Advanced Microeconomics I: Lecture Notes 9

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1 Externalities

1 An externality is present whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy.

In particular, an externality is a cost or a benefit imposed upon someone by actions taken by others.

在具体的分析中，对外部性的界定是比较复杂的。

2 When we say "directly," we mean to exclude any effects that are mediated by prices.

3 An externality imposed benefit is a positive externality:

- a well-maintained garden next door,
- the pleasant scent of the perfume wore by a lady seated next to you,
- etc.

4 An externality imposed cost is a negative externality:

- gas emission,
- water pollution,
- second-hand cigarette smoke,
- etc.

5 Goal: We explore the implications of external effects for competitive equilibria and public policy.

6 There are two consumers (1 and 2) and L goods. Consumer i's initial wealth is w_i.

We suppose that the actions of these consumers do not affect the prices $p \in \mathbb{R}^L$.

7 We assume that each consumer has preference not only over her consumption $x_i = (x_{1i}, x_{2i}, \ldots, x_{Li})$ but also over some action $h \in \mathbb{R}_+$ taken by consumer 1.

Consumer i's utility function takes the form $u_i(x_i, h)$. We assume that $\frac{\partial u_2}{\partial h} \neq 0$. Consumer 1's choice of h affects consumer 2's well-being, then it generates an externality.

8 It is convenient to define a derived utility function:

$$\begin{aligned} v_i(p, w_i, h) = \underset{x_i}{\text{maximize}} \quad & u_i(x_i, h) \\ \text{subject to} \quad & p \cdot x_i \leq w_i. \end{aligned}$$

9 We also assume that consumers' utility functions take a quasilinear form. Thus, v_i has the form of $\phi_i(p,h)+w_i$.

Since prices are assumed to be unaffected, $\phi_i(p,h)$ can be simply rewritten as $\phi_i(h)$.

We assume that ϕ_i is twice differentiable with $\phi_i'' < 0$.

10 Suppose that there is a competitive equilibrium with prices p. Therefore, consumer 1 will choose h to maximize $\phi_1(h)$.

Since $\phi_1'' < 0$, the equilibrium level h^* satisfies the necessary and sufficient first-order condition

$$\phi_1'(h^*) \leq 0 \text{ with equality if } h^* > 0.$$

11 In any Pareto optimal allocation, the optimal level h^o maximizes the joint surplus of the two consumers, and so solves:

$$\max_{h\geq 0} \phi_1(h)+\phi_2(h).$$

h^o satisfies the necessary and sufficient first-order condition

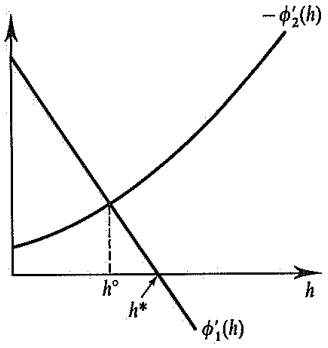
$$\phi_1'(h^o) \leq -\phi_2'(h^o) \text{ with equality if } h^o > 0.$$

12 When external effects are present, so that $\phi_2'(h) \neq 0$ at all h, the equilibrium level is not optimal unless $h^o = h^* = 0$ (角点解).

13 Suppose that we have interior solutions.

Case 1: If $\phi_2' < 0$, so h generates a negative externality. Then we have $\phi_1'(h^o) = -\phi_2'(h^o) > 0$. Since ϕ_1' is decreasing and $\phi_1'(h^*) = 0$, we have $h^* > h^o$.

Case 2: If $\phi_2' > 0$, so h generates a positive externality. Then we have $\phi_1'(h^o) = -\phi_2'(h^o) < 0$. Since ϕ_1' is decreasing and $\phi_1'(h^*) = 0$, we have $h^* < h^o$.



In the presence of externality, the competitive equilibrium outcome could be inefficient.

1.1 Solutions to externality problem

14 Quotas.

Suppose that h generates a negative externality, so $h^o < h^*$. The most direct sort of government intervention to achieve efficiency is the direct control of the externality-generating activity itself.

The government can simply mandate that h be no larger than h^o. With this constraint, consumer 1 will indeed fix the level at h^o.

15 Pigouvian taxation (庇古税).

之所以出现了上述偏离社会最优的结果，是因为消费者面对错误的价格，所以采取了错误的行为。因此，只要实施某个税收，纠正这种价格的错误，就可以导致有效的资源配置。

Suppose that consumer 1 is made to pay a tax of $t_h = -\phi_2'(h^o)$ per unit of h.

