Advanced Microeconomics I: Lecture Notes 3

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1 Signaling for job market

1 The key to resolve the adverse selection: some "mechanisms/procedures" to help distinguish among workers.

2 Signaling is one of such mechanisms, which was first investigated by Spence (1973, 1974).

Basic idea: The high-ability workers may have (costly or costless) actions to distinguish themselves from low-ability workers.

3 The ideal case: Workers can take a costless test that reveals their types.

Then in any SPE, all workers with ability greater than $\underline{\theta}$ will take the test and the market will achieve the full information outcome.

4 In general, no procedure exists that directly reveals a worker's type.

5 There are two types of workers with productivities θ_L and θ_H, where $0 < \theta_L < \theta_H$ and $\lambda = \text{Prob}(\theta = \theta_H) \in (0, 1)$.

6 Before entering the job market, a worker can get some education, and the amount of education that a worker receives is observable.

The cost of obtaining education level e for a type-θ worker is given by $c(e, \theta)$. We assume $c(e, \theta)$ is twice continuously differentiable and $c(0, \theta) = 0$, $c_e(e, \theta) > 0$, $c_{ee}(e, \theta) > 0$, $c_\theta(e, \theta) < 0$ for all $e > 0$, and $c_{e\theta}(e, \theta) < 0$.

Assumption: The education does nothing for a worker's productivity.

7 Utility for a type-θ worker who chooses education level e and receives wage w is $w - c(e, \theta)$.

A type-θ worker can earn $r(\theta)$ by working at home.

8 For simplicity, assume $r(\theta) = 0$.

Thus, the unique equilibrium in the absence of the ability to signal: $w^* = \mathbf{E}[\theta]$.

9 Game

- A random move of nature determines whether the worker is of high or low ability.
- Conditional her type, the worker chooses how much education level to obtain. After that, the worker enters the market.
- Conditional the observed education level, two firms simultaneously make wage offers.
- The worker decides whether to work for a firm and, if so, which one.

Remark: Here we model only a single worker of unknown type. The model with many workers can be thought of as simply having many of these single-worker games going on simultaneously, with the fraction of high-ability workers in the market being λ.

2 PBE

10 A typical strategy of worker: $e(\theta)$. A typical strategy of firm: $w(e)$.

11 Perfect Bayesian equilibrium: a pair of strategy profiles and a belief function $\mu(e) \in [0, 1]$ giving the firms' common probability assessment that the worker is of high ability after observing education level e such that

- The worker's strategy $e^*(\theta)$ is optimal given the firms' strategies $w_1^*(e)$ and $w_2^*(e)$.
- The belief $\mu^*(e)$ is derived from the workers' strategies $e^*(\theta)$ via Bayes' rule when possible.
- Following each e (i.e., given each $\mu^*(e)$), the firms' wage offers $w_1^*(e)$ and $w_2^*(e)$ constitute a NE.

12 We focus on pure-strategy PBE.

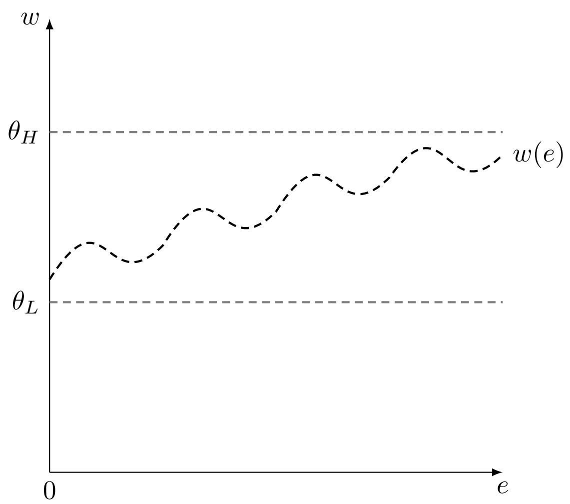
13 At the end of the game:

(1) After seeing the education level e, the firms have belief $\mu(e)$ that the worker is type θ_H.

(2) The expected productivity is $\mu(e)\theta_H + (1 - \mu(e))\theta_L$.

(3) Like Bertrand pricing game,[1] in any PBE, both firms offer wage $w(e) = \mu(e)\theta_H + (1 - \mu(e))\theta_L$.

For any e, $w(e) \in [\theta_L, \theta_H]$.



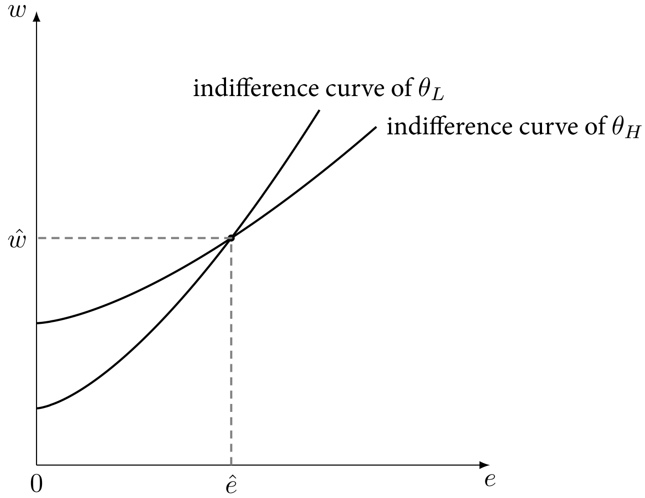
14 Single-crossing property: Due to the assumptions on $c(e, \theta)$, an indifference curve of type-θ_H worker and an indifference curve of type-θ_L worker cross only once.

