Advanced Microeconomics III: Lecture Notes 4

1 Screening: Uninformed parties take step to distinguish/screen the types of informed parties.

2 Literature: Rothschild and Stiglitz (1976) and Wilson (1977).

3 There are two firms.

4 There are two types of workers, θ_H and θ_L, with $\theta_H > \theta_L > 0$ and the fraction of type-θ_H workers is $\lambda \in (0, 1)$.

Workers earn nothing if working at home, i.e., $r(\theta_H) = r(\theta_L) = 0$.

5 Jobs may differ in the "task level" required of the worker.

We assume that the task levels do not affect the output; rather, their only effect is to lower the utility of the worker.

6 The utility of a type-θ worker who faces task level $t \geq 0$ and receives wage w is $w - c(t, \theta)$.

We assume $c(t, \theta)$ is twice continuously differentiable and $c(0, \theta) = 0$, $c_t(t, \theta) > 0$, $c_{tt}(t, \theta) > 0$, $c_\theta(t, \theta) < 0$ for all $t > 0$, and $c_{t\theta}(t, \theta) < 0$.

7 Game:

- Two firms simultaneously announce (finite) sets of contracts. A contract is a pair (w, t).
- Given the offers made by the firms and their types, workers choose whether to accept a contract and, if so, which one.

1 Complete information

8 When types are observable, we allow firms to condition their offer on a worker's type, i.e., a firm can offer a contract (w_L, t_L) solely to type-θ_L workers and another contract (w_H, t_H) solely to type-θ_H workers.

9 Proposition: In any SPE of the screening game with observable types, a type-θ_i worker accepts contract $(w_i^*, t_i^*) = (\theta_i, 0)$, and firms earn zero profits.

10 *Proof.* Step 1: Any contract (w_i^*, t_i^*) accepted by type-θ_i workers in SPE will produce zero profits, and $w_i^* = \theta_i$.

- If $w_i^* > \theta_i$, then the firm who offers (w_i^*, t_i^*) is making a loss and can do better by not offering any contract to type-θ_i workers.
- Assume that $w_i^* < \theta_i$.
  (1) Let $\Pi > 0$ be the aggregate profits earned by two firms on type-θ_i workers.
  (2) There is one firm earning no more than $\frac{\Pi}{2}$, say firm j.
  (3) Firm j can deviate by offering a contract $(w_i^* + \varepsilon, t_i^*)$ for sufficiently small $\varepsilon > 0$.
  (4) Then all type-θ_i workers will accept this contract.

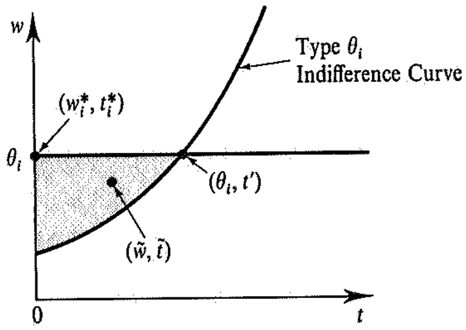
(5) Thus, the profit of firm j is close to Π. That is, the deviation increases its profit.

- Therefore, $w_i^* = \theta_i$.

Step 2: The SPE task level of type-θ_i workers is 0.

(1) Suppose that $(w_i^*, t_i^*) = (\theta_i, t')$ for some $t' > 0$.

(2) Then either firm could deviate to offer contract $(\tilde{w}, \tilde{t})$ (for type-θ_i-workers):



- Firm: the wage $\tilde{w}$ is lower than $w_i^* = \theta_i$.
- Type-θ_i worker: the utility $\tilde{w} - c(\tilde{t}, \theta_i)$ is larger than $\theta_i - c(t', \theta_i)$.

Contradiction.

(3) The only contract at which there are no profitable deviations is $(\theta_i, 0)$.

□

2 Incomplete information

11 The workers' types are not observable. So each contract can be accepted by workers of either type.

12 The outcome in the complete information case $(\theta_H, 0)$ and $(\theta_L, 0)$ cannot arise when types are unobservable: the type-θ_L worker prefers the high-ability contract $(\theta_H, 0)$ to contract $(\theta_L, 0)$.

