

ADVANCED MICROECONOMICS I: LECTURE NOTES 5

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- 1 In a principal-agent problem (or an agency model), one party, called an agent (代理人), acts on behalf of another party, called the principal (委托人).
 - In adverse selection models (or hidden information), the agent has private information about his type before the contract is written.
 - In moral hazard models (or hidden actions), the agent becomes privately informed after the contract is written.
- 2 Screening: Uninformed parties take step to distinguish/screen the types of informed parties.
 - In competitive screening, there are several competing firms.
 - In monopolistic screening, there is a single firm screening workers.

1 Adverse selection

- 3 An owner (principal) wishes to hire a manager (agent) to run a one-time project.

If the agent's effort level is $e \in [0, \infty)$, then principal's income is $\pi(e)$, with $\pi(0) = 0$, $\pi'(e) > 0$, and $\pi''(e) < 0$ for all e .

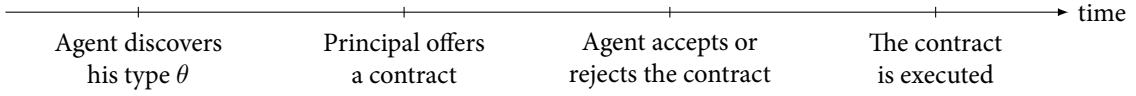
If the principal pays wage w to the agent, his utility/profit is $\pi(e) - w$.
- 4 The agent is an expected utility maximizer with utility $v(w - g(e, \theta))$.
 - $\theta \in \{\theta_L, \theta_H\}$ represents agent's ability. Here, $\theta_H > \theta_L$ and $\text{Prob}(\theta_H) = \lambda \in (0, 1)$.
 - $g(e, \theta)$ measures the cost/disutility of effort.
 - $g(0, \theta) = 0, g_e(e, \theta) \begin{cases} > 0, & \text{if } e > 0 \\ = 0, & \text{if } e = 0 \end{cases}, g_{ee} > 0, g_\theta < 0, g_{e\theta}(e, \theta) \begin{cases} < 0, & \text{if } e > 0 \\ = 0, & \text{if } e = 0 \end{cases}$.

\Rightarrow The agent's indifference curves have single-crossing property.

 - The agent is risk averse: $v' > 0$ and $v'' < 0$.¹
 - The agent has a reservation utility \bar{u} .
- 5 The economic variables are effort level e and the wage w . These variables are both observable and verifiable by a third party such as a benevolent court of law.

A contract is a pair (e, w) . Let \mathcal{A} be the set of all feasible contracts, that is, $\mathcal{A} = \{(e, w) \mid e \in \mathbb{R}_+, w \in \mathbb{R}\}$.
- 6 The sequence of play is as follows:

¹Question: How about when the manager is risk neutral?



2 Complete information

- 7 First suppose that there is no asymmetry of information between the principal and the agent, i.e., θ is observable.
- 8 The principal will try to maximize her utility subject to inducing the agent to accept the proposed contract. Clearly, the agent obtains \bar{u} if he does not take the principal's contract. So the principal will solve the following problem:

$$\begin{aligned} & \underset{(e_i, w_i) \in \mathcal{A}}{\text{maximize}} && \pi(e_i) - w_i \\ & \text{subject to} && v(w_i - g(e_i, \theta_i)) \geq \bar{u}. \end{aligned}$$

- 9 In any solution, the IR constraint must bind; otherwise, the principal could lower the wage offered and still have the agent accept the contract. Thus, the maximization problem becomes:

$$\max_{(e_i, w_i) \in \mathcal{A}} \pi(e_i) - v^{-1}(\bar{u}) - g(e_i, \theta_i).$$

Clearly, $\pi'' - g_{ee} < 0$. Then the solution (e_i^*, w_i^*) must satisfy the first-order condition:

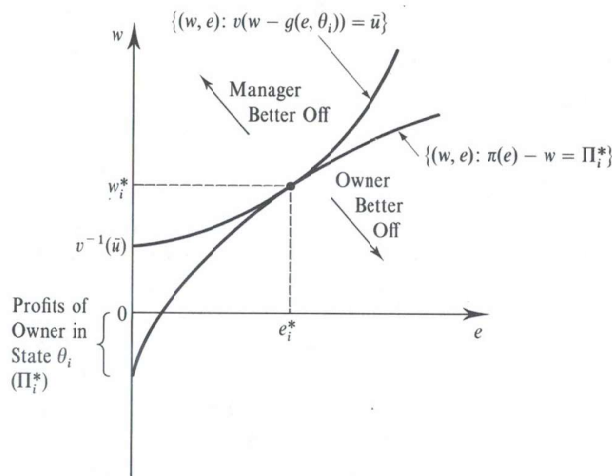
$$\pi'(e_i^*) \begin{cases} \leq g_e(e_i^*, \theta_i), \\ = g_e(e_i^*, \theta_i), & \text{if } e_i^* > 0. \end{cases}$$

Since $\pi'(0) > 0$ and $g_e(0, \theta_i) = 0$, we have that $e_i^* > 0$. Thus,

$$\pi'(e_i^*) = g_e(e_i^*, \theta_i).$$

Interpretation: The optimal level of effort e_i^* (for θ_i agent) equals the principal's marginal value and the agent's marginal cost.

- 10 Graphic illustration



- Agent's reservation utility is \bar{u} , which is equivalent to the contract $(0, v^{-1}(\bar{u}))$.
- Principal seeks to find the most profitable point on the indifference curve with utility \bar{u} , i.e., through the point $(0, v^{-1}(\bar{u}))$.
- For a θ_i agent, principal pays the wage w_i^* such that $w_i^* - g(e_i^*, \theta_i) = v^{-1}(\bar{u})$.
- For a θ_i agent, principal's profit is $\Pi_i^* = \pi(e_i^*) - v^{-1}(\bar{u}) - g(e_i^*, \theta_i)$.

This profit is exactly equal to the distance from the origin to the intersection point between the indifference curve through (e_i^*, w_i^*) and the vertical axis: letting $e = 0$ in the indifference curve $\pi(e) - w = \Pi_i^*$, we have $-w = \Pi_i^*$.

- If \bar{u} is small (especially, $\bar{u} = 0$), then this profit could be strictly positive. If \bar{u} is very large, this profit could be negative; in this case, the principal will not provide such a contract—the shutdown occurs.

Interpretation: If agent's reservation utility is low, principal can attract him to accept some contract; otherwise, agent will not accept any contract that is acceptable for principal.

11 Note: This equation $\pi'(e_i^*) = g_e(e_i^*, \theta_i)$ may not have a solution. For example, we consider the following case

- $\pi'(\cdot)$ is strictly decreasing and has a lower bound $\underline{\pi}' > 0$;
- $g_e(\cdot, \theta)$ is strictly increasing and has an upper bound $\bar{g}_e > 0$;
- $\underline{\pi}' > \bar{g}_e$.

Then such an equation does not have a solution. To guarantee the existence of a solution, we have to have more assumptions.

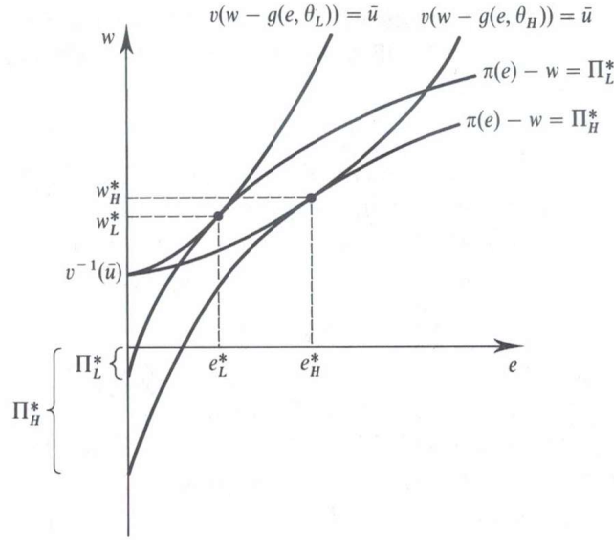
12 Since $\theta_H > \theta_L$, $\pi'' < 0$, $g_{e\theta} < 0$, $g_{ee} > 0$, $\pi'(e_i^*) = g_e(e_i^*, \theta_i)$ for $i \in \{H, L\}$, we have $e_H^* > e_L^*$:

- It is impossible that $e_H^* = e_L^*$.
- If $e_H^* < e_L^*$, then we have

$$\pi'(e_H^*) > \pi'(e_L^*) \text{ and } g_e(e_H^*, \theta_H) < g_e(e_L^*, \theta_H) < g_e(e_L^*, \theta_L).$$

Contradiction.