A typical indifference curve of θ-worker is $w - c(e, \theta) =$ constant, i.e., $w = c(e, \theta) +$ constant. Then, at any (w, e), the marginal rate of substitution between wages and education is

$$\frac{\mathrm{d}w}{\mathrm{d}e} = c_e(e, \theta),$$

[1] In this model, we indeed assume that workers have all the bargaining power. When the firm has all the bargaining power, the equilibrium wage is $w = 0$ no matter what the workers' productivity is. In this case, it is not in the workers' interest to acquire costy education so as to signal his productivity.

which is decreasing in θ since $c_{e\theta}(e, \theta) < 0$.



15 Preview of the result: The unique outcome of "good" PBE is the best separating PBE outcome:

- High-ability worker: $(\tilde{e}, \theta_H)$.
- Low-ability worker: $(0, \theta_L)$.

3 Separating PBE

16 In a separating PBE (if exists), two types of workers choose different education levels.

17 Lemma: In any separating PBE (if exists), $w^*\big(e^*(\theta_H)\big) = \theta_H$ and $w^*\big(e^*(\theta_L)\big) = \theta_L$.

Proof. (1) Bayes' rule: After seeing $e^*(\theta_H)$, the firms should believe that the worker is of high ability θ_H, given worker's strategy $e^*(\theta)$; otherwise, the firms should believe that the worker is of low ability θ_L.

(2) The resulting wages are θ_H and θ_L, respectively.

□

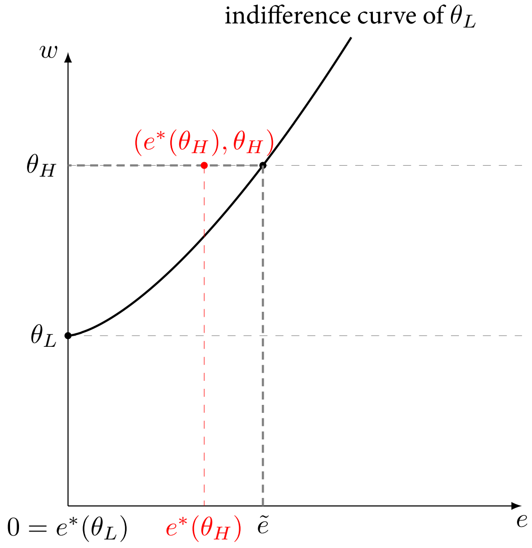
18 Lemma: In any separating PBE (if exists), $e^*(\theta_L) = 0$.

Proof. (1) The type-θ_L worker always receives wage θ_L.

(2) Thus, choosing $e = 0$ will save her cost of education, and is optimal.

□

19 Let $(\tilde{e}, \theta_H)$ be the intersection point of the curve $\theta_L = w - c(e, \theta_L)$ and the curve $w = \theta_H$.



Lemma: In any separating PBE (if exists), $e^*(\theta_H) \geq \tilde{e}$.

Proof. (1) Suppose $e^*(\theta_H) < \tilde{e}$.

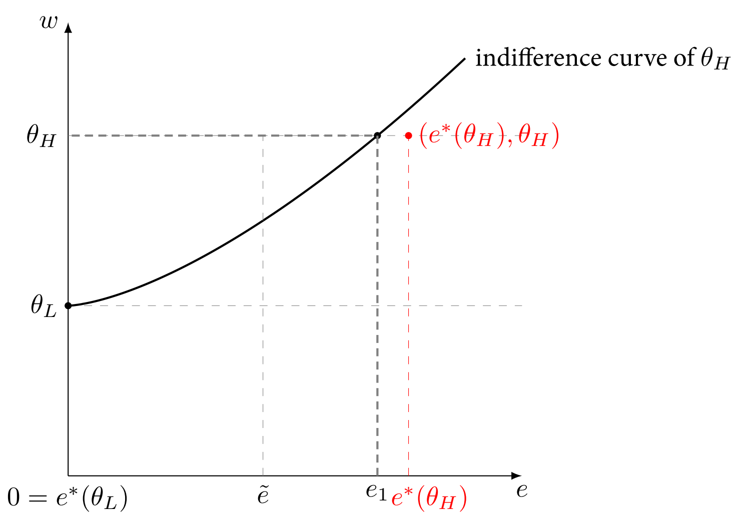
(2) Then the type-θ_L worker will mimic the type-θ_H worker by choosing $e^*(\theta_H)$ (the red point):

$$\theta_L = \theta_H - c(\tilde{e}, \theta_L) < \theta_H - c(e^*(\theta_H), \theta_L).$$

(3) It is not an equilibrium. Contradiction.