Then consumer 1 will choose the level that solves

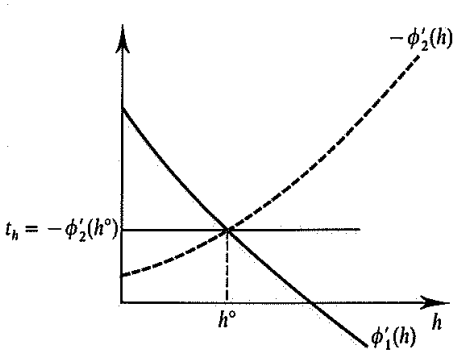
$$\max_h \phi_1(h) - t_h h.$$

The necessary and sufficient first-order condition is

$$\phi_1'(h) \leq t_h \text{ with equality if } h > 0.$$

Since $t_h = -\phi_2'(h^o)$, h^o satisfies the above condition.

When faced with this tax, consumer 1 carries out an individual cost-benefit computation that internalizes the externality that she imposes on consumer 2.



16 Property rights (产权).

We establish enforceable property rights with regard to the externality-generating activity.

For example, we assign the right to an externality-free environment to consumer 2. In this case, consumer 1 is unable to engage in the externality-producing activity without consumer 2's permission.

We assume that consumer 2 makes consumer 1 a take-it-or-leave-it offer, demanding a payment T in return for permission to generate externality level h.

- Consumer 1 will agree to this demand T if and only if she will be at least as well off as she would be by rejecting it, i.e., $\phi_1(h) - T \geq \phi_1(0)$.
- Hence consumer 2 will choose her offer (h, T) to solve

$$\begin{aligned} \underset{(h,T)}{\text{maximize}} \quad & \phi_2(h) + T \\ \text{subject to} \quad & \phi_1(h) - T \geq \phi_1(0). \end{aligned}$$

- Since the constraint is binding at the optimum, $T = \phi_1(h) - \phi_1(0)$. Therefore, consumer 2's optimal offer involves h that solves

$$\max_h \phi_2(h) + \phi_1(h) - \phi_1(0).$$

  Thus, the solution is precisely h^0.

If consumer 1 instead has the right to generate as much of the externality as she wants, the efficiency can be still achieved.

Coase Theorem: If trade of the externality can occur, then bargaining will lead to an efficient outcome no matter how property rights are allocated.

当出现外部性时，不论初始的产权是怎样分配的，市场交易都可以导致有效的结果。

2 Coase theorem

17 "Coase Theorem" is named after Ronald H. Coase, a winner of Nobel Prize.

Coase proposes two important concepts "transaction costs" and "property rights" in his two articles published in 1937 and 1960:

- The Nature of the Firm
- The Problem of Social Cost

The term "Coase Theorem" is first introduced by George Stigler in 1966.

18 Theorem: If trade of an externality is possible and there are no transaction costs, bargaining among agents will lead to an efficient outcome regardless of the initial allocation of property rights.

- Efficiency: the efficient outcome (or allocation) can be achieved through bargaining. (本质是通过市场交易将外部性内生化)
- Invariance: the allocation of property rights will not affect the bargaining outcome (which is always efficient).

19 Suppose both agents bargain over the choice of h through a Nash bargaining process: the bargaining power of agent 1 is α and agent 2 is $1-\alpha$, where $\alpha \in [0,1]$. The relative bargaining power of each agent can be affected by the ownership (or property rights).

Without loss of generality, let agent 2 claim a transfer T from agent 1, where T can be negative.

We will see the initial allocation of property rights does not affect the efficiency outcome, but will only impact the distribution of surplus.

20 Case 1: the right of engaging the activity (property right) is allocated to agent 1. If they cannot achieve an agreement, then the outcome is h^*.

- The action h and transfer T solve

$$\max_{h,T} \left[\phi_1(h) - T - \phi_1(h^*)\right]^{\alpha} \left[\phi_2(h) + T - \phi_2(h^*)\right]^{1-\alpha}.$$

- The bargaining outcome is (h^1, T^1), where

$$h^1 = h^o \text{ and } T^1 = -\alpha\left[\phi_2(h^o) - \phi_2(h^*)\right] + (1-\alpha)\left[\phi_1(h^o) - \phi_1(h^*)\right].$$

- Two agents receive

$$v_1^1 = \alpha \left[\phi_1(h^o) + \phi_2(h^o) - \phi_1(h^*) - \phi_2(h^*)\right] + \phi_1(h^*),$$
$$v_2^1 = (1-\alpha) \left[\phi_1(h^o) + \phi_2(h^o) - \phi_1(h^*) - \phi_2(h^*)\right] + \phi_2(h^*).$$

21 Case 2: the right of engaging the activity (property right) is allocated to agent 2. If they cannot achieve an agreement, then the outcome is 0.