13 Lemma: In any (separating or pooling) SPE, both firms earn zero profits.

Proof. (1) Let (w_L, t_L) and (w_H, t_H) are the contracts (could be the same) signed by low- and high-ability workers in a SPE, and suppose that the two firms' aggregate profits are $\Pi > 0$.

(2) Then $[w_L - c(t_L, \theta_L)] - [w_H - c(t_H, \theta_L)] \geq 0$ and $[w_H - c(t_H, \theta_H)] - [w_L - c(t_L, \theta_H)] \geq 0$.

(3) The one firm must make no more then $\frac{\Pi}{2}$.

(4) This firm will deviate to offer contracts $(w_L + \varepsilon, t_L)$ and $(w_H + \varepsilon, t_H)$ for sufficiently small $\varepsilon > 0$.

(5) Contract $(w_L + \varepsilon, t_L)$ will attract all type-θ_L workers, and contract $(w_H + \varepsilon, t_H)$ will attract all type-θ_H workers:

- Type-θ_L workers: $[w_L + \varepsilon - c(t_L, \theta_L)] - [w_H + \varepsilon - c(t_H, \theta_L)] = [w_L - c(t_L, \theta_L)] - [w_H - c(t_H, \theta_L)] \geq 0$.
- Type-θ_H workers: $[w_H + \varepsilon - c(t_H, \theta_H)] - [w_L + \varepsilon - c(t_L, \theta_H)] = [w_H - c(t_H, \theta_H)] - [w_L - c(t_L, \theta_H)] \geq 0$.

(6) Such a deviation will make this firm have profit close to Π. It is profitable. Contradiction.

(7) Thus, $\Pi \leq 0$, and hence $\Pi = 0$.

□

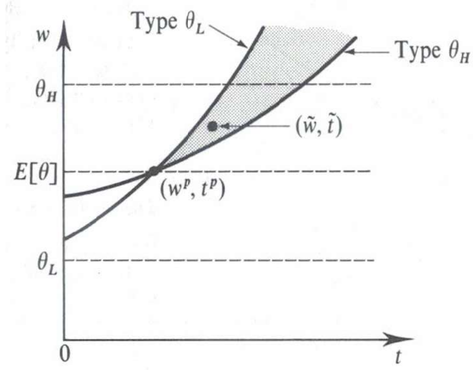
14 Lemma: No pooling SPE exists.

Proof. (1) Suppose that there is a pooling SPE contract (w^p, t^p); firm j offers this contract, and both type-θ_L and type-θ_H workers accept it.

(2) Thus, the expected productivity is $\mathbf{E}[\theta]$.

(3) Since the firms have zero profit in SPE, $w^p = \mathbf{E}[\theta]$.

(4) Firm k can deviate to offer a single contract $(\tilde{w}, \tilde{t})$.



(5) This contract will attract all the type-θ_H workers and none of the type-θ_L workers (they prefer contract (w^p, t^p)).

(6) Since $\tilde{w} < \theta_H$, firm k makes strictly positive profit $\theta_H - \tilde{w}$.

(7) Contradiction.

□

15 Lemma: If (w_L, t_L) and (w_H, t_H) are the contracts signed by low- and high-ability workers in a separating SPE, then both contracts yield zero profits, i.e., $w_L = \theta_L$ and $w_H = \theta_H$.

Proof. Step 1: $w_L \geq \theta_L$.

(1) Suppose that $w_L < \theta_L$ and firm j offers contract (w_L, t_L).

(2) Then firm k can deviate by only offering contract $(\tilde{w}_L, t_L)$, where $\theta_L > \tilde{w}_L > w_L$.

(3) The deviating firm will earn strictly positive profit.

- All low-ability workers will accept this contract $\Rightarrow$ positive profit.
- If high-ability workers do not accept this contract $\Rightarrow$ zero profit.
- If high-ability workers accept this contract $\Rightarrow$ positive profit.

(4) Contradiction. Thus, $w_L \geq \theta_L$.

Step 2: $w_H \geq \theta_H$.

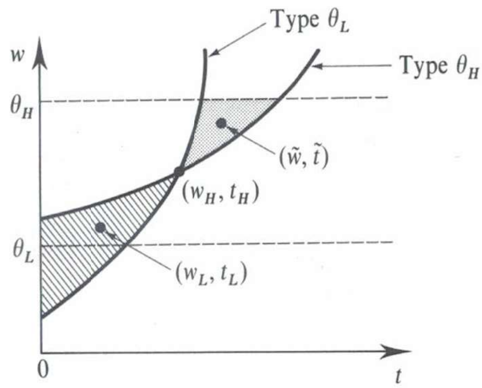
(1) Suppose that $w_H < \theta_H$.