□

20 Let (e_1, θ_H) be the intersection point of the curve $\theta_L = w - c(e, \theta_H)$ and the curve $w = \theta_H$.



Lemma: In any separating PBE (if exists), $e^*(\theta_H) \leq e_1$.

Proof. (1) Suppose $e^*(\theta_H) > e_1$.

(2) Then the type-θ_H worker (the red point) will mimic the type-θ_L worker by choosing 0:

$$\theta_L = \theta_H - c(e_1, \theta_H) > \theta_H - c(e^*(\theta_H), \theta_H).$$

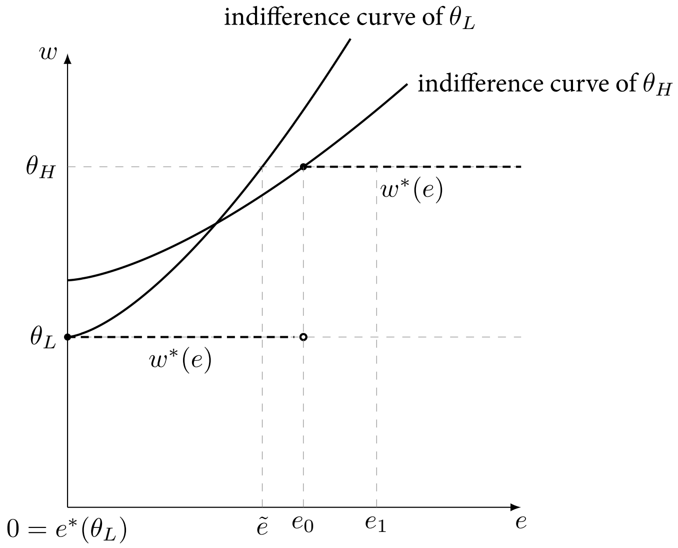
(3) It is not an equilibrium. Contradiction.

□

21 Remark: The above analysis is only heuristic, since we have not proved the existence of separating PBE.

22 Proposition: For each $e_0 \in [\tilde{e}, e_1]$, there is a separating PBE:

$$e^*(\theta_H) = e_0,\ e^*(\theta_L) = 0,\ \mu^*(e) = \begin{cases} 0, & \text{if } e = 0, \\ 0, & \text{if } 0 < e < e_0, \\ 1, & \text{if } e = e_0, \\ 1, & \text{if } e > e_0. \end{cases},\ w^*(e) = \begin{cases} \theta_L, & \text{if } e = 0, \\ \theta_L, & \text{if } 0 < e < e_0, \\ \theta_H, & \text{if } e = e_0, \\ \theta_H, & \text{if } e > e_0. \end{cases}.$$

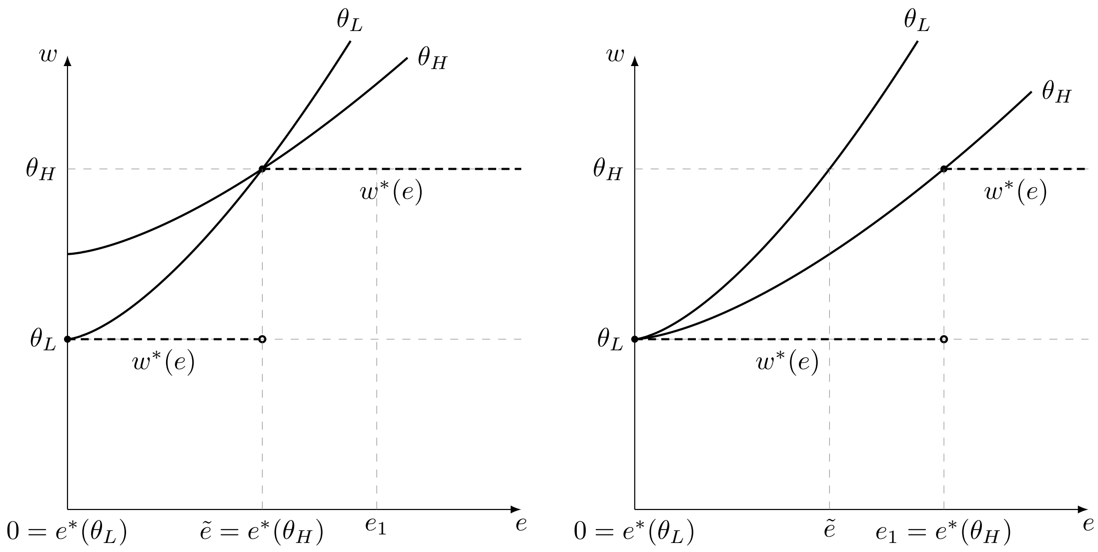


Proof.