- The action h and transfer T solve

$$\max_{h,T} \left[\phi_1(h) - T - \phi_1(0)\right]^{\alpha} \left[\phi_2(h) + T - \phi_2(0)\right]^{1-\alpha}.$$

- The bargaining outcome is (h^2, T^2), where

$$h^2 = h^o \text{ and } T^2 = -\alpha \left[\phi_2(h^o) - \phi_2(0)\right] + (1-\alpha) \left[\phi_1(h^o) - \phi_1(0)\right].$$

- Two agents receive

$$v_1^2 = \alpha \left[\phi_1(h^o) + \phi_2(h^o) - \phi_1(0) - \phi_2(0)\right] + \phi_1(0),$$
$$v_2^2 = (1-\alpha) \left[\phi_1(h^o) + \phi_2(h^o) - \phi_1(0) - \phi_2(0)\right] + \phi_2(0).$$

22 In both Cases 1 and 2, the efficient outcome is achieved, i.e., $h^1 = h^2 = h^o$.

The initial allocation of property rights can affect agents' payoffs, i.e., $v_i^1 \neq v_i^2$ for $i = 1, 2$.

23 Remark:

- Aside from transaction costs, the efficient allocation can be achieved (i.e., the externality is internalized).
- As long as property rights are clearly defined, the allocation of these property rights does not matter.
- The allocation outcome is not affected by the institutional arrangements:
  - contracts, quota, taxes (subsides) or a market.

24 Proposition: If transaction costs are not neglectable, bargaining among agents may not lead to an efficient outcome. The allocation would depend on the initial allocation of property rights.

25 Suppose there is a positive cost C if both agents want to negotiate over the choice of h.

26 Consider following three cases:

- Case 1: no transaction and agent 1 has the ownership:
  - The outcome is h^* and the total surplus is $\phi_1(h^*) + \phi_2(h^*)$.
- Case 2: no transaction and agent 2 has the ownership:
  - The outcome is $h = 0$ and the total surplus is $\phi_1(0) + \phi_2(0)$.
- Case 3: both agents bargain:
  - The outcome is h^o and the total surplus is $\phi_1(h^o) + \phi_2(h^o) - C$.

Each of the three outcomes can be the most efficient one!

27 Remark:

- When there are positive transaction costs, the efficient allocation outcome may not always be achieved.
- The initial allocation of property rights and institutional arrangements matter a lot in affecting the allocative efficiencies of resources.
- Transaction costs play a crucial role in determining the optimal institutional arrangement and the way how property rights should be legally determined.

3 Public goods

28 A public good is a commodity for which use of a unit of the good by one agent does not preclude its use by other agents.

29 Example:

- Knowledge,
- Broadcast radio and TV programs,
- Public highways,
- Clean air,
- National parks,
- etc.

30 There are I consumers, 1 (pure) public good, and L usual goods.

31 We assume that each consumer's utility function is quasilinear. Therefore we can define, for each consumer, a derived utility function over the level of the public good.

Let x denote the quantity of the public good, we denote consumer i's utility from the public good by $\phi_i(x)$. We also assume that it is twice differentiable and $\phi_i''(x) < 0$ for all $x \geq 0$.

32 The cost of supplying q units of the public good is $c(q)$. We assume that $c(\cdot)$ is twice differentiable with $c''(q) > 0$ at all $q \geq 0$.

- The production of a desirable public good is costly: $\phi_i'(\cdot) > 0$ for all i and $c'(\cdot) > 0$.
- The reduction of a public bad is costly: $\phi_i'(\cdot) < 0$ for all i and $c'(\cdot) < 0$.

33 Any Pareto optimal allocation maximizes the aggregate surplus, and involve a level of the public good that solves:

$$\max_q \sum_{i=1}^{I} \phi_i(q) - c(q).$$

The necessary and sufficient first-order condition for q^o is

$$\sum_{i=1}^{I} \phi_i'(q^o) \leq c'(q^o) \text{ with equality if } q^o > 0.$$

社会最优的公共品供给应该使得其边际成本与所有人从该公共品中得到的边际收益之和相等。

34 Consider the case in which the public good is provided by means of private purchases by consumers. There is a market for the public good and each consumer i chooses how much of the public good to buy (x_i).

