

# ADVANCED MICROECONOMICS: LECTURE NOTES 5

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- 1 Given desired objectives, how to design economic mechanisms or incentives in strategic settings such that players's strategic choices approach to those objectives?
- 2 In a principal-agent problem (or an agency model), one party, called an agent (代理人), acts on behalf of another party, called the principal (委托人).
  - In adverse selection models (or hidden information/hidden characteristic), the agent has private information about his type before the contract is written.
  - In moral hazard models (or hidden actions), the agent becomes privately informed after the contract is written.
  - Nonverifiability.
- 3 Screening: Uninformed parties take step to distinguish/screen the types of informed parties.
  - In competitive screening, there are several competing firms.
  - In monopolistic screening, there is a single firm screening workers.

## 1 Adverse selection

- 4 An owner (principal) wishes to hire a manager (agent) to run a one-time project.

If the agent's effort level is  $e \in [0, \infty)$ , then principal's income is  $\pi(e)$ , with  $\pi(0) = 0$ ,  $\pi'(e) > 0$ , and  $\pi''(e) < 0$  for all  $e$ .

If the principal pays wage  $w$  to the agent, his utility/profit is  $\pi(e) - w$ .

- 5 The agent is an expected utility maximizer with utility  $v(w - g(e, \theta))$ .
  - $\theta \in \{\theta_L, \theta_H\}$  represents agent's ability. Here,  $\theta_H > \theta_L$  and  $\text{Prob}(\theta_H) = \lambda \in (0, 1)$ .
  - $g(e, \theta)$  measures the cost/disutility of effort.
  - $g(0, \theta) = 0, g_e(e, \theta) \begin{cases} > 0, & \text{if } e > 0 \\ = 0, & \text{if } e = 0 \end{cases}, g_{ee} > 0, g_\theta < 0, g_{e\theta}(e, \theta) \begin{cases} < 0, & \text{if } e > 0 \\ = 0, & \text{if } e = 0 \end{cases}$ .

$\Rightarrow$  The agent's indifference curves have single-crossing property.

  - The agent is risk averse:  $v' > 0$  and  $v'' < 0$ .<sup>1</sup>
  - The agent has a reservation utility  $\bar{u}$ .

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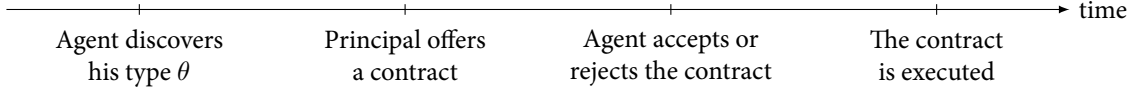
<sup>1</sup>Question: How about when the manager is risk neutral?

- 6 The economic variables are effort level  $e$  and the wage  $w$ . These variables are both observable and verifiable by a third party such as a benevolent court of law.

A contract is a pair  $(e, w)$ . Let  $\mathcal{A}$  be the set of all feasible contracts, that is,  $\mathcal{A} = \{(e, w) \mid e \in \mathbb{R}_+, w \in \mathbb{R}\}$ .

Once a contract is signed, it must be implemented—commitment.

- 7 The sequence of play is as follows:



## 2 Complete information

- 8 First suppose that there is no asymmetry of information between the principal and the agent, i.e.,  $\theta$  is observable.

- 9 There are two questions for principal:

- To induce agent's acceptance, what is the optimal contract?
- Does the obtained optimal contract guarantee principal to get better than doing nothing?

- 10 The principal will try to maximize her utility subject to inducing the agent to accept the proposed contract. Clearly, the agent obtains  $\bar{u}$  if he does not take the principal's contract. So the principal will solve the following problem:

$$\begin{aligned} & \underset{(e_i, w_i) \in \mathcal{A}}{\text{maximize}} && \pi(e_i) - w_i \\ & \text{subject to} && v(w_i - g(e_i, \theta_i)) \geq \bar{u}. \end{aligned}$$

- 11 In any solution, the IR constraint must bind; otherwise, the principal could lower the wage offered and still have the agent accept the contract. Thus, the maximization problem becomes:

$$\max_{(e_i, w_i) \in \mathcal{A}} \pi(e_i) - v^{-1}(\bar{u}) - g(e_i, \theta_i).$$

Clearly,  $\pi'' - g_{ee} < 0$ . Then the solution  $(e_i^*, w_i^*)$  must satisfy the first-order condition:

