\theta] \geq r$, and hence firms will be willing to hire workers at a wage that they are willing to accept $\Rightarrow$ Too many workers are employed—the worker θ with $r > \theta$ should work at home in a Pareto improvement.

(6) If there is a few fraction of high-productivity workers, then $w^* = \mathbf{E}[\theta] < r$, and hence firms will be unwilling to hire any workers at a wage that is sufficient to have workers accept employment (i.e., a wage of at least r) $\Rightarrow$ Too few workers are employed—the worker θ with $r < \theta$ should accept an employment in a Pareto improvement.

Because firms are unable to distinguish among workers of differing conductivities, the market is unable to allocate workers efficiently between firms and home production.

3 Adverse selection

11 Suppose that $r(\theta) \leq \theta$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$ and $r(\cdot)$ is strictly increasing. Then we have

- Pareto optimal allocation makes every type worker employed by a firm.
- Workers who are more productive at a firm are also more productive at home.

  Since the payoff of home production is greater for more capable workers, only less capable workers ($\{\theta \mid r(\theta) \leq w\}$) accept employment at any given wage w.

12 Given wage w, consider the expected productivity of workers accepting employment

$$\mathbf{E}[\theta \mid r(\theta) \leq w].$$

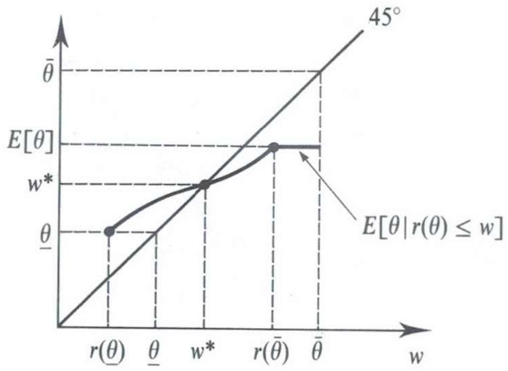
As wage w increases, more productive workers become willing to accept employment at a firm, and the average productivity of those workers accepting employment rises. That is, $\mathbf{E}[\theta \mid r(\theta) \leq w]$ is increasing in w.

For simplicity, we assume that $\mathbf{E}[\theta \mid r(\theta) \leq w]$ is continuous in $w \in [r(\underline{\theta}), \infty]$.

13 The equilibrium wage w^* satisfies

$$w^* = \mathbf{E}[\theta \mid r(\theta) \leq w^*].$$

14 Graph:



- There is no employment when $w < r(\underline{\theta})$. In this case, $\mathbf{E}[\theta \mid r(\theta) \leq w] = \mathbf{E}[\theta]$ does not make any sense.
- The graph of $\mathbf{E}[\theta \mid r(\theta) \leq w]$ is increasing on $[r(\underline{\theta}), \infty)$.
- Minimum value: $\mathbf{E}[\theta \mid r(\theta) \leq r(\underline{\theta})] = \mathbf{E}[\underline{\theta}] = \underline{\theta}$ at $w = r(\underline{\theta})$.
- Maximum value: $\mathbf{E}[\theta]$ for $w \geq r(\bar{\theta})$.
- Be constant on $[r(\bar{\theta}), \infty)$.
- w^*: the intersection point between $\mathbf{E}[\theta \mid r(\theta) \leq w]$ and 45-degree line.

15 Numerical example 1: $r(\theta) = \frac{2}{3}\theta$. θ is uniformly distributed on $[0, 2]$.

(1) Largest possible wage: $r(\bar{\theta}) = \frac{4}{3}$.

(2) Smallest possible wage: $r(\underline{\theta}) = 0$.

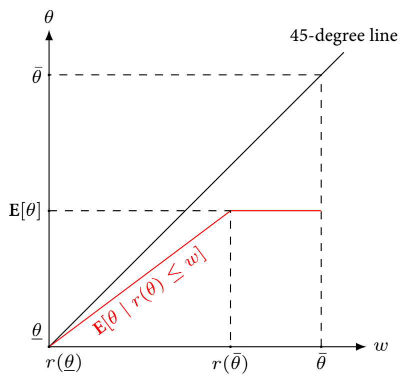
(3) If $w \in [0, \frac{4}{3}]$, then $\mathbf{E}[\theta \mid r(\theta) \leq w] = \mathbf{E}[\theta \mid \frac{2}{3}\theta \leq w] = \frac{3}{4}w$.