Interpretation: the optimal effort level of a high-ability agent is greater than that of a low-ability agent.



- 13 In the figure, the wage w_H^* is greater than w_L^* , but we note that w_H^* can be greater or smaller than w_L^* depending on the curvature of the functions π , g , and v , as it can be easily seen graphically.
- 14 Every agent (no matter θ_H or θ_L) obtains exactly \bar{u} from principal, just balancing his reservation utility.
- 15 The principal's profit:

$$\Pi_H^* = \overbrace{\pi(e_H^*) - g(e_H^*, \theta_H) - v^{-1}(\bar{u})}^{e_H^* \text{ maximizes } \pi(e) - v^{-1}(\bar{u}) - g(e, \theta_H)} \geq \underbrace{\pi(e_L^*) - g(e_L^*, \theta_H) - v^{-1}(\bar{u})}_{\theta_L < \theta_H} \geq \pi(e_L^*) - g(e_L^*, \theta_L) - v^{-1}(\bar{u}) = \Pi_L^*.$$

- 16 For contract to be always carried out, it is thus enough that profit is positive for a θ_L agent, i.e., the following condition must be satisfied

$$\Pi_L^* = \pi(e_L^*) - g(e_L^*, \theta_L) - v^{-1}(\bar{u}) \geq 0,$$

i.e., $\bar{u} \leq v(\pi(e_L^*) - g(e_L^*, \theta_L))$. We will maintain this hypothesis hereafter.

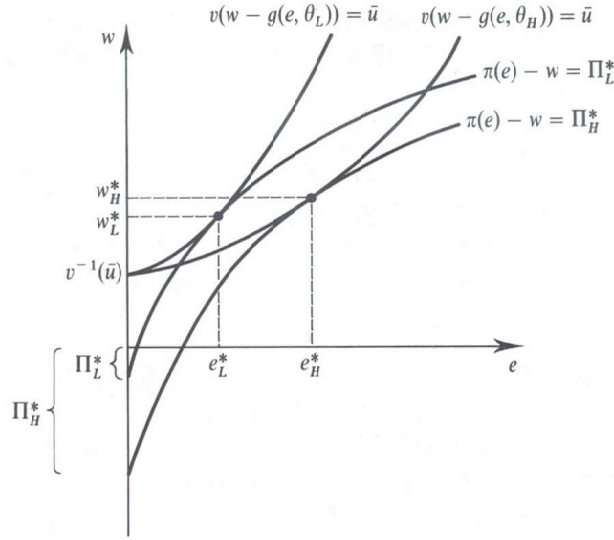
- 17 First-best contract menu $\{(e_i^*, w_i^*)\}_{i=H,L}$.

To implement the first-best effort levels e_H^* and e_L^* , the principal can make the following take-it-or-leave-it offers to the agent: If $\theta = \theta_H$ (resp. θ_L), the principal offers the wage w_H^* (resp. w_L^*) for the effort level e_H^* (resp. e_L^*) with $w_i^* - g(e_i^*, \theta_i) = v^{-1}(\bar{u})$.

Whatever his type, agent accepts the offer and makes utility \bar{u} . The complete-information optimal contracts are thus (e_H^*, w_H^*) if $\theta = \theta_H$ and (e_L^*, w_L^*) if $\theta = \theta_L$.

3 Incomplete information

- 18 Suppose that θ is the agent's private information.
- 19 Consider the case where the principal offers the menu of first-best contracts $\{(e_H^*, w_H^*), (e_L^*, w_L^*)\}$ hoping that an agent with type θ_L will select (e_L^*, w_L^*) and an agent with type θ_H will select instead (e_H^*, w_H^*) .



We see that (e_L^*, w_L^*) is preferred to (e_H^*, w_H^*) by both types of agents:

- The θ_H -agent's isoutility curve that passes through (e_L^*, w_L^*) corresponds to a utility level higher than \bar{u} at (e_H^*, w_H^*) .
- The θ_L -agent's isoutility curve that passes through (e_H^*, w_H^*) corresponds to a utility level lower than \bar{u} at (e_L^*, w_L^*) .

Offering the menu of contracts $\{(e_H^*, w_H^*), (e_L^*, w_L^*)\}$ fails to have the agents self-selecting properly within this menu. The high-ability agent mimics the low-ability one and selects also contract (e_L^*, w_L^*) . The complete information optimal contracts can no longer be implemented under asymmetric information.