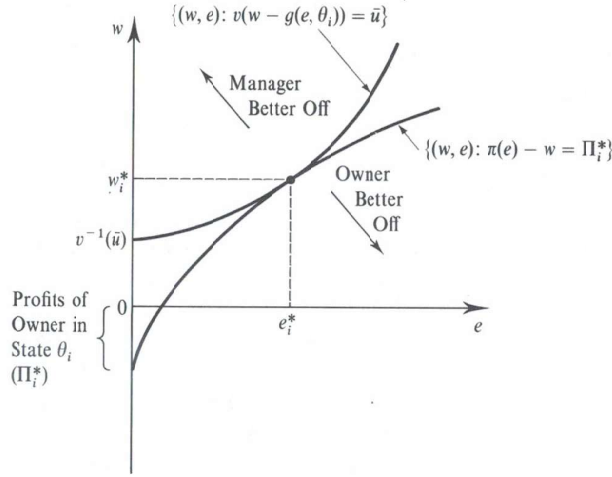
$$\pi'(e_i^*) \begin{cases} \leq g_e(e_i^*, \theta_i), \\ = g_e(e_i^*, \theta_i), & \text{if } e_i^* > 0. \end{cases}$$

Since  $\pi'(0) > 0$  and  $g_e(0, \theta_i) = 0$ , we have that  $e_i^* > 0$ . Thus,

$$\pi'(e_i^*) = g_e(e_i^*, \theta_i).$$

Interpretation: The optimal level of effort  $e_i^*$  (for  $\theta_i$  agent) equals the principal's marginal value and the agent's marginal cost.

- 12 Graphic illustration



- Agent's reservation utility is  $\bar{u}$ , which is equivalent to the contract  $(0, v^{-1}(\bar{u}))$ .
- Principal seeks to find the most profitable point on the indifference curve with utility  $\bar{u}$ , i.e., through the point  $(0, v^{-1}(\bar{u}))$ .
- For a  $\theta_i$  agent, principal pays the wage  $w_i^*$  such that  $w_i^* - g(e_i^*, \theta_i) = v^{-1}(\bar{u})$ .
- For a  $\theta_i$  agent, principal's profit is  $\Pi_i^* = \pi(e_i^*) - v^{-1}(\bar{u}) - g(e_i^*, \theta_i)$ .

This profit is exactly equal to the distance from the origin to the intersection point between the indifference curve through  $(e_i^*, w_i^*)$  and the vertical axis: letting  $e = 0$  in the indifference curve  $\pi(e) - w = \Pi_i^*$ , we have  $-w = \Pi_i^*$ .

- If  $\bar{u}$  is small (especially,  $\bar{u} = 0$ ), then this profit could be strictly positive.
- If  $\bar{u}$  is very large, this profit could be negative; in this case, the principal will not provide such a contract—the shutdown occurs.

Interpretation: If agent's reservation utility is low, principal can attract him to accept some contract; otherwise, agent will not accept any contract that is acceptable for principal.

13 Note: This equation  $\pi'(e_i^*) = g_e(e_i^*, \theta_i)$  may not have a solution. For example, we consider the following case

- $\pi'(\cdot)$  is strictly decreasing and has a lower bound  $\underline{\pi}' > 0$ ;
- $g_e(\cdot, \theta)$  is strictly increasing and has an upper bound  $\bar{g}_e > 0$ ;
- $\underline{\pi}' > \bar{g}_e$ .

Then such an equation does not have a solution.

For example, let  $f(x) = \frac{1}{x}$  and  $g(x) = -\frac{1}{x}$ . Clearly,  $g(x) - f(x)$  is concave on  $(0, +\infty)$ . FOC implies that  $x^{-2} \leq 0$  with equality if  $x > 0$ . It is easy to see that there is interior solution.

To guarantee the existence of a solution, we have to have more assumptions.

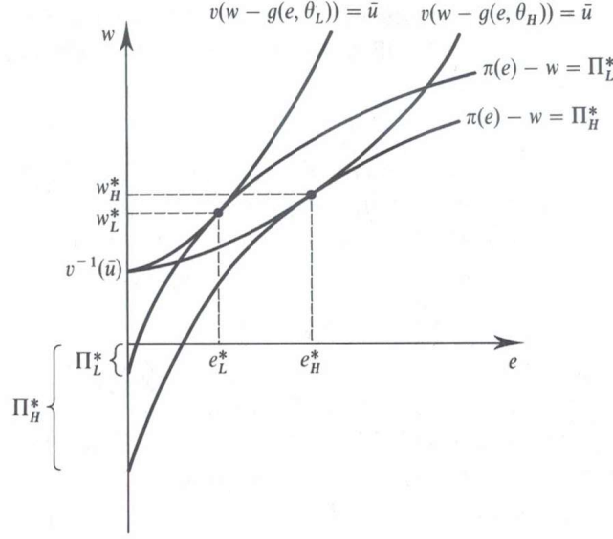
14 Since  $\theta_H > \theta_L$ ,  $\pi'' < 0$ ,  $g_{e\theta} < 0$ ,  $g_{ee} > 0$ ,  $\pi'(e_i^*) = g_e(e_i^*, \theta_i)$  for  $i \in \{H, L\}$ , we have  $e_H^* > e_L^*$ :

- It is impossible that  $e_H^* = e_L^*$ .
- If  $e_H^* < e_L^*$ , then we have

$$\pi'(e_H^*) > \pi'(e_L^*) \text{ and } g_e(e_H^*, \theta_H) < g_e(e_L^*, \theta_H) < g_e(e_L^*, \theta_L).$$

Contradiction.