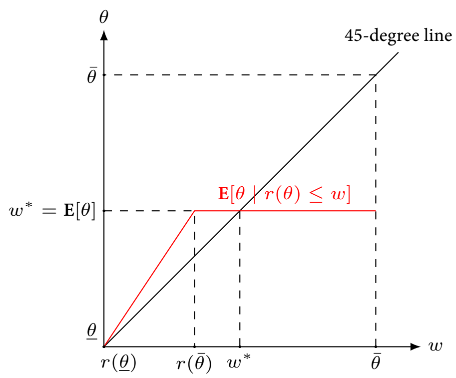
(4) Minimum value: $\underline{\theta} = 0$ at $r(\underline{\theta}) = 0$.

(5) Maximum value: $\mathbf{E}[\theta] = 1$ at $r(\bar{\theta}) = \frac{4}{3}$.

(6) For all $w > 0$, $\mathbf{E}[\theta \mid r(\theta) \leq w] = \frac{3}{4}w < w$.

(7) A zero-measure set of workers are hired.





16 Numerical example 2: $r(\theta) = \frac{1}{3}\theta$. θ is uniformly distributed on $[0, 2]$.

(1) Largest possible wage: $r(\bar{\theta}) = \frac{2}{3}$.

(2) Smallest possible wage: $r(\underline{\theta}) = 0$.

(3) If $w \in [0, \frac{2}{3}]$, $\mathbf{E}[\theta \mid r(\theta) \le w] = \mathbf{E}[\theta \mid \frac{1}{3}\theta \le w] = \frac{3}{2}w$.

(4) Minimum value: $\underline{\theta} = 0$ at $r(\underline{\theta}) = 0$.

(5) Maximum value: $\mathbf{E}[\theta] = 1$ at $r(\bar{\theta}) = \frac{2}{3}$.

(6) For all $w > 0$, $\mathbf{E}[\theta \mid r(\theta) \le w] = \frac{3}{2}w > w$.

(7) All workers are hired, and competitive equilibrium wage is $\mathbf{E}[\theta] = 1$.

(8) There is the other competitive equilibrium wage 0. (why?)

17 Numerical example 3: $r(\theta) = \begin{cases} \frac{1}{3}\theta, & \text{if } 0 \le \theta \le 1 \\ \frac{3}{4}(\theta - 1) + \frac{1}{3}, & \text{if } 1 \le \theta \le 2 \end{cases}$. θ is uniformly distributed on $[0, 2]$.

(1) $\mathbf{E}[\theta \mid r(\theta) \le w] = \begin{cases} \mathbf{E}[\theta \mid \frac{1}{3}\theta \le w] = \frac{3}{2}w, & \text{if } 0 \le w \le \frac{1}{3} \\ \mathbf{E}[\theta \mid \frac{3}{4}(\theta - 1) + \frac{1}{3} \le w] = \frac{12w+5}{18}, & \text{if } \frac{1}{3} \le w \le \frac{13}{12} \end{cases}$.

(2) Minimum value: $\underline{\theta} = 0$ at $r(\underline{\theta}) = 0$.

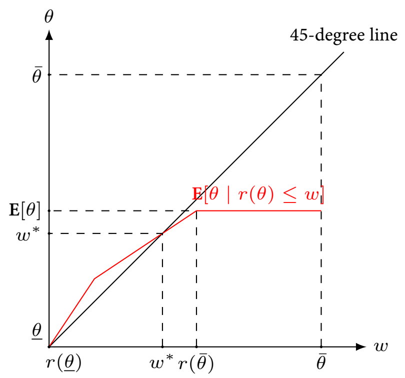
(3) Maximum value: $\mathbf{E}[\theta] = 1$ at $r(\bar{\theta}) = \frac{13}{12}$.

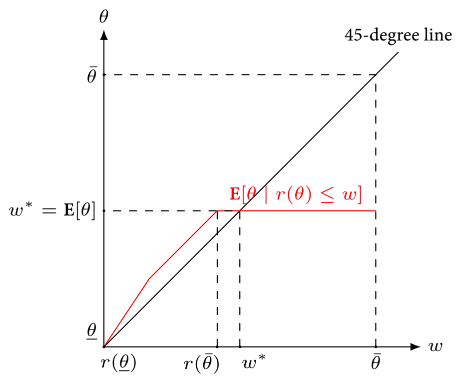
(4) Intersection point occurs at $w^* = \frac{5}{6}$.