20 Definition: A menu of contracts $\{(e_L, w_L), (e_H, w_H)\}$ is incentive compatible when (e_L, w_L) is weakly preferred to (e_H, w_H) by the type- θ_L agent and (e_H, w_H) is weakly preferred to (e_L, w_L) by the type- θ_H agent.

Mathematically,

$$w_L - g(e_L, \theta_L) \geq w_H - g(e_H, \theta_L), \quad (\text{IC}_L)$$

$$w_H - g(e_H, \theta_H) \geq w_L - g(e_L, \theta_H). \quad (\text{IC}_H)$$

21 If a menu of contracts $\{(e_L, w_L), (e_H, w_H)\}$ is incentive compatible, then $e_H \geq e_L$, which is called the monotonicity constraint. Indeed,

$$\int_{e_L}^{e_H} g_e(e, \theta_L) de = \overbrace{g(e_H, \theta_L) - g(e_L, \theta_L)}^{\text{By Equation (IC}_L\text{)}} \geq \underbrace{w_H - w_L}_{\text{By Equation (IC}_H\text{)}} \geq g(e_H, \theta_H) - g(e_L, \theta_H) = \int_{e_L}^{e_H} g_e(e, \theta_H) de,$$

and hence $e_H \geq e_L$.

If $e_H \neq e_L$, only one of (IC_L) and (IC_H) can bind.

22 Definition: A menu of contracts $\{(e_L, w_L), (e_H, w_H)\}$ is individually rational if

$$w_L - g(e_L, \theta_L) \geq v^{-1}(\bar{u}), \quad (\text{IR}_L)$$

$$w_H - g(e_H, \theta_H) \geq v^{-1}(\bar{u}). \quad (\text{IR}_H)$$

- 23 Information rent: Under complete information, the principal is able to maintain all types of agents at their reservation utility. Their respective utility levels at the first-best contracts satisfy

$$w_H^* - g(e_H^*, \theta_H) = v^{-1}(\bar{u}) \text{ and } w_L^* - g(e_L^*, \theta_L) = v^{-1}(\bar{u}).$$

Generally this will not be possible anymore under incomplete information, at least when the principal wants both types of agents to be active.

Let $r_H = w_H - g(e_H, \theta_H) - v^{-1}(\bar{u})$ and $r_L = w_L - g(e_L, \theta_L) - v^{-1}(\bar{u})$ denote the respective information rent (the utility in excess of the reservation utility) of each type.

信息租金是由于 agent 拥有比 principal 更多的信息而获得的额外收益。Principal 的问题是选择一份最明智的方式，对某些类型的 agent 让渡一些信息租金，而获得一个尽可能大的利润。

- 24 The principal's problem is to solve

$$\begin{aligned} & \underset{(e_L, w_L), (e_H, w_H)}{\text{maximize}} && \lambda(\pi(e_H) - w_H) + (1 - \lambda)(\pi(e_L) - w_L) \\ & \text{subject to} && \text{Equations (IC}_L\text{)} - \text{(IR}_H\text{)}. \end{aligned}$$

We can rewrite as $\lambda(\pi(e_H) - g(e_H, \theta_H) - v^{-1}(\bar{u})) + (1 - \lambda)(\pi(e_L) - g(e_L, \theta_L) - v^{-1}(\bar{u})) - \underbrace{[\lambda r_H + (1 - \lambda)r_L]}_{\text{expected information rent}}.$

上面这个表达式清楚地揭示了：principal 希望最大化“配置效率减去信息租金”。Principal 愿意接受一定程度上的配置扭曲，以便减少支付给 agent 的信息租金。

3.1 Solving the principal's problem

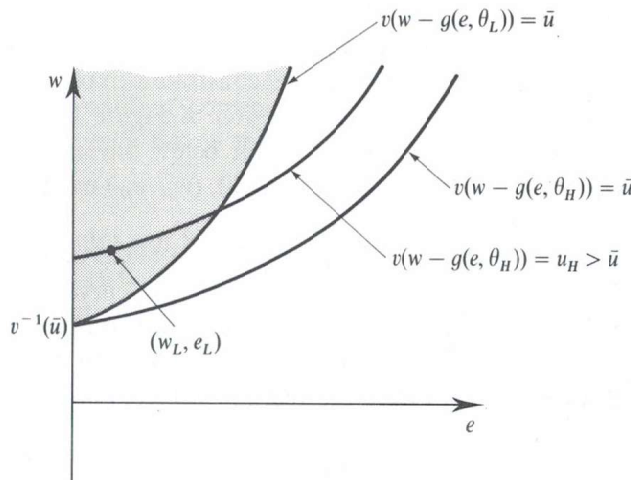
- 25 Lemma: The constraint (IR_H) is always satisfied due to constraints (IC_H) and (IR_L).