- Type-θ_L worker:
  - Deviation $e \in (0, e_0)$: worse off since $\theta_L - c(e, \theta_L) < \theta_L$.
  - Deviation $e \geq e_0$: not better off since $\theta_H - c(e, \theta_L) \leq \theta_H - c(\tilde{e}, \theta_L) = \theta_L$.
- Type-θ_H worker:
  - Deviation $e < e_0$: not better off since $\theta_L = \theta_H - c(e_1, \theta_H) \leq \theta_H - c(e_0, \theta_H)$.
  - Deviation $e > e_0$: worse off since $\theta_H - c(e, \theta_H) < \theta_H - c(e_0, \theta_H)$.
- Belief: $\mu^*(0) = 0$ and $\mu^*(e_0) = 1$. For $e \notin \{0, e_0\}$, set $\mu^*(e)$ as in the statement.
- Wage: Given the belief, it is optimal.

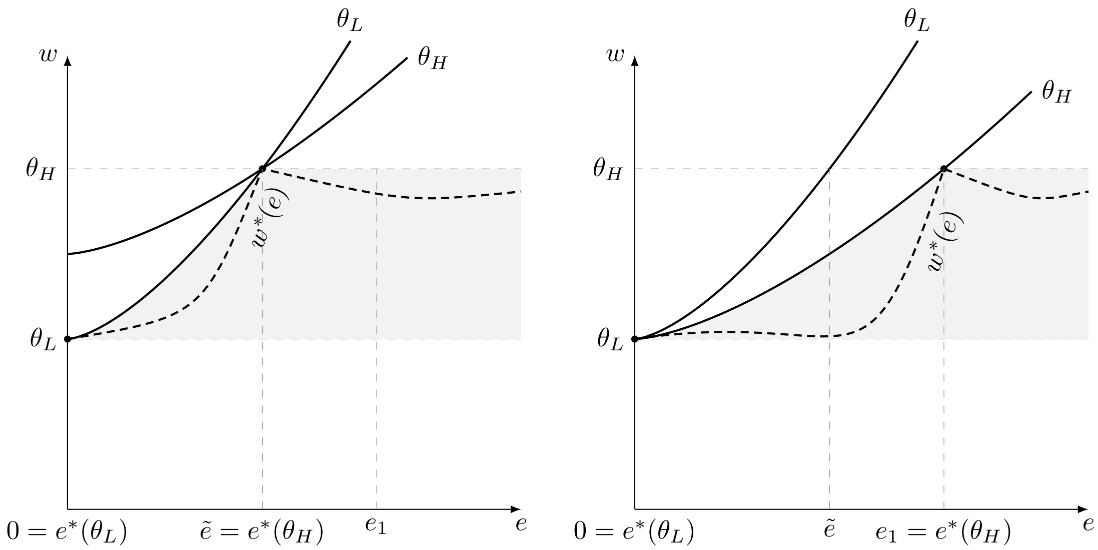
□

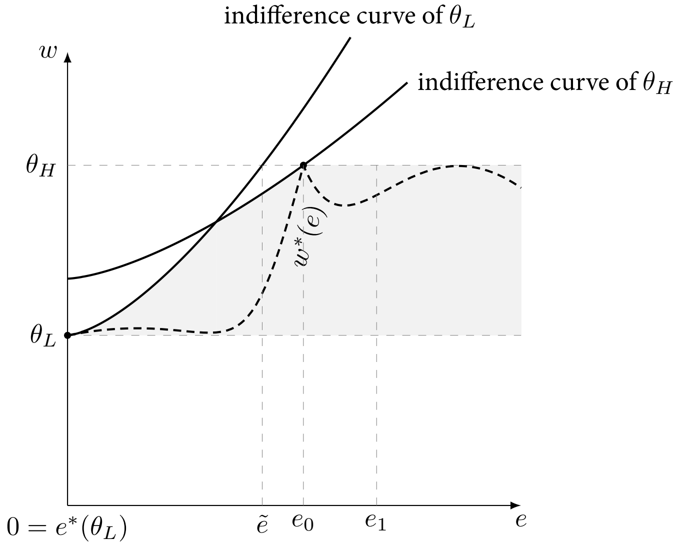
23 Two extreme separating PBE:



24 Notice:

- The Bayes' rule only requires that $\mu^*(0) = 0$ and $\mu^*(e_0) = 1$.
- However, after seeing $e \notin \{0, e_0\}$, the belief $\mu^*(e)$ could be arbitrary. It leads to multiple equilibria.





25 Key: The useless education can serve as a signal because the marginal cost of education is higher for a low-ability worker.

- a type-θ_H worker may find it worthwhile to get some positive level of education to raise her wage by some amount,
- a type-θ_L worker may be unwilling to get this same level of education in return for the same wage increase.

26 Pareto efficiency among all the separating PBEs:

- Firms earn zero profits.
- A type-θ_L worker's utility is θ_L.
- A type-θ_H worker does strictly better in separating PBE where she gets a lower level of education.