(5) Workers $\{\theta \mid r(\theta) \le \frac{5}{6}\} = \{\theta \mid 0 \le \theta \le \frac{5}{3}\}$ are employed.

(6) Although workers in $[0, \frac{5}{3}]$ accept the employment, workers in $[0, \frac{5}{6}]$ earn more than the output they produce and workers in $[\frac{5}{6}, \frac{5}{3}]$ produce more than they earn.

(7) There is the other competitive equilibrium wage 0.





18 Numerical example 4: $r(\theta) = \begin{cases} \frac{1}{3}\theta, & \text{if } 0 \le \theta \le 1 \\ \frac{1}{2}(\theta - 1) + \frac{1}{3}, & \text{if } 1 \le \theta \le 2 \end{cases}$. θ is uniformly distributed on $[0, 2]$.

(1) $\mathbf{E}[\theta \mid r(\theta) \le w] = \begin{cases} \mathbf{E}[\theta \mid \frac{1}{3}\theta \le w] = \frac{3}{2}w, & \text{if } 0 \le w \le \frac{1}{3} \\ \mathbf{E}[\theta \mid \frac{1}{2}(\theta - 1) + \frac{1}{3} \le w] = w + \frac{1}{6}, & \text{if } \frac{1}{3} \le w \le \frac{5}{6} \end{cases}$.

(2) Minimum value: $\underline{\theta} = 0$ at $r(\underline{\theta}) = 0$.

(3) Maximum value: $\mathbf{E}[\theta] = 1$ at $r(\bar{\theta}) = \frac{5}{6}$.

(4) Intersection point occurs at $w^* = 1$. (how about $r(\bar{\theta}) = \frac{5}{6}$?)

(5) Workers $\{\theta \mid r(\theta) \le 1\} = \{\theta \mid 0 \le \theta \le 2\}$ are employed.

(6) Although workers in $[0, 2]$ accept the employment, workers in $[0, 1]$ earns more than the output they produce and workers in $[1, 2]$ produce more than they earn.

(7) There is the other competitive equilibrium wage 0.

19 Adverse selection:

(1) To get the best workers to accept employment at a firm, the wage needs to be at least $r(\bar{\theta})$.

(2) Firm cannot do that, because their inability to distinguish among different types of workers leaves them receiving only an expected output of $\mathbf{E}[\theta] < r(\bar{\theta})$.

The presence of enough low-productivity workers forces the wage down below $r(\bar{\theta})$.

(3) This drives the best workers out of the market.

(4) Once the best workers are driven out of the market, the average productivity falls, thereby further lowering the wage that firms are willing to pay.

(5) Then the next-best workers may be driven out of the market.

(6) And so on.

(7) Termination: at some competitive equilibrium wage.

20 Recall numerical example 1: $r(\theta) = \frac{2}{3}\theta$. θ is uniformly distributed on $[0, 2]$.

(1) $\mathbf{E}[\theta \mid r(\theta) \leq w] = \mathbf{E}[\theta \mid \frac{2}{3}\theta \leq w] = \frac{3}{4}w$.

(2) For all $w > 0$, $\mathbf{E}[\theta \mid r(\theta) \leq w] = \frac{3}{4}w < w$.

(3) A zero-measure set of workers are hired.

(4) On the other hand, $r(\theta) = \frac{2}{3}\theta < \theta$ for all $\theta > 0$.

(5) The Pareto optimal allocation calls for all workers to be hired.

21 Recall numerical example 3: $r(\theta) = \begin{cases} \frac{1}{3}\theta, & \text{if } 0 \leq \theta \leq 1 \\ \frac{3}{4}(\theta - 1) + \frac{1}{3}, & \text{if } 1 \leq \theta \leq 2 \end{cases}$. θ is uniformly distributed on $[0, 2]$.

(1) $\mathbf{E}[\theta \mid r(\theta) \leq w] = \begin{cases} \mathbf{E}[\theta \mid \frac{1}{3}\theta \leq w] = \frac{3}{2}w, & \text{if } 0 \leq w \leq \frac{1}{3} \\ \mathbf{E}[\theta \mid \frac{3}{4}(\theta - 1) + \frac{1}{3} \leq w] = \frac{12w+5}{18}, & \text{if } \frac{1}{3} \leq w \leq \frac{13}{12} \end{cases}$.