Proof.

$$w_H - g(e_H, \theta_H) \geq w_L - g(e_L, \theta_H) \geq w_L - g(e_L, \theta_L) \geq v^{-1}(\bar{u}).$$

□

- 26 Graphic illustration.



- (1) By constraint (IR_L), (e_L, w_L) must lie in the shaded region.
- (2) By constraint (IC_H), (e_H, w_H) must lie on or above the θ_H -indifference curve through (e_L, w_L) .
- (3) This implies that θ_H -agent's utility is at least \bar{u} .

27 Lemma: The constraint (IR_L) is binding at the optimal.

Proof. Suppose that $w_L - g(e_L, \theta_L) - v^{-1}(\bar{u}) = \varepsilon > 0$. Then the principal can decrease w_L by ε and consequently also w_H by ε and gain ε .

Notice that the constraint (IR_L) is still satisfied. In addition, constraints (IC_H) and (IC_L) are also satisfied. \square

It implies that $r_L = w_L - g(e_L, \theta_L) - v^{-1}(\bar{u}) = 0$ —no information rent for θ_L -agents.

28 Lemma: The constraint (IC_H) is binding at the optimal.

Proof. Suppose that $[w_H - g(e_H, \theta_H)] - [w_L - g(e_L, \theta_H)] = \varepsilon > 0$. Then the principal can decrease w_H by ε and gain $\lambda\varepsilon$. \square

It implies that $r_H = w_H - g(e_H, \theta_H) - v^{-1}(\bar{u}) = (w_L - g(e_L, \theta_H)) - (w_L - g(e_L, \theta_L)) = g(e_L, \theta_L) - g(e_L, \theta_H) > 0$.

29 Ignoring constraint (IC_L), we obtain a reduced program

$$\max_{e_L, e_H} \lambda(\pi(e_H) - g(e_H, \theta_H) - v^{-1}(\bar{u}) + \underbrace{g(e_L, \theta_H) - g(e_L, \theta_L)}_{-r_H}) + (1 - \lambda)(\pi(e_L) - g(e_L, \theta_L) - v^{-1}(\bar{u})).$$

Compared with the full information setting, asymmetric information alters the principal's optimization simply by the subtraction of the expected rent that has to be given up to the efficient type (θ_H). The inefficient type (θ_L) gets no rent, but the efficient type θ_H gets the information rent that he could obtain by mimicking the inefficient type θ_L . This rent depends only on the effort level requested from this inefficient type.

30 The first order condition on e_H implies

$$\pi'(e_H^{\text{SB}}) = g_e(e_H^{\text{SB}}, \theta_H), \text{ that is, } e_H^{\text{SB}} = e_H^*.$$

Hence, there is no distortion away from the first-best output for the efficient type.

Notice that: $\pi'(0) > 0$, $\pi'' < 0$, $g_e(0, \theta_H) = 0$, and $g_{ee} > 0$, such a $e_H^{\text{SB}} > 0$ exists.

31 The first order condition on e_L implies

$$(1 - \lambda) \cdot (\pi'(e_L^{\text{SB}}) - g_e(e_L^{\text{SB}}, \theta_L)) = \lambda \cdot (g_e(e_L^{\text{SB}}, \theta_L) - g_e(e_L^{\text{SB}}, \theta_H)).$$

This equation expresses the important trade-off between efficiency and rent extraction which arises under asymmetric information. The expected marginal efficiency gain (resp. cost) and the expected marginal cost (resp. gain) of the rent brought about by an infinitesimal increase (resp. decrease) of θ_L agent's output are equated. Thus, the principal is neither willing to increase nor to decrease θ_L agent's effort.

Notice: Such a $e_L^{\text{SB}} > 0$ exists: $\pi'(0) > 0$, $g_e(0, \theta_L) = g_e(0, \theta_H) = 0$, $\pi'' < 0$, and $g_{ee} > 0$.