Thus, the separating PBE in which the high-ability worker gets $\tilde{e}$ Pareto dominate all the others.

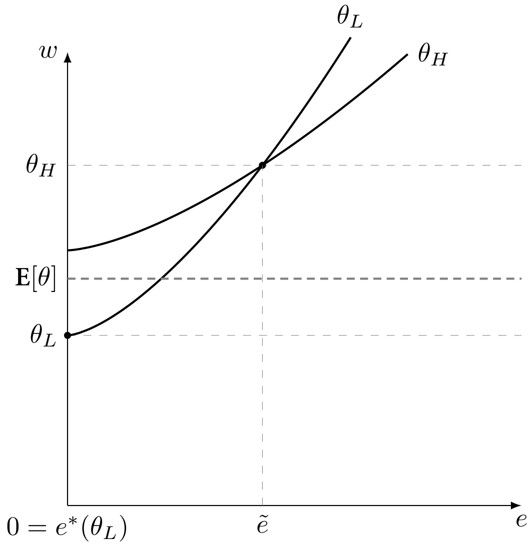
On the other hand, the Pareto dominated separating PBE are sustained because of the high-ability worker's fear: if she chooses a lower level of education than equilibrium education, firms will believe that she is not a high-ability worker. These beliefs can be maintained because in PBE they are never disconfirmed (off-equilibrium path).

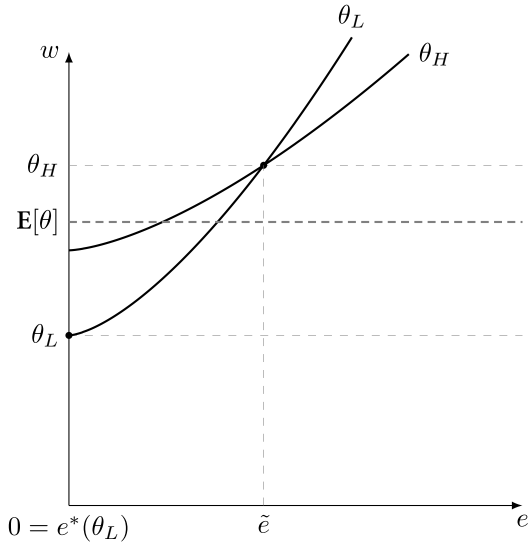
27 Welfare for type-θ_L workers: they are strictly worse off when signaling is possible, i.e., $\mathbf{E}[\theta] > \theta_L$.

28 Welfare for type-θ_H workers: they may be either better or worse off when signaling is possible.

- If $\mathbf{E}[\theta] < \theta_H - c(\tilde{e}, \theta_H)$, then the high-ability workers are better off because of the increase in their wages arising through signaling.
- If $\mathbf{E}[\theta] > \theta_H - c(\tilde{e}, \theta_H)$, then the high-ability workers are worse off than when signaling is impossible.

  In a separating PBE, the outcome $(0, \mathbf{E}[\theta])$ from no-signaling situation is no longer available to the high-ability workers.





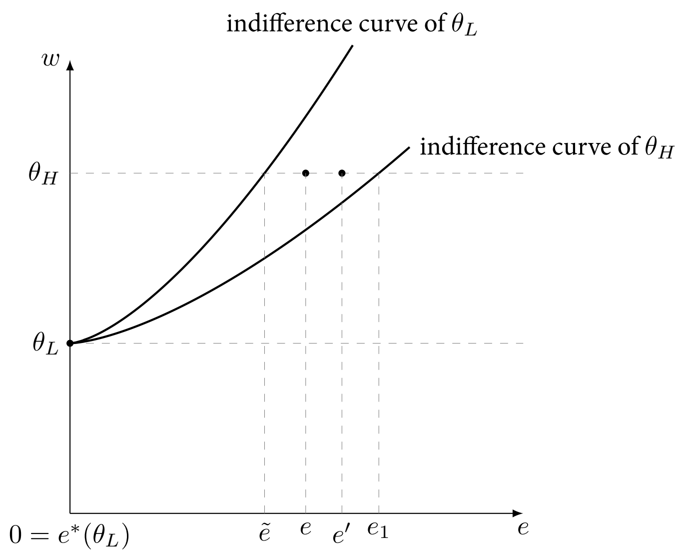
Summary:

- The set of separating PBE is completely unaffected by the fraction λ.
- As λ grows, it becomes more likely that the high-ability workers are worse off by the possibility of signaling.

29 Comparison with complete-information case:

- Complete-information case: $(0, \theta_L)$ for θ_L-worker and $(0, \theta_H)$ for θ_H-worker.
- Signaling: $(0, \theta_L)$ for θ_L-worker and $(\tilde{e}, \theta_H)$ for θ_H-worker.
- $\tilde{e}$ is the cost, paid by the beneficiary (i.e., θ_H-worker).

30 Refinement:



(1) For any $e' \in (\tilde{e}, e_1]$, consider the PBE: θ_L worker chooses education 0 and receives wage θ_L, and θ_H worker chooses education e' and receives wage θ_H.