(2) Intersection point occurs at $w^* = \frac{5}{6}$.

(3) To get the best workers to accept employment at a firm, the wage needs to be at least $w_1 = r(\bar{\theta}) = \frac{13}{12}$.

(4) Firm cannot do that, because their inability to distinguish among different types of workers leaves them receiving only an expected output of $\mathbf{E}[\theta] = 1 < \frac{13}{12} = r(\bar{\theta})$.

(5) The presence of enough low-productivity workers forces the wage down to $w_2 = 1$.

(6) This drives the best workers ($(\frac{17}{9}, 2]$) out of the market.

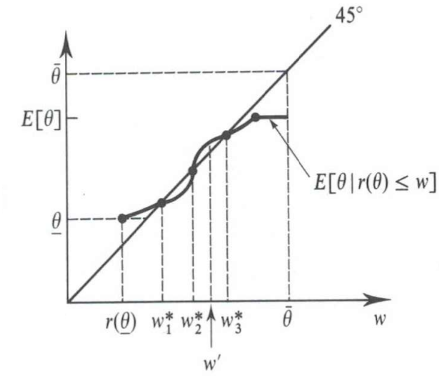
(7) Once the best workers are driven out of the market, the average productivity falls $\mathbf{E}[\theta \mid r(\theta) \leq w_2] = \frac{17}{18}$, thereby further lowering the wage to $w_3 = \frac{17}{18}$.

(8) Then the next-best workers ($(\frac{49}{27}, \frac{17}{9}]$) may be driven out of the market.

(9) And so on.

(10) Termination: at some competitive equilibrium wage $w^* = \frac{5}{6}$.

22 Example (multiple competitive equilibrium)



The low-wage (w_1^*) competitive equilibria arise because of a coordination problem:

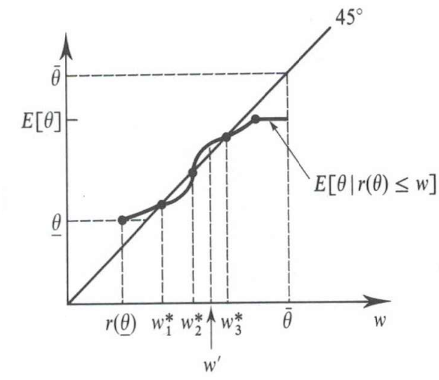
- the wage w_1^* is too low because firms expect that the productivity of workers employment is poor,
- only bad workers accept employment because the wage w_1^* is low.

4 Competitive equilibria vs. subgame Nash equilibrium

23 In the competitive equilibria, firms know only the average productivity of the workers who accept employment at the equilibrium wage; they need not have any idea of the market mechanism.

How about the equilibrium when firms understand the structure of the market, including the relationship between the wage and the quality of employed workers? In other words, can the competitive equilibria be viewed as the outcome of a richer model where firms could change their offered wages but choose not to in some version of equilibrium (subgame perfect equilibrium)?

24 Example:



- Consider the competitive equilibrium with wage w_2^*.
- Firm can profitably deviate by setting wage $w' > w_2^*$: it would attract workers with an average productivity $\mathbf{E}[\theta \mid r(\theta) \leq w'] > w'$.

$\Rightarrow$ When firms could choose wages, then this competitive equilibrium is unlikely to be a (subgame perfect) equilibrium.

Similarly for the competitive equilibrium with wage w_1^*.

25 Game-theoretical model:

- Two firms.
- $r(\cdot)$ is strictly increasing with $r(\theta) \leq \theta$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.
- $F(\cdot)$ has a density function $f(\cdot)$ with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.
- In stage 1: two firms simultaneously announce their wages.
- In stage 2: workers decide whether to work for a firm and, if so, which one.

26 Proposition: Let W^* denote the set of competitive equilibrium wages, and let $w^* = \max\limits_{w \in W^*} w$.

(1) If $w^* > r(\underline{\theta})$ and there is an $\varepsilon > 0$ such that $\mathbf{E}[\theta \mid r(\theta) \leq w'] > w'$ for all $w' \in (w^* - \varepsilon, w^*)$ (从上方穿越 45 度线), then there is a unique pure-strategy SPE. In this SPE, employed workers receive wage w^*, and workers with type in the set $\Theta(w^*) = \{\theta \mid r(\theta) \leq w^*\}$ accept employment in firms.