Interpretation: The optimal effort level of a high-ability agent is greater than that of a low-ability agent.



15 In the figure, the wage  $w_H^*$  is greater than  $w_L^*$ , but we note that  $w_H^*$  can be greater or smaller than  $w_L^*$  depending on the curvature of the functions  $\pi$ ,  $g$ , and  $v$ , as it can be easily seen graphically.

16 Every agent (no matter  $\theta_H$  or  $\theta_L$ ) obtains exactly  $\bar{u}$  from principal, just balancing his reservation utility.

17 The principal's profit:

$$\Pi_H^* = \overbrace{\pi(e_H^*) - g(e_H^*, \theta_H) - v^{-1}(\bar{u})}^{e_H^* \text{ maximizes } \pi(e) - v^{-1}(\bar{u}) - g(e, \theta_H)} \geq \underbrace{\pi(e_L^*) - g(e_L^*, \theta_H) - v^{-1}(\bar{u})}_{\theta_L < \theta_H} \geq \pi(e_L^*) - g(e_L^*, \theta_L) - v^{-1}(\bar{u}) = \Pi_L^*.$$

18 For contract to be always carried out, it is thus enough that profit is positive for a  $\theta_L$  agent, i.e., the following condition must be satisfied

$$\Pi_L^* = \pi(e_L^*) - g(e_L^*, \theta_L) - v^{-1}(\bar{u}) \geq 0,$$

i.e.,  $\bar{u} \leq v(\pi(e_L^*) - g(e_L^*, \theta_L))$ . We will maintain this hypothesis hereafter.

19 First-best contract menu  $\{(e_i^*, w_i^*)\}_{i=H,L}$ .

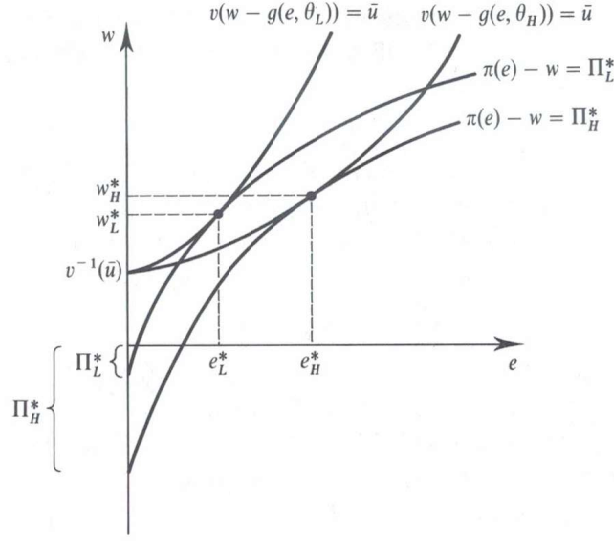
To implement the first-best effort levels  $e_H^*$  and  $e_L^*$ , the principal can make the following take-it-or-leave-it offers to the agent: If  $\theta = \theta_H$  (resp.  $\theta_L$ ), the principal offers the wage  $w_H^*$  (resp.  $w_L^*$ ) for the effort level  $e_H^*$  (resp.  $e_L^*$ ) with  $w_i^* - g(e_i^*, \theta_i) = v^{-1}(\bar{u})$ .

Whatever his type, agent accepts the offer and makes utility  $\bar{u}$ . The complete-information optimal contracts are thus  $(e_H^*, w_H^*)$  if  $\theta = \theta_H$  and  $(e_L^*, w_L^*)$  if  $\theta = \theta_L$ .

### 3 Incomplete information

20 Suppose that  $\theta$  is the agent's private information.

- 21 Consider the case where the principal offers the menu of first-best contracts  $\{(e_H^*, w_H^*), (e_L^*, w_L^*)\}$  hoping that an agent with type  $\theta_L$  will select  $(e_L^*, w_L^*)$  and an agent with type  $\theta_H$  will select instead  $(e_H^*, w_H^*)$ .