(2) Then the low-ability contract (w_L, t_L) must lie in the hatched region:

- High-ability workers will choose $(w_H, t_H) \Rightarrow (w_L, t_L)$ is below the θ_H-indifference curve through (w_H, t_H).
- Low-ability workers will choose $(w_L, t_L) \Rightarrow (w_L, t_L)$ is above the θ_L-indifference curve through (w_H, t_H).
- Since firms earn strictly positive profits on high-ability workers, $w_L > \theta_L$.

(3) Suppose that firm j is offering the low-ability contract (w_L, t_L).

(4) Then firm $k \neq j$ can deviate by only offering a contract $(\tilde{w}, \tilde{t})$ lying in the shaded region.



(5) This contract will be accepted by all the θ_H workers and none of θ_L workers. θ_L workers will accept the contract (w_L, t_L) offered by firm j.

(6) This deviation leads to a strictly positive profit for firm k, since $\tilde{w} < \theta_H$. Contradiction.

(7) Thus, $w_H \geq \theta_H$.

Step 3: each firm earns zero profit, so $w_L = \theta_L$ and $w_H = \theta_H$. □

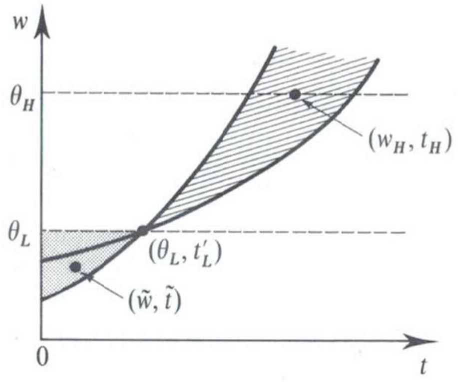
16 Lemma: In any separating SPE, the low-ability workers accept contract $(\theta_L, 0)$; that is, they receive the same contract as when no informational asymmetry is present.

Proof. (1) In any separating SPE, $w_L^* = \theta_L$.

(2) Suppose that the low-ability contract is (θ_L, t_L') with $t_L' > 0$.

(3) Suppose that firm j is offering the high-ability contract (w_H, t_H), which lies on the segment of the line $w = \theta_H$ lying in the hatched region.

(4) Then firm k can deviate by only offering a contract $(\tilde{w}, \tilde{t})$ lying in the shaded region.



(5) This contract will be accepted by all the θ_L workers and none of θ_H workers. θ_H workers will accept the contract (w_H, t_H) offered by firm j.

(6) This deviation leads to a strictly positive profit for firm k, since $\tilde{w} < \theta_L$. Contradiction.

□

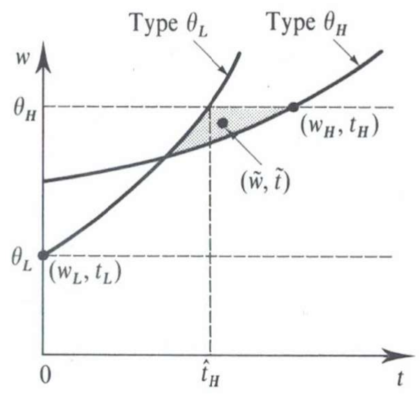
17 Lemma: In any separating SPE, the high-ability workers accept contract $(\theta_H, \hat{t}_H)$, where $\hat{t}_H$ satisfies $\theta_H - c(\hat{t}_H, \theta_L) = \theta_L - c(0, \theta_L)$.

Proof. (1) In any separating SPE, $(\theta_L, 0)$ is the contract for θ_L workers and (θ_H, t_H) is the contract for θ_H workers. In the following, we shall determine t_H.

(2) For θ_L workers, $t_H \geq \hat{t}_H$; otherwise, θ_L workers will choose the contract (θ_H, t_H).

(3) Suppose that $t_H > \hat{t}_H$.

(4) Then either firm can deviate by offering, in addition to its current contracts, a contract $(\tilde{w}, \tilde{t})$ lying in the shaded region.