(2) If $w^* = r(\underline{\theta})$, then there are multiple pure-strategy SPE each agent's payoff exactly equals her payoff in the highest-wage competitive equilibrium.

27 *Proof of (1).* Assume $w^* > r(\underline{\theta})$.

Claim 1: In any SPE, both firms earn zero.

(1) Suppose that there is an SPE where M workers are hired at wage $\bar{w}$ (the highest wage offered by either of two firms) and then aggregate profit is

$$\pi = M \cdot (\mathbf{E}[\theta \mid r(\theta) \leq \bar{w}] - \bar{w}) > 0.$$

(2) Then $M > 0$, and hence $\bar{w} \geq r(\underline{\theta})$ (otherwise no workers accept $\bar{w}$).

(3) Consider the (weakly) less-profitable firm, say firm j. Firm j earns no more than $\frac{\pi}{2}$.

(4) However, firm j would be better off via setting wage $\bar{w} + \alpha$ for sufficiently small $\alpha > 0$: the profit is at least

$$M \cdot (\mathbf{E}[\theta \mid r(\theta) \leq \bar{w} + \alpha] - \bar{w} - \alpha) \approx \pi.$$

Contradiction.

(5) $\pi \leq 0 \Rightarrow \pi = 0$.

Claim 2: The highest wage offered by either of two firms equals w^* in any SPE.

(1) Let $\bar{w}$ be the highest wage offered by either of two firms in an SPE, then
   - either $\mathbf{E}[\theta \mid r(\theta) \leq \bar{w}] - \bar{w} = 0$

   $\Rightarrow$ $\bar{w} \in W^*$ (it must be a competitive equilibrium wage),
   - or $M = 0$

   $\Rightarrow$ $\bar{w} < r(\underline{\theta}) < w^*$ (it must be low that no workers accept employment).

(2) If $\bar{w} < w^* = \max\limits_{w \in W^*} w$, then either firm can earn a strictly positive expected profit by deviating and offering any wage $w' \in (w^* - \varepsilon, w^*)$.

(3) Thus, $\bar{w} = w^*$.

(4) Actually, both firms choose wage w^*: otherwise, the high-wage firm may choose $w' \in (w^* - \varepsilon, w^*)$ to get a strictly positive expected profit.

Claim 3: SPE: both firms choose wage w^*, and type-θ worker accepts employment iff the wage is at least $r(\theta)$.

(1) Neither firm can earn a positive profit by lowering its wage: it gets no workers if it does so.

(2) Neither firm can earn a positive profit by increasing its wage:

i. w^* is the highest competitive equilibrium wage, that is, there is no $w > w^*$ such that $\mathbf{E}[\theta \mid r(\theta) \le w] = w$.

ii. Since $\mathbf{E}[\theta \mid r(\theta) \le w] - w$ is continuous in w, $\mathbf{E}[\theta \mid r(\theta) \le w] - w$ should have the same sign for all $w \in (w^*, \infty)$.

iii. Since $\lim_{w\to\infty} \mathbf{E}[\theta \mid r(\theta) \le w] = \mathbf{E}[\theta]$ is finite, $\mathbf{E}[\theta \mid r(\theta) \le w] - w < 0$ for all $w \in (w^*, \infty)$.

□

28 *Proof of (2).* Assume $w^* = r(\underline{\theta})$.

(1) By the Claim 3 above, $\mathbf{E}[\theta \mid r(\theta) \le w] - w < 0$ for all $w \in (w^*, \infty)$.

(2) Then any firm setting wage in excess of w^* incurs losses.

(3) A firm earn zero by setting wage $w \le w^*$.

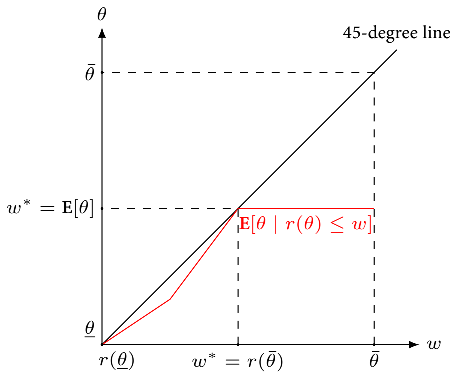
(4) Hence, the set of wages (w_1, w_2) that can arise in an SPE is $\{(w_1, w_2) \mid w_1, w_2 \le w^*\}$.