32 Note: In several other setups, the equation for e_L^{SB} may not have a positive solution. In that case, θ_L -agent will shut down in the optimal contract for asymmetric environment. See Section 2.6.3 in Laffont and Martimont (2002).

33 Since $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$ and $\pi'(e_L^{SB}) = g_e(e_L^{SB}, \theta_L) + \frac{\lambda}{1-\lambda}[g_e(e_L^{SB}, \theta_L) - g_e(e_L^{SB}, \theta_H)]$, we have the following inequality

$$e_H^{SB} = e_H^* > \underbrace{e_L^*}_{\pi'' < 0} > e_L^{SB},$$

and hence

$$\begin{aligned} w_L^{SB} - g(e_L^{SB}, \theta_L) - w_H^{SB} + g(e_H^{SB}, \theta_L) &= g(e_L^{SB}, \theta_H) - g(e_H^{SB}, \theta_H) - g(e_L^{SB}, \theta_L) + g(e_H^{SB}, \theta_L) \\ &= \int_{e_L^{SB}}^{e_H^{SB}} [g_e(e, \theta_L) - g_e(e, \theta_H)] de \geq 0. \end{aligned}$$

That is, the constraint (IC_L) is strictly satisfied.

这点说明：向上的激励相容条件（upward incentive compatibility, θ_L 模仿 θ_H ）不是问题。另一方面，向下的激励相容条件（downward incentive compatibility, θ_H 模仿 θ_L ）更为关键，需要谨慎处理。

34 Proposition: Under asymmetric information, the optimal menu of contracts entails:

- No output distortion for the high-ability agent with respect to the first-best, $e_H^{SB} = e_H^*$. A downward output distortion for the low-ability agent, $e_L^{SB} < e_L^*$ with

$$\pi'(e_L^{SB}) = g_e(e_L^{SB}, \theta_L) + \frac{\lambda}{1-\lambda}[g_e(e_L^{SB}, \theta_L) - g_e(e_L^{SB}, \theta_H)].$$

- The second-best wages are respectively given by

$$\begin{aligned} w_H^{SB} &= g(e_H^{SB}, \theta_H) + v^{-1}(\bar{u}) + \underbrace{g(e_L^{SB}, \theta_L) - g(e_L^{SB}, \theta_H)}_{r_H} > g(e_H^*, \theta_H) + v^{-1}(\bar{u}) = w_H^*, \\ w_L^{SB} &= g(e_L^{SB}, \theta_L) + v^{-1}(\bar{u}) < g(e_L^*, \theta_L) + v^{-1}(\bar{u}) = w_L^*. \end{aligned}$$

$$\text{Moreover, } w_H^{SB} = g(e_H^{SB}, \theta_H) + \underbrace{v^{-1}(\bar{u}) + g(e_L^{SB}, \theta_L) - g(e_L^{SB}, \theta_H)}_{w_L^{SB}} > w_L^{SB}.$$

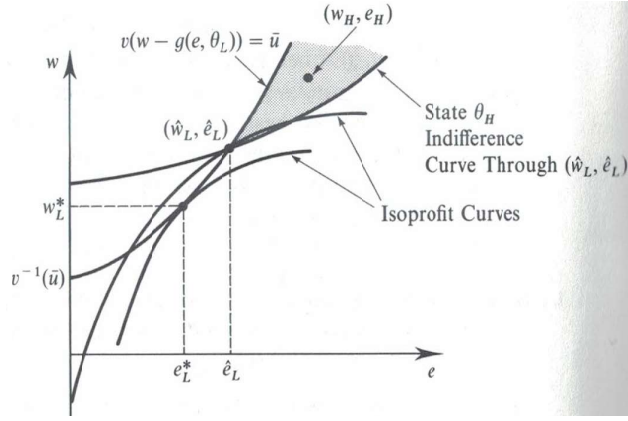
- Only the high-ability agent gets a positive information rent given by