(2) Pick any $e \in (\tilde{e}, e')$, a type-θ_L worker will never be better off by choosing e than 0 regardless of what firms believe about her as a result.

(3) Upon seeing $e \in (\tilde{e}, e')$, any belief other than $\mu(e) = 1$ seems unreasonable.

(4) Thus, $w^*(e) = \theta_H$.

(5) As a consequence, type-θ_H worker will deviate from e' to e. The given PBE is problematic.

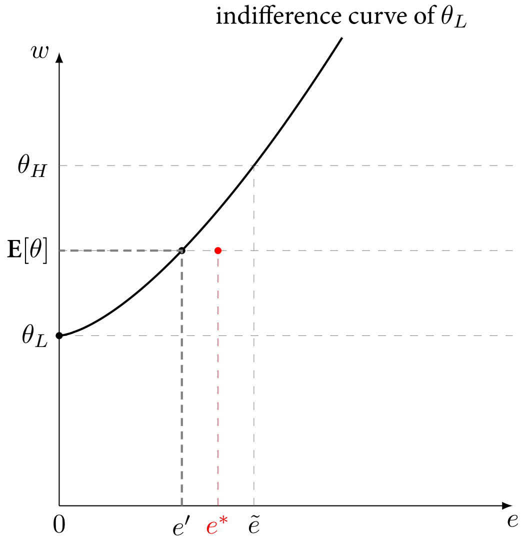
By this logic, the only reasonable separating SPE outcome is $(0, \theta_L)$ for θ_L workers and $(\tilde{e}, \theta_H)$ for θ_H workers.

4 Pooling PBE

31 In a pooling PBE, the two types of workers choose the same level of education, $e^*(\theta_L) = e^*(\theta_H) = e^*$.

32 After seeing e^* (on the equilibrium path), the firms should believe the worker is of high ability with probability λ. Thus, the wage $w^*(e^*) = \lambda\theta_H + (1-\lambda)\theta_L = \mathbf{E}[\theta]$.

33 Let $(e', \mathbf{E}[\theta])$ be the intersection point between the curve $\theta_L = w - c(e, \theta_L)$ and the curve $w = \mathbf{E}[\theta]$.



Lemma: In a pooling PBE, $e^* \leq e'$.

Proof. (1) Suppose $e^* > e'$.

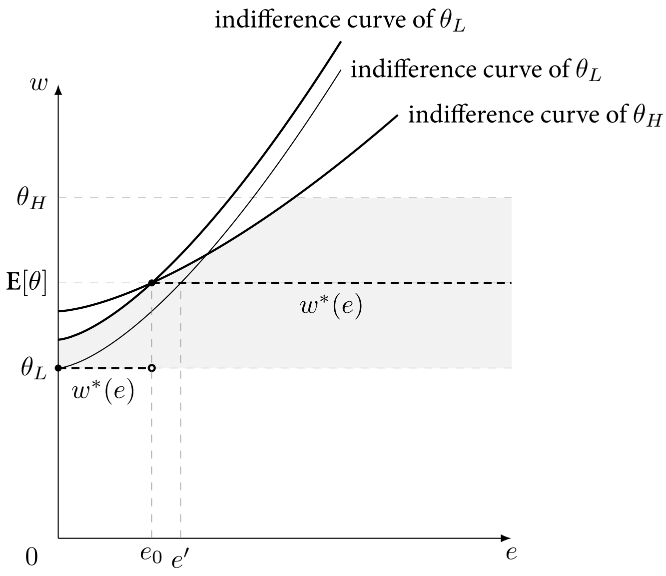
(2) Then the type-θ_L worker will deviate to 0: $\theta_L = \mathbf{E}[\theta] - c(e', \theta_L) > \mathbf{E}[\theta] - c(e^*, \theta_L)$.

(3) Thus, it is not an equilibrium. Contradiction.