We see that  $(e_L^*, w_L^*)$  is preferred to  $(e_H^*, w_H^*)$  by both types of agents:

- The  $\theta_H$ -agent's isoutility curve that passes through  $(e_L^*, w_L^*)$  corresponds to a utility level higher than  $\bar{u}$  at  $(e_H^*, w_H^*)$ .
- The  $\theta_L$ -agent's isoutility curve that passes through  $(e_H^*, w_H^*)$  corresponds to a utility level lower than  $\bar{u}$  at  $(e_L^*, w_L^*)$ .

Offering the menu of contracts  $\{(e_H^*, w_H^*), (e_L^*, w_L^*)\}$  fails to have the agents self-selecting properly within this menu. The high-ability agent mimics the low-ability one and selects also contract  $(e_L^*, w_L^*)$ . The complete information optimal contracts can no longer be implemented under asymmetric information.

- 22 Definition: A menu of contracts  $\{(e_L, w_L), (e_H, w_H)\}$  is incentive compatible when  $(e_L, w_L)$  is weakly preferred to  $(e_H, w_H)$  by the type- $\theta_L$  agent and  $(e_H, w_H)$  is weakly preferred to  $(e_L, w_L)$  by the type- $\theta_H$  agent.

Mathematically,

$$w_L - g(e_L, \theta_L) \geq w_H - g(e_H, \theta_L), \quad (\text{IC}_L)$$

$$w_H - g(e_H, \theta_H) \geq w_L - g(e_L, \theta_H). \quad (\text{IC}_H)$$

- 23 If a menu of contracts  $\{(e_L, w_L), (e_H, w_H)\}$  is incentive compatible, then  $e_H \geq e_L$ , which is called the monotonicity constraint. Indeed,

$$\int_{e_L}^{e_H} g_e(e, \theta_L) de = \overbrace{g(e_H, \theta_L) - g(e_L, \theta_L)}^{\text{By Equation (IC}_L\text{)}} \geq \overbrace{w_H - w_L}^{\text{By Equation (IC}_H\text{)}} \geq \overbrace{g(e_H, \theta_H) - g(e_L, \theta_H)}^{\text{By Equation (IC}_H\text{)}} = \int_{e_L}^{e_H} g_e(e, \theta_H) de,$$

and hence  $e_H \geq e_L$ .

If  $e_H \neq e_L$ , only one of  $(\text{IC}_L)$  and  $(\text{IC}_H)$  can bind.

24 Definition: A menu of contracts  $\{(e_L, w_L), (e_H, w_H)\}$  is individually rational if

$$w_L - g(e_L, \theta_L) \geq v^{-1}(\bar{u}), \quad (\text{IR}_L)$$

$$w_H - g(e_H, \theta_H) \geq v^{-1}(\bar{u}). \quad (\text{IR}_H)$$

25 Information rent: Under complete information, the principal is able to maintain all types of agents at their reservation utility. Their respective utility levels at the first-best contracts satisfy

$$w_H^* - g(e_H^*, \theta_H) = v^{-1}(\bar{u}) \text{ and } w_L^* - g(e_L^*, \theta_L) = v^{-1}(\bar{u}).$$

Generally this will not be possible anymore under incomplete information, at least when the principal wants both types of agents to be active.

Let  $r_H = w_H - g(e_H, \theta_H) - v^{-1}(\bar{u})$  and  $r_L = w_L - g(e_L, \theta_L) - v^{-1}(\bar{u})$  denote the respective information rent (the utility in excess of the reservation utility) of each type.

信息租金是由于 agent 拥有比 principal 更多的信息而获得的额外收益。Principal 的问题是选择一份最明智的方式，对某些类型的 agent 让渡一些信息租金，而获得一个尽可能大的利润。

26 The principal's problem is to solve

$$\begin{aligned} & \underset{(e_L, w_L), (e_H, w_H)}{\text{maximize}} && \lambda(\pi(e_H) - w_H) + (1 - \lambda)(\pi(e_L) - w_L) \\ & \text{subject to} && \text{Equations (IC}_L\text{)-(IR}_H\text{)}. \end{aligned}$$

We can rewrite as  $\underbrace{\lambda(\pi(e_H) - g(e_H, \theta_H) - v^{-1}(\bar{u})) + (1 - \lambda)(\pi(e_L) - g(e_L, \theta_L) - v^{-1}(\bar{u}))}_{\text{Total surplus=efficiency}} - \underbrace{[\lambda r_H + (1 - \lambda)r_L]}_{\text{expected information rent}}.$

上面这个表达式清楚地揭示了：principal 希望最大化“配置效率减去信息租金”。Principal 愿意接受一定程度上的配置扭曲，以便减少支付给 agent 的信息租金。