(5) In every SPE, all agents earn exactly what they earn at the competitive equilibrium with w^*.

□

29 The coordination problem disappears: If the wage is too low, some firm will find it profitable to offer a higher wage. Then the highest-wage competitive outcome must then arise.

30 There are several other cases where we cannot use SPE to remove bad CE. For example,



5 Constrained Pareto optima

31 The presence of asymmetric information results in competitive equilibria that fail to be Pareto optimal.

If there is a central planner who knows all agents' private information and can engage in lump-sum transfers (一次性支付转移) among agents in the economy, then a Pareto improvement can be achieved.

In practice, however, a central planner may be no more able to observe agents' private information than are market participants. Without this information, the central planner will face additional constraints in trying to achieve a Pareto improvement.

32 An allocation that cannot be Pareto improved by an central planner who is unable to observe agents' private information is known as a constrained (or second-best) Pareto optimum.

A constrained Pareto optimal allocation need not be fully Pareto optimal.

33 We shall study whether Pareto-improving intervention is possible, or whether the competitive equilibria are constrained Pareto optima.

- $r(\cdot)$ is strictly increasing with $r(\theta) \leq \theta$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.
- $F(\cdot)$ has a density function $f(\cdot)$ with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

34 Observations:

- Simplification: The central planner runs the firm itself and tries to achieve a Pareto improvement for the workers (the firms' owners will earn zero profits in competitive equilibria).
- The central planner cannot tell the types' of workers. Thus, the intervention schemes can depend only on whether the worker is employed.

  The intervention scheme should be: offer wage w_e to the employed workers and w_u to the unemployed workers.

35 Consider the Pareto dominated competitive equilibrium.

(1) The central planner can always implement the best (highest-wage) competitive equilibrium outcome (or a Pareto improvement) by setting $w_e = w^*$ and $w_u = 0$.

(2) All workers in $\Theta(w^*)$ accept employment and the firms balance the budget.

(3) The Pareto dominated competitive equilibrium outcome is not a constrained Pareto optimum.

The central planner is able to solve the coordination problem as well.

36 Consider the highest-wage competitive equilibrium.

Proposition: In the adverse selection labor market model, the highest-wage competitive equilibrium is a constrained Pareto optimum.

37 *Proof.* If all workers are employed in the highest-wage competitive equilibrium, then the outcome is fully (and hence constrained) Pareto optimal.

In the following, suppose some workers are not employed.

Step 1: Each intervention (w_e, w_u) can be characterized by a unique $\hat{\theta}$.

(1) For any wages (w_e, w_u), the set of worker types accepting employment has the form $[\underline{\theta}, \hat{\theta}]$ for some $\hat{\theta}$ (or $\{\theta \mid w_u + r(\theta) \leq w_e\}$).

(2) For some $\hat{\theta}$, to implement the outcome where worker types $\theta \in [\underline{\theta}, \hat{\theta}]$ accept, the central planner/firm should choose $(w_e(\hat{\theta}), w_u(\hat{\theta}))$ so that
$$w_u(\hat{\theta}) + r(\hat{\theta}) = w_e(\hat{\theta}).$$

(3) To balance the budget,
$$w_e(\hat{\theta})F(\hat{\theta}) + w_u(\hat{\theta})(1 - F(\hat{\theta})) = \int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta)\, \mathrm{d}\theta.$$

(4) Then

$$
\begin{aligned}
w_u(\hat{\theta}) &= \int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta)\,\mathrm{d}\theta - r(\hat{\theta})F(\hat{\theta}) = F(\hat{\theta})\left(\mathbf{E}[\theta \mid \theta \le \hat{\theta}] - r(\hat{\theta})\right), \\
w_e(\hat{\theta}) &= \int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta)\,\mathrm{d}\theta + r(\hat{\theta})(1 - F(\hat{\theta})) = F(\hat{\theta})\left(\mathbf{E}[\theta \mid \theta \le \hat{\theta}] - r(\hat{\theta})\right) + r(\hat{\theta}).
\end{aligned}
$$

Step 2: As long as the central planner/firm sets wage $(w^*, 0)$, the outcome with intervention will be the highest-wage competitive equilibrium.