□

34 Proposition: For any $e_0 \in [0, e']$, there is a pooling PBE:

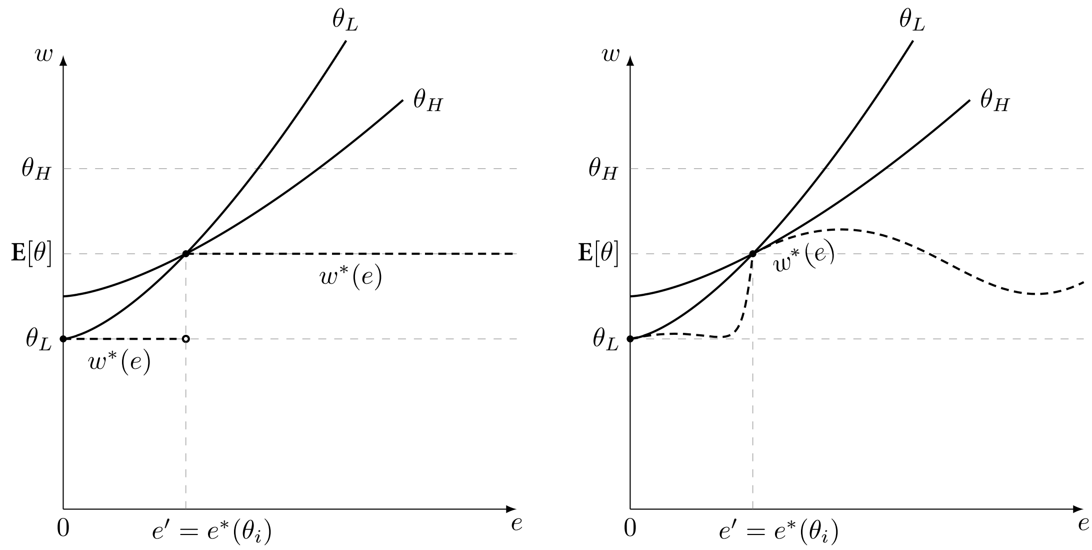
$$e^*(\theta_L) = e^*(\theta_H) = e_0,\ \mu^*(e) = \begin{cases} 0, & \text{if } e < e_0, \\ \lambda, & \text{if } e = e_0, \\ \lambda, & \text{if } e > e_0. \end{cases},\ w^*(e) = \begin{cases} \theta_L, & \text{if } e < e_0, \\ \mathbf{E}[\theta], & \text{if } e = e_0, \\ \mathbf{E}[\theta], & \text{if } e > e_0. \end{cases}.$$

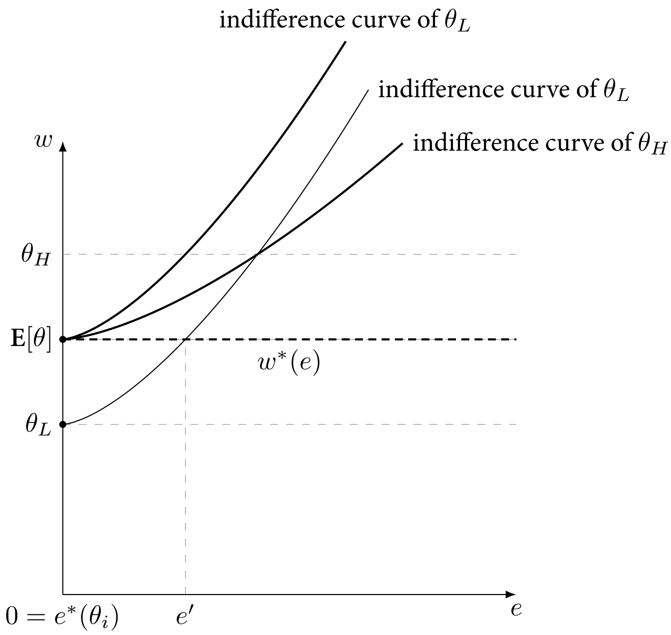


Proof.
- For type-θ_L worker:
  - Deviation $e < e_0$: not better off since $\theta_L = \mathbf{E}[\theta] - c(e', \theta_L) \leq \mathbf{E}[\theta] - c(e_0, \theta_L)$.
  - Deviation $e > e_0$: worse off since $\mathbf{E}[\theta] - c(e, \theta_L) < \mathbf{E}[\theta] - c(e_0, \theta_L)$.
- For type-θ_H worker:
  - Deviation $e < e_0$: worse off since $\theta_L = \mathbf{E}[\theta] - c(e', \theta_L) < \mathbf{E}[\theta] - c(e_0, \theta_H)$.
  - Deviation $e > e_0$: worse off since $\mathbf{E}[\theta] - c(e, \theta_H) < \mathbf{E}[\theta] - c(e_0, \theta_H)$.
- Belief: $\mu^*(e_0) = \lambda$. For $e \neq e_0$, $\mu^*(e)$ could be arbitrary. We set $\mu^*(e)$ as in the statement.
- Wage: Given the belief, it is optimal.

$\square$

35 Two extreme pooling PBE:





36 Remark: $e' < \tilde{e} < e_1$.

37 Pareto efficiency:

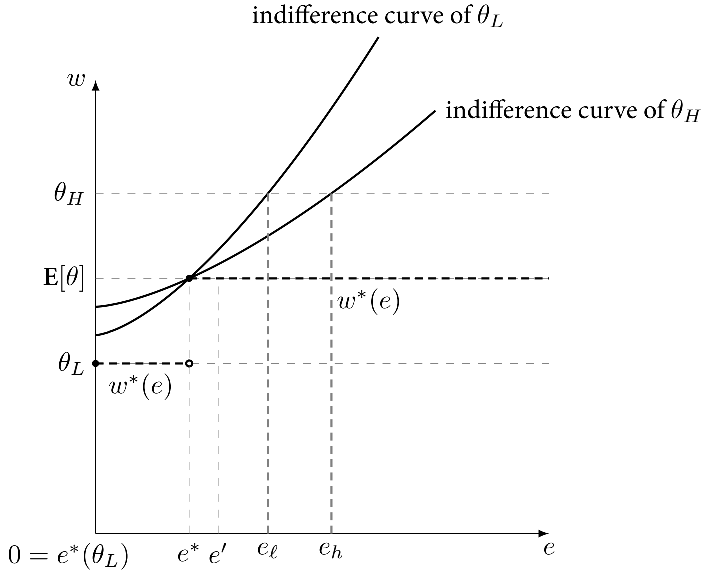
A pooling PBE in which both types of worker get no education Pareto dominates any pooling PBE with a positive education level.