### 3.1 Solving the principal's problem

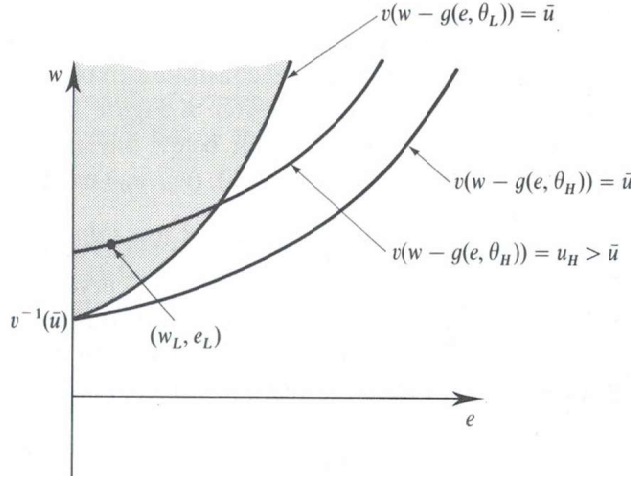
27 Lemma: The constraint (IR<sub>H</sub>) is always satisfied due to constraints (IC<sub>H</sub>) and (IR<sub>L</sub>).

*Proof.*

$$w_H - g(e_H, \theta_H) \geq w_L - g(e_L, \theta_H) \geq w_L - g(e_L, \theta_L) \geq v^{-1}(\bar{u}).$$

□

28 Graphic illustration.



- (1) By constraint (IR<sub>L</sub>),  $(e_L, w_L)$  must lie in the shaded region.
- (2) By constraint (IC<sub>H</sub>),  $(e_H, w_H)$  must lie on or above the  $\theta_H$ -indifference curve through  $(e_L, w_L)$ .
- (3) This implies that  $\theta_H$ -agent's utility is at least  $\bar{u}$ .

29 Lemma: The constraint (IR<sub>L</sub>) is binding at the optimal.

*Proof.* Suppose that  $w_L - g(e_L, \theta_L) - v^{-1}(\bar{u}) = \varepsilon > 0$ . Then the principal can decrease  $w_L$  by  $\varepsilon$  and consequently also  $w_H$  by  $\varepsilon$  and gain  $\varepsilon$ .

Notice that the constraint (IR<sub>L</sub>) is still satisfied. In addition, constraints (IC<sub>H</sub>) and (IC<sub>L</sub>) are also satisfied.  $\square$

It implies that  $r_L = w_L - g(e_L, \theta_L) - v^{-1}(\bar{u}) = 0$ —no information rent for  $\theta_L$ -agents.

30 Lemma: The constraint (IC<sub>H</sub>) is binding at the optimal.

*Proof.* Suppose that  $[w_H - g(e_H, \theta_H)] - [w_L - g(e_L, \theta_H)] = \varepsilon > 0$ . Then the principal can decrease  $w_H$  by  $\varepsilon$  and gain  $\lambda\varepsilon$ .  $\square$

It implies that  $r_H = w_H - g(e_H, \theta_H) - v^{-1}(\bar{u}) = (w_L - g(e_L, \theta_H)) - (w_L - g(e_L, \theta_L)) = g(e_L, \theta_L) - g(e_L, \theta_H) > 0$ , which depends on  $e_L$ .

31 Ignoring constraint (IC<sub>L</sub>), we obtain a reduced program

$$\max_{e_L, e_H} \lambda(\pi(e_H) - g(e_H, \theta_H) - v^{-1}(\bar{u}) + \underbrace{g(e_L, \theta_H) - g(e_L, \theta_L)}_{-r_H}) + (1 - \lambda)(\pi(e_L) - g(e_L, \theta_L) - v^{-1}(\bar{u})).$$

Compared with the full information setting, asymmetric information alters the principal's optimization simply by the subtraction of the expected rent that has to be given up to the efficient type ( $\theta_H$ ). The inefficient type ( $\theta_L$ ) gets no rent, but the efficient type  $\theta_H$  gets the information rent that he could obtain by mimicking the inefficient type  $\theta_L$ . This rent depends only on the effort level requested from this inefficient type.

32 The first order condition on  $e_H$  implies

$$\pi'(e_H^{SB}) = g_e(e_H^{SB}, \theta_H), \text{ that is, } e_H^{SB} = e_H^*.$$

Hence, there is no distortion away from the first-best output for the efficient type.

Notice that:  $\pi'(0) > 0$ ,  $\pi'' < 0$ ,  $g_e(0, \theta_H) = 0$ , and  $g_{ee} > 0$ , such a  $e_H^{SB} > 0$  exists.

33 The first order condition on  $e_L$  implies

$$(1 - \lambda) \cdot (\pi'(e_L^{SB}) - g_e(e_L^{SB}, \theta_L)) = \lambda \cdot (g_e(e_L^{SB}, \theta_L) - g_e(e_L^{SB}, \theta_H)).$$

This equation expresses the important trade-off between efficiency and rent extraction which arises under asymmetric information. The expected marginal efficiency gain (resp. cost) and the expected marginal cost (resp. gain) of the rent brought about by an infinitesimal increase (resp. decrease) of  $\theta_L$  agent's output are equated. Thus, the principal is neither willing to increase nor to decrease  $\theta_L$  agent's effort.