- The supply side is a single profit-maximizing firm with cost function $c(\cdot)$.

  At a competitive equilibrium with price p^*, each consumer i's purchase x_i^* must maximize her utility and solve

  $$\max_{x_i} \phi_i\Big(x_i + \sum_{k \neq i} x_k^*\Big) - p^* x_i.$$

  Thus, x_i^* satisfy the necessary and sufficient condition

  $$\phi_i'(x^*) \leq p^* \text{ with the equality if } x_i^* > 0,$$

  where $x^* = \sum x_i^*$.

- The firm's supply q^* solve

  $$\max_q p^* q - c(q),$$

  and satisfy the necessary and sufficient condition

  $$p^* \leq c'(q^*) \text{ with equality if } q^* > 0.$$

- At a competitive equilibrium, we have $q^* = x^*$. Then we have

  $$\sum_i I_{x_i^* > 0}[\phi_i'(q^*) - c'(q^*)] = 0.$$

- Whenever $I > 1$ and $q^* > 0$ (so that $x_i^* > 0$ for some i), we have

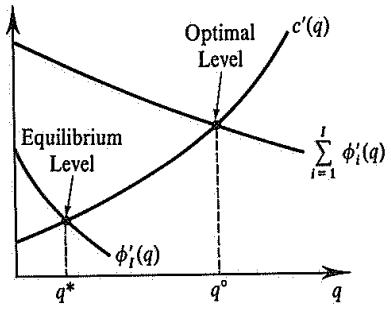
  $$\sum_i \phi_i'(q^*) > \phi_i'(q^*) = c'(q^*).$$

  Thus, $q^* < q^o$.

35 Inefficiency: each consumer's purchase provides a direct benefit not only to the consumer herself but also to every other consumer. Hence, private provision creates a situation in which externalities are present.

The failure of each consumer to consider the benefits for others of her public good provision is referred to as the free-rider problem: Each consumer has an incentive to enjoy the benefits of the public good provided by others while providing it insufficiently herself.

36 Graph



37 Quantity-based intervention (direct governmental provision) or price-based intervention in the form of taxes or subsidies.

38 Lindahl equilibria: Suppose that each consumer's consumption of the public good is a distinct commodity with its own market. We denote the price of this personalized good by p_i.

Given the equilibrium price p_i^{**}, each consumer i sees herself as deciding the total amount of the public good she will consume, x_i, so as to solve

$$\max_{x_i} \phi_i(x_i) - p_i^{**} x_i.$$

Her equilibrium consumption level x_i^{**} satisfies the necessary and sufficient condition

$$\phi_i'(x_i^{**}) \leq p_i^{**} \text{ with equality } x_i^{**} > 0.$$

The firm is viewed as producing a bundle of I goods with a fixed-proportions technology (i.e., the level of production of each personalized good is necessarily the same). Thus, the firm sloves

$$\max_{q} \sum_i p_i^{**} q - c(q).$$

The firm's equilibrium level of q^{**} satisfies the necessary and sufficient condition

$$\sum_i p_i^{**} \leq c'(q^{**}) \text{ with equality if } q^{**} > 0.$$

The market-clearing condition implies that $x_i^{**} = q^{**}$ for all i. And hence,

$$\sum_i \phi_i'(q^{**}) \leq c'(q^{**}) \text{ with equality if } q^{**} > 0.$$

Thus, $q^{**} = q^o$.

林达尔均衡使人们对公共品的供给水平取得了一致（$x_i^{**} = q^{**} = q^o$），同时分摊的成本 p_i^{**} 与边际收益 $\phi_i'(x_i^{**})$ 成比例。处于均衡状态时，均衡价格 p_i^{**} 使每个人需要的公共品的数量相同，并与应该提供的公共品的数量保持一致。因为每个人购买并消费了公共品的总产量，按照这些价格的供给恰好就是各个个人支付价格的总和。（类比众筹）

39 Problem of Lindahl equilibria: it is impossible to exclude a consumer from use of the public good from others' purchase.

4 Homework

- Reading: Chapter 11.2–11.3 in MWG.
- Homework: 11.B.3, 11.C.1