$$r_H^{SB} = g(e_L^{SB}, \theta_L) - g(e_L^{SB}, \theta_H).$$

35 “顶部无扭曲”与“单向扭曲”是两条最基本的规律。

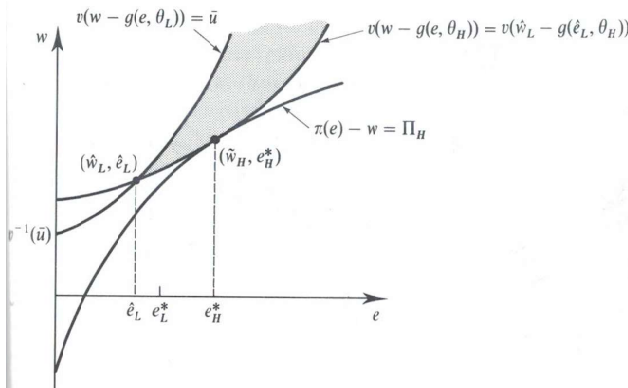
- 对于 θ_H ，不存在劳动水平的扭曲（其劳动水平与完全信息最优时的劳动水平一致），但代价是需要给其支付信息租金。
- 对于 θ_L ，其付出的劳动水平低于完全信息最优时的劳动水平，但没有信息租金。

36 Graphic illustration for $e_L^{SB} \leq e_L^*$.



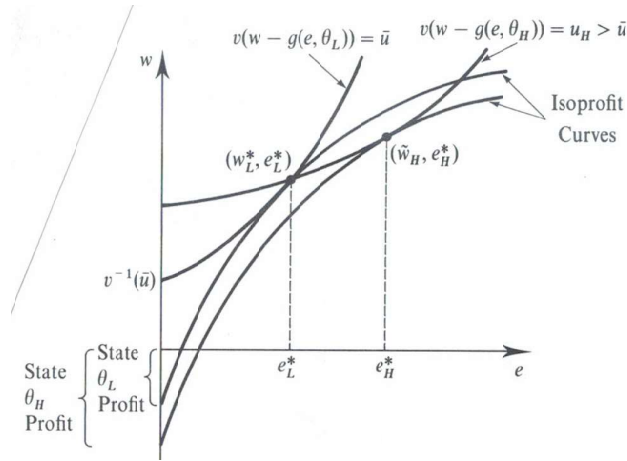
- (1) Suppose that $e_L^{SB} > e_L^*$.
- (2) Since θ_L -IR binds, (e_L^{SB}, w_L^{SB}) lies on the indifference curve through $v^{-1}(\bar{u})$.
- (3) To make θ_L -IC and θ_H -IC hold, (e_H^{SB}, w_H^{SB}) lies in the shade region.
- (4) Principal can raise her profit by moving (e_L^{SB}, w_L^{SB}) to (e_L^*, w_L^*) : θ_L -IC and θ_H -IC still hold.
- (5) Thus, $e_L^{SB} > e_L^*$ cannot be optimal.