(1) Let θ^* be the highest worker type who accepts employment in the highest-wage competitive equilibrium. Then $r(\theta^*) = w^* = \mathbf{E}[\theta \mid r(\theta) \le w^*] = \mathbf{E}[\theta \mid r(\theta) \le r(\theta^*)] = \mathbf{E}[\theta \mid \theta \le \theta^*]$.

(2) Meanwhile,

$$
\begin{aligned}
w_u(\theta^*) &= F(\theta^*)\left(\mathbf{E}[\theta \mid \theta \le \theta^*] - r(\theta^*)\right) = 0, \\
w_e(\theta^*) &= F(\theta^*)\left(\mathbf{E}[\theta \mid \theta \le \theta^*] - r(\theta^*)\right) + r(\theta^*) = r(\theta^*).
\end{aligned}
$$

Step 3: No Pareto improvement can be achieved by setting $\hat{\theta} \neq \theta^*$.

(1) First consider $\hat{\theta} < \theta^*$.

(2) Since $r(\hat{\theta}) < r(\theta^*)$,

$$
w_e(\hat{\theta}) = \int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta)\,\mathrm{d}\theta + r(\hat{\theta})(1 - F(\hat{\theta})) \le \int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta)\,\mathrm{d}\theta + r(\theta^*)(1 - F(\hat{\theta})),
$$

and hence

$$
\begin{aligned}
w_e(\hat{\theta}) - r(\theta^*) &\le \int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta)\,\mathrm{d}\theta - r(\theta^*)F(\hat{\theta}) = F(\hat{\theta})\left(\mathbf{E}[\theta \mid \theta \le \hat{\theta}] - r(\theta^*)\right) \\
&= F(\hat{\theta})\left(\mathbf{E}[\theta \mid \theta \le \hat{\theta}] - \mathbf{E}[\theta \mid \theta \le \theta^*]\right) < 0.
\end{aligned}
$$

(3) Thus, for workers with type in $[\underline{\theta}, \hat{\theta}]$, the intervention makes them worse off.

(4) Next consider $\hat{\theta} > \theta^*$, then $r(\hat{\theta}) > r(\theta^*) = w^*$.

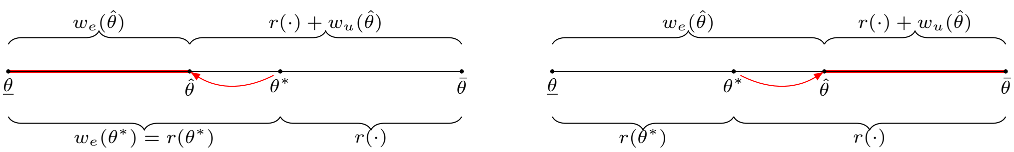
(5) We have the fact that $\mathbf{E}[\theta \mid r(\theta) \le w] < w$ for all $w > w^*$.

(6) Thus, $\mathbf{E}[\theta \mid r(\theta) \le r(\hat{\theta})] < r(\hat{\theta})$.

(7) Then $\mathbf{E}[\theta \mid \theta \le \hat{\theta}] - r(\hat{\theta}) = \mathbf{E}[\theta \mid r(\theta) \le r(\hat{\theta})] - r(\hat{\theta}) < 0$, and hence $w_u(\hat{\theta}) < 0$.

(8) Thus, for workers with type in $[\hat{\theta}, \bar{\theta}]$, the intervention makes them worse off.

□



38 Summary:

- There is no Pareto improvement of intervention if the market has achieved the highest-wage (w^*) competitive equilibrium.
- The highest-wage (w^*) competitive equilibrium is constrained Pareto optimal.

39 To resolve the adverse selection:

- Signaling (informed agents send information to uninformed agents).
- Screening (uninformed agents screen information from informed agents).

6 Homework

- Key:
  - What is the adverse selection? How does it occur? What's problem of it?
  - What is the competitive equilibrium of the labor market?
  - What is the SPE of the labor market? What is the advantage of SPE in comparison with CE?
  - Which outcome is constrained Pareto efficient?
- Reading: 13.B