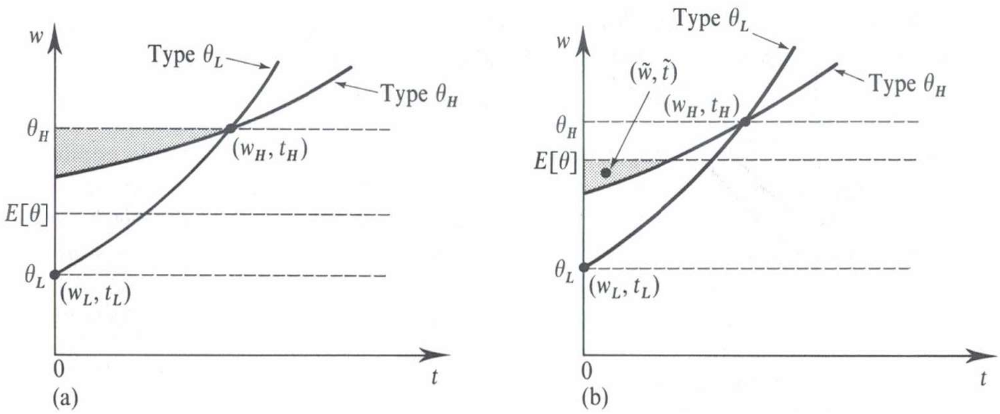
(5) This contract attracts all the θ_H workers and does not change the choice of θ_L workers.

(6) This deviation leads to a strictly positive profit, since $\tilde{w} < \theta_H$. Contradiction.

□

18 The existence of separating SPE: We just know what any equilibrium must look like, but we do not know whether one exists.

19 Example 1 on nonexistence.



(1) Assume both firms offer contracts $(\theta_i, 0)$ and (θ_H, t_H) as in Lemmas.

(2) Either firm can deviate to offer a single contract $(\tilde{w}, \tilde{t})$ (right figure).

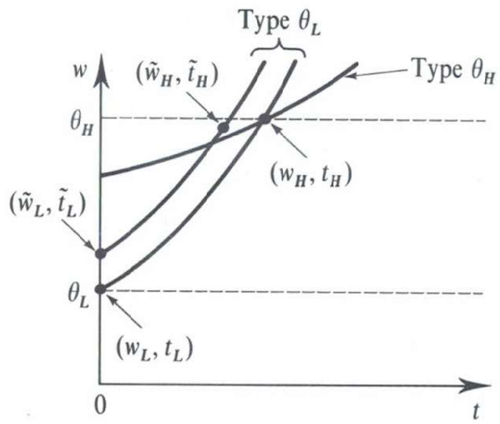
(3) This contract attracts all the workers.

(4) On the other hand, the deviating firm earns strictly positive profit: $\mathbf{E}[\theta] > \tilde{w}$.

Note that the single contract attracts all the workers if and only if the contract lies in the shaded region. If the line $w = \mathbf{E}[\theta]$ is below the shaded region, then the single contract does not give a strictly positive profit for the deviating firms. (left figure)

Note that no firm can earn strictly positive profits by deviating in a manner that attracts either only high-ability workers or only low-ability workers.

20 Example 2 on nonexistence.



(1) Assume both firms offer contracts $(\theta_i, 0)$ and (θ_H, t_H) as in Lemmas.

(2) Either firm can deviate to offer $(\tilde{w}_L, \tilde{t}_L)$ and $(\tilde{w}_H, \tilde{t}_H)$.

(3) θ_L workers will choose $(\tilde{w}_L, \tilde{t}_L)$ and θ_H workers will choose $(\tilde{w}_H, \tilde{t}_H)$.

(4) If the profit is strictly positive, then this deviation breaks the separating contracts $(\theta_i, 0)$ and (θ_H, t_H).

21 Welfare: We focus on the case when a SPE exists.

- Asymmetric information leads to Pareto inefficient outcomes: high-ability workers end up signing contracts that make them engage in useless tasks merely to distinguish themselves from low-ability workers.

- The low-ability workers are worse off when screening is possible than when it is not.
- Since a SPE exists, the high-ability workers are better off when screening is possible.
- The SPE outcome is constrained Pareto optimal.

3 Homework

- Key: The SPE contracts in competitive screening.
- Reading: 13.D