37 Graphic illustration for $e_H^{SB} = e_H^*$.

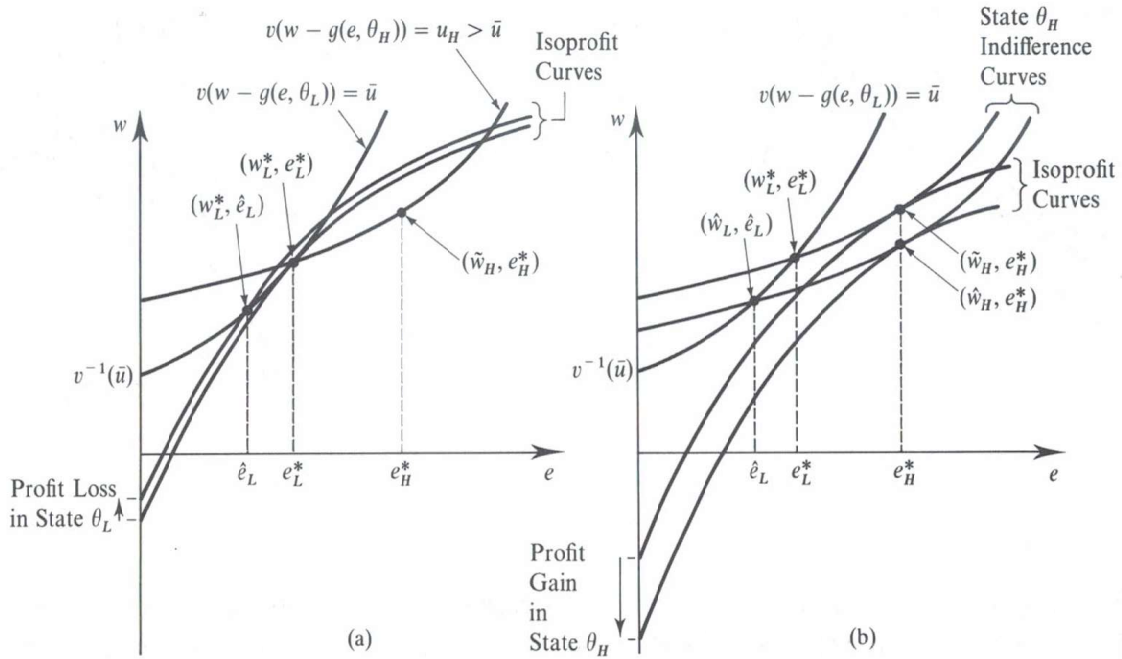


- (1) Suppose that $e_L^{SB} \leq e_L^*$.
- (2) To make θ_L -IC and θ_H -IC hold, (e_H^{SB}, w_H^{SB}) lies in the shade region.
- (3) Principal's problem is to find the allocation of (e_H^{SB}, w_H^{SB}) that maximizes her profit.
- (4) The optimal solution occurs at a point of tangency between the indifference curve of θ_H agent through (e_H^{SB}, w_H^{SB}) and an isoprofit curve for principal.
- (5) All points of tangency between indifference curves of θ_H -agent and isoprofit curves of principal occur at e_H^* .

38 Graphic illustration for the optimal contracts.



- (1) Suppose that principal starts with (e_L^*, w_L^*) for θ_L agents, which lies on the θ_L indifference curve through $(0, v^{-1}(\bar{u}))$.
- (2) Since θ_H -IC binds, principal could choose (e_H^*, \tilde{w}_H) for θ_H agents, which lies on θ_H indifference curve through (e_L^*, w_L^*) .
- (3) The menu $\{(e_H^*, \tilde{w}_H), (e_L^*, w_L^*)\}$ is IC. However, principal can do better.



- (1) Principal firstly moves (e_L^*, w_L^*) to (e_L^{SB}, w_L^{SB}) , where $e_L^* > e_L^{SB}$. Note that both (e_L^*, w_L^*) and (e_L^{SB}, w_L^{SB}) lie on θ_L indifference curve through $v^{-1}(\bar{u})$.
- (2) This change lowers the profit that principal earns from θ_L agents.
- (3) On the other hand, it relaxes θ_H -IC.
- (4) Principal then moves (e_H^*, \tilde{w}_H) to (e_H^*, \hat{w}_H) .
- (5) This change increases the profit that principal earns from θ_H agents.
- (6) Comparison:

- The derivative of principal's profit from θ_L agent with respect to e_L at e_L^* is zero:

$$\frac{d}{de_L} [\pi(e_L) - g(e_L, \theta_L) - v^{-1}(\bar{u})] \Big|_{e_L=e_L^*} = 0.$$

- The derivative of principal's profit from θ_H agent with respect to e_L at e_L^* is strictly negative:

$$\frac{d}{de_L} [\pi(e_H^*) - g(e_H^*, \theta_H) - v^{-1}(\bar{u}) + g(e_L, \theta_H) - g(e_L, \theta_L)] \Big|_{e_L=e_L^*} < 0.$$

(7) How far should principal go in lowering e_L —When the marginal loss from θ_L agent equals the marginal gain from θ_H agent, i.e.,

$$(1 - \lambda) \cdot [\pi'(e_L^{SB}) - g_e(e_L^{SB}, \theta_L)] = \lambda \cdot [g_e(e_L^{SB}, \theta_L) - g_e(e_L^{SB}, \theta_H)].$$

39 配置效率与信息租金之间的权衡：

- 为了让 θ_H 选择为其设计的劳动水平，需要给他一定好处的信息租金；该信息租金取决于 θ_L 的劳动水平，以及 θ_H 和 θ_L 。
- 之所以降低 θ_L 的劳动水平，是为了尽可能减少支付给 θ_H 的信息租金。

4 Homework

- Key: The optimal contracts in monopolistic screening.

偏离最优的次优合约将导致劳动水平的扭曲，委托人需要让渡一些信息租金给最有效率的代理人。

- Reading: 14.C in MWG, 2.1–2.9 in *The Theory of Incentives*
- Optional reading: 16.1–16.2 in 高级微观经济学 (田国强)