Notice: Such a  $e_L^{SB} > 0$  exists:  $\pi'(0) > 0$ ,  $g_e(0, \theta_L) = g_e(0, \theta_H) = 0$ ,  $\pi'' < 0$ , and  $g_{ee} > 0$ .

34 Note: In several other setups, the equation for  $e_L^{SB}$  may not have a positive solution. In that case,  $\theta_L$ -agent will shut down in the optimal contract for asymmetric environment. See Section 2.6.3 in Laffont and Martimont (2002).

35 Since  $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$  and  $\pi'(e_L^{SB}) = g_e(e_L^{SB}, \theta_L) + \frac{\lambda}{1-\lambda}[g_e(e_L^{SB}, \theta_L) - g_e(e_L^{SB}, \theta_H)]$ , we have the following inequality

$$e_H^{SB} = e_H^* > \underbrace{e_L^*}_{\pi'' < 0} > e_L^{SB},$$

and hence

$$\begin{aligned} w_L^{SB} - g(e_L^{SB}, \theta_L) - w_H^{SB} + g(e_H^{SB}, \theta_L) &= g(e_L^{SB}, \theta_H) - g(e_H^{SB}, \theta_H) - g(e_L^{SB}, \theta_L) + g(e_H^{SB}, \theta_L) \\ &= \int_{e_L^{SB}}^{e_H^{SB}} [g_e(e, \theta_L) - g_e(e, \theta_H)] de \geq 0. \end{aligned}$$

That is, the constraint (IC<sub>L</sub>) is strictly satisfied.

这点说明：向上的激励相容条件（upward incentive compatibility,  $\theta_L$  模仿  $\theta_H$ ）不是问题。另一方面，向下的激励相容条件（downward incentive compatibility,  $\theta_H$  模仿  $\theta_L$ ）更为关键，需要谨慎处理。

36 Proposition: Under asymmetric information, the optimal menu of contracts entails:

- No output distortion for the high-ability agent with respect to the first-best,  $e_H^{SB} = e_H^*$ . A downward output distortion for the low-ability agent,  $e_L^{SB} < e_L^*$  with

$$\pi'(e_L^{SB}) = g_e(e_L^{SB}, \theta_L) + \frac{\lambda}{1-\lambda}[g_e(e_L^{SB}, \theta_L) - g_e(e_L^{SB}, \theta_H)].$$

- The second-best wages are respectively given by

$$\begin{aligned} w_H^{SB} &= g(e_H^{SB}, \theta_H) + v^{-1}(\bar{u}) + \underbrace{g(e_L^{SB}, \theta_L) - g(e_L^{SB}, \theta_H)}_{r_H} > g(e_H^*, \theta_H) + v^{-1}(\bar{u}) = w_H^*, \\ w_L^{SB} &= g(e_L^{SB}, \theta_L) + v^{-1}(\bar{u}) < g(e_L^*, \theta_L) + v^{-1}(\bar{u}) = w_L^*. \end{aligned}$$

$$\text{Moreover, } w_H^{SB} = g(e_H^{SB}, \theta_H) + v^{-1}(\bar{u}) + \underbrace{g(e_L^{SB}, \theta_L) - g(e_L^{SB}, \theta_H)}_{w_L^{SB}} > w_L^{SB}.$$

- Only the high-ability agent gets a positive information rent given by

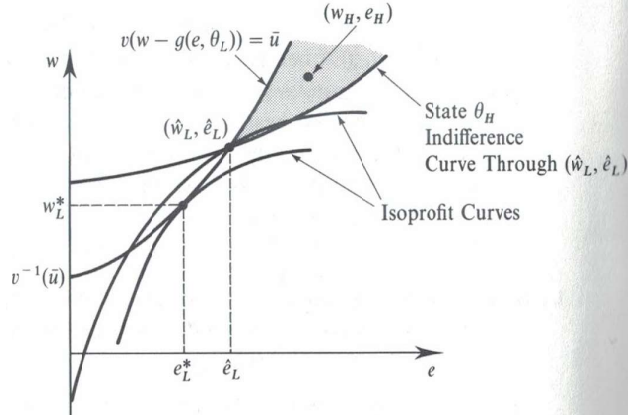
$$r_H^{SB} = g(e_L^{SB}, \theta_L) - g(e_L^{SB}, \theta_H).$$