The Pareto-dominated pooling PBE are sustained by the worker's fear: A deviation will lead firms to have an unfavorable impression of her ability.

38 For any pooling PBE (e^*, μ^*, w^*) where $e^* \in [0, e']$,

- let (e_ℓ, θ_H) be the intersection point between the curve $\mathbf{E}[\theta] - c(e^*, \theta_L) = w - c(e, \theta_L)$ and the curve $w = \theta_H$,
- let (e_h, θ_H) be the intersection point between the curve $\mathbf{E}[\theta] - c(e^*, \theta_H) = w - c(e, \theta_H)$ and the curve $w = \theta_H$.

39 Refinement (intuition criterion):



(1) To support the education choice e^* as a pooling PBE outcome, we must have $\mu(e) < 1$ after seeing $e \in (e_\ell, e_h)$:

- If $\mu(e) = 1$ for some $e \in (e_\ell, e_h)$, then the wage should be θ_H, and the type-θ_H worker will be better off by deviating to e:

$$\theta_H - c(e, \theta_H) > \theta_H - c(e_h, \theta_H) = \mathbf{E}[\theta] - c(e^*, \theta_H) \geq \mathbf{E}[\theta].$$

(2) Consider the off-equilibrium path: Suppose that a firm is confronted with a deviation to some education level $e \in (e_\ell, e_h)$ when it was expecting the equilibrium level of education e^* to be chosen.

(3) The firm will reason as follows:

- a type-θ_L worker would be worse off deviating to e regardless of what beliefs firms have after that:

$$\mathbf{E}[\theta] - c(e^*, \theta_L) = \theta_H - c(e_\ell, \theta_L) > \theta_H - c(e, \theta_L).$$

- a type-θ_H worker might be better off by doing this:

$$\mathbf{E}[\theta] - c(e^*, \theta_H) = \theta_H - c(e_h, \theta_H) < \theta_H - c(e, \theta_H).$$

Thus, this must not be a low-ability worker.

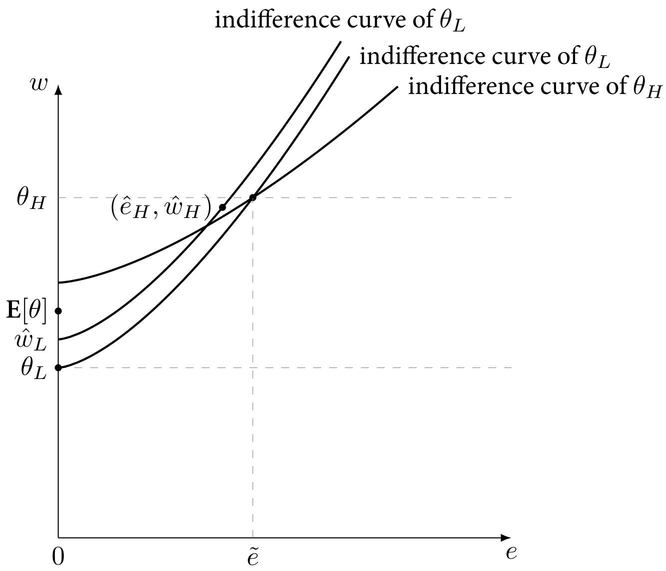
(4) Thus, e^* cannot be a pooling PBE education level. No pooling PBE survives.

5 Second-best intervention

40 In the presence of signaling, although the central planner cannot observe workers' types, it may be able to achieve a Pareto improvement relative to the market outcome.

41 Case 1: When the best separating PBE is Pareto dominated by the no-signaling outcome, a Pareto improvement can be achieved simply by banning the signaling activity.

42 Case 2: When the no-signaling outcome does not Pareto dominate the best separating PBE, a Pareto improvement can be achieved by "cross-subsidization":



The outcomes $(0, \hat{w}_L)$ and $(\hat{e}_H, \hat{w}_H)$ can be achieved by mandating

- workers with education levels below $\hat{e}_H$ receive wage $\hat{w}_L$,
- workers with education levels of at least $\hat{e}_H$ receive wage $\hat{w}_H$.

Thus, low-ability workers will choose $e = 0$ and high-ability workers will choose $e = \hat{e}_H$.

6 Homework

- Key:
  - When and why can the high-type workers separate themselves from the low-type workers?
  - What is the favorite between the pooling PBE and separating SPE for low-type workers?
- Reading: 13.C