37 “顶部无扭曲”与“单向扭曲/向下扭曲”是两条最基本的规律。

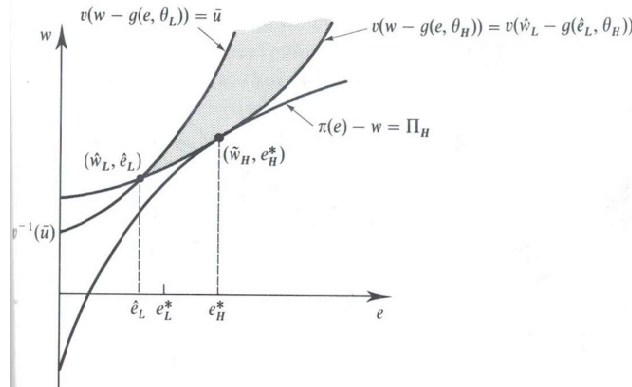
- 对于  $\theta_H$ ，不存在劳动水平的扭曲（其劳动水平与完全信息最优时的劳动水平一致），但代价是需要给其支付信息租金。
- 对于  $\theta_L$ ，其付出的劳动水平低于完全信息最优时的劳动水平，但没有信息租金。

38 Graphic illustration for  $e_L^{SB} \leq e_L^*$ .



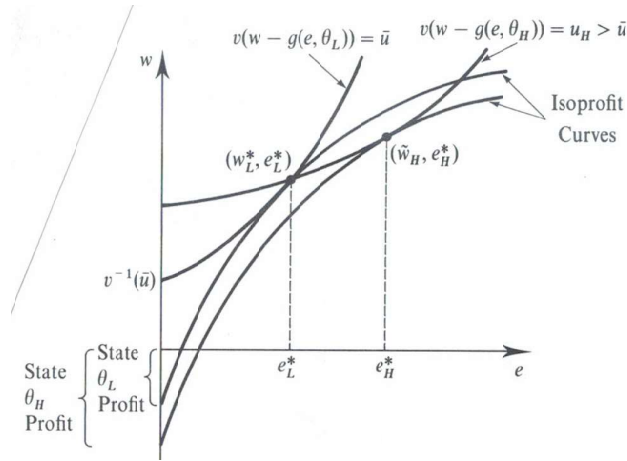
- (1) Suppose that  $e_L^{SB} > e_L^*$ .
- (2) Since  $\theta_L$ -IR binds,  $(e_L^{SB}, w_L^{SB})$  lies on the indifference curve through  $v^{-1}(\bar{u})$ .
- (3) To make  $\theta_L$ -IC and  $\theta_H$ -IC hold,  $(e_H^{SB}, w_H^{SB})$  lies in the shade region.
- (4) Principal can raise her profit by moving  $(e_L^{SB}, w_L^{SB})$  to  $(e_L^*, w_L^*)$ :  $\theta_L$ -IC and  $\theta_H$ -IC still hold.
- (5) Thus,  $e_L^{SB} > e_L^*$  cannot be optimal.

39 Graphic illustration for  $e_H^{SB} = e_H^*$ .

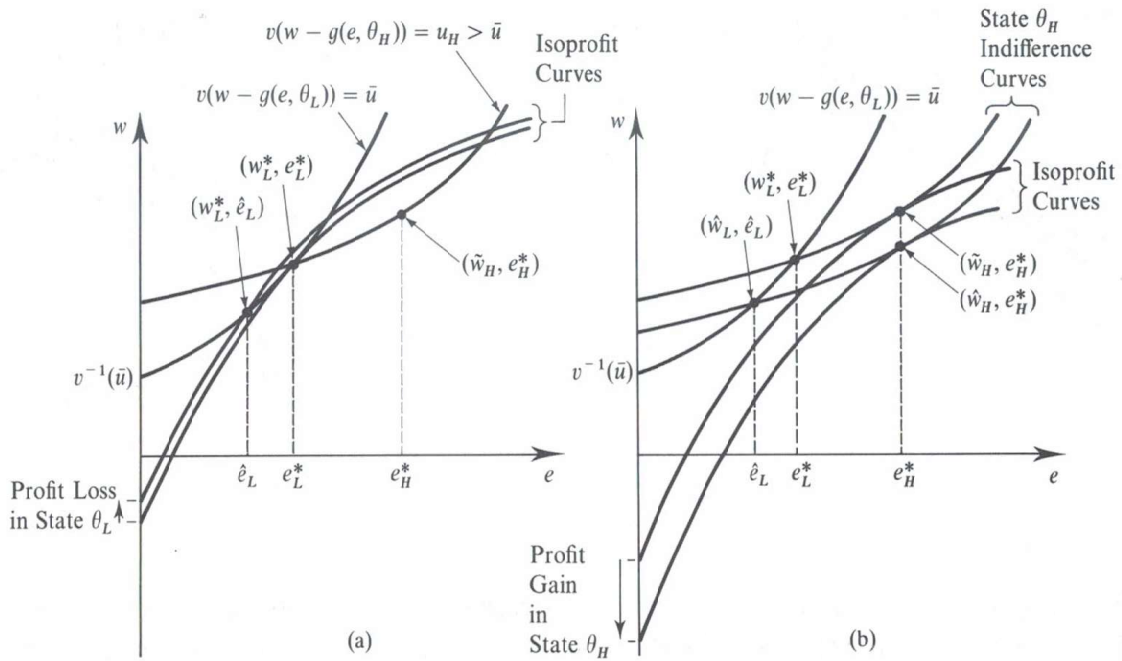


- (1) Suppose that  $e_L^{SB} \leq e_L^*$ .
- (2) To make  $\theta_L$ -IC and  $\theta_H$ -IC hold,  $(e_H^{SB}, w_H^{SB})$  lies in the shade region.
- (3) Principal's problem is to find the allocation of  $(e_H^{SB}, w_H^{SB})$  that maximizes her profit.
- (4) The optimal solution occurs at a point of tangency between the indifference curve of  $\theta_H$  agent through  $(e_H^{SB}, w_H^{SB})$  and an isoprofit curve for principal.
- (5) All points of tangency between indifference curves of  $\theta_H$ -agent and isoprofit curves of principal occur at  $e_H^*$ .

40 Graphic illustration for the optimal contracts.



- (1) Suppose that principal starts with  $(e_L^*, w_L^*)$  for  $\theta_L$  agents, which lies on the  $\theta_L$  indifference curve through  $(0, v^{-1}(\bar{u}))$ .
- (2) Since  $\theta_H$ -IC binds, principal could choose  $(e_H^*, \tilde{w}_H)$  for  $\theta_H$  agents, which lies on  $\theta_H$  indifference curve through  $(e_L^*, w_L^*)$ .
- (3) The menu  $\{(e_H^*, \tilde{w}_H), (e_L^*, w_L^*)\}$  is IC. However, principal can do better.



- (1) Principal firstly moves  $(e_L^*, w_L^*)$  to  $(e_L^{SB}, w_L^{SB})$ , where  $e_L^* > e_L^{SB}$ . Note that both  $(e_L^*, w_L^*)$  and  $(e_L^{SB}, w_L^{SB})$  lie on  $\theta_L$  indifference curve through  $v^{-1}(\bar{u})$ .
- (2) This change lowers the profit that principal earns from  $\theta_L$  agents.
- (3) On the other hand, it relaxes  $\theta_H$ -IC.
- (4) Principal then moves  $(e_H^*, \tilde{w}_H)$  to  $(e_H^*, \hat{w}_H)$ .
- (5) This change increases the profit that principal earns from  $\theta_H$  agents.
- (6) Comparison:

- The derivative of principal's profit from  $\theta_L$  agent with respect to  $e_L$  at  $e_L^*$  is zero:

$$\frac{d}{de_L} [\pi(e_L) - g(e_L, \theta_L) - v^{-1}(\bar{u})] \Big|_{e_L=e_L^*} = 0.$$

- The derivative of principal's profit from  $\theta_H$  agent with respect to  $e_L$  at  $e_L^*$  is strictly negative:

$$\frac{d}{de_L} [\pi(e_L^*) - g(e_L^*, \theta_H) - v^{-1}(\bar{u}) + g(e_L, \theta_H) - g(e_L, \theta_L)] \Big|_{e_L=e_L^*} < 0.$$

(7) How far should principal go in lowering  $e_L$ —When the marginal loss from  $\theta_L$  agent equals the marginal gain from  $\theta_H$  agent, i.e.,

$$(1 - \lambda) \cdot [\pi'(e_L^{SB}) - g_e(e_L^{SB}, \theta_L)] = \lambda \cdot [g_e(e_L^{SB}, \theta_L) - g_e(e_L^{SB}, \theta_H)].$$

41 配置效率与信息租金之间的权衡：

- 为了让  $\theta_H$  选择为其设计的劳动水平，需要给他一定好处的信息租金；该信息租金取决于  $\theta_L$  的劳动水平，以及  $\theta_H$  和  $\theta_L$ 。
- 之所以降低  $\theta_L$  的劳动水平，是为了尽可能减少支付给  $\theta_H$  的信息租金。

## 4 Homework

- Key: The optimal contracts in monopolistic screening.

偏离最优的次优合约将导致劳动水平的扭曲，委托人需要让渡一些信息租金给最有效率的代理人。

- Reading: 14.C in MWG, 2.1–2.9 in *The Theory of Incentives*
- Optional reading: 16.1–16.2 in 高级微观经济学 (田国强)