Advanced Microeconomics: Lecture Notes 8

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1 When two parties engage in a relationship, it is often the case that they are uncertain about the value of some parameter that will affect their future gains from trade. This uncertainty is represented by assuming that the parameter can take several values, each value corresponding to different states of nature whose probability distribution is common knowledge.

Even though they will both learn the value of the parameter in the future, the trading partners cannot write ex ante contracts contingent on the state of nature, because this state of nature is not verifiable by a third party that could enforce their contract. That is the nonverifiability (不可验证性) of the state of nature.

不可验证性问题在现实中并不少见：代理人有私有信息，但由于某些原因（比如两者都对某所在的行业比较了解），委托人也可以观察到该信息。但双方都不可能对此提供客观证据，而外界又不掌握了解这一行业所需的专业知识，从而无法验证私有信息。

2 The goal is to assess whether the nonverifiability significantly affects the ability of the contractual partners to realize the full gains from trade.

3 An owner (principal) wishes to hire a manager (agent) to run a one-time project.

If the agent's effort level is $e \in [0, \infty)$, then principal's income is $\pi(e)$, with $\pi(0) = 0$, $\pi'(e) > 0$, and $\pi''(e) < 0$ for all e.

If the principal pays wage w to the agent, his utility is $\pi(e) - w$.

4 The agent is an expected utility maximizer with utility $w - g(e, \theta)$.

- $\theta \in \{\theta_L, \theta_H\}$ represents agent's ability. Here, $\theta_H > \theta_L$ and Prob$(\theta_H) = \lambda \in (0, 1)$.
- $g(e, \theta)$ measures the disutility/cost of effort.
- $g(0, \theta) = 0$, $g_e(e, \theta) \begin{cases} > 0, & \text{if } e > 0 \\ = 0, & \text{if } e = 0 \end{cases}$, $g_{ee} > 0$, $g_\theta < 0$, $g_{e\theta}(e, \theta) \begin{cases} < 0, & \text{if } e > 0 \\ = 0, & \text{if } e = 0 \end{cases}$.

$\Rightarrow$ The agent's indifference curves have single-crossing property.

- The agent is risk neutral. The agent has a reservation utility 0.

5 First-best outcome:

- $\pi'(e_H^*) = g_e(e_H^*, \theta_H)$ when θ_H.
- $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$ when θ_L.

Note that $\pi(e_H^*) - g(e_H^*, \theta_H) > 0$ and $\pi(e_L^*) - g(e_L^*, \theta_L) > 0$. That is, delegation is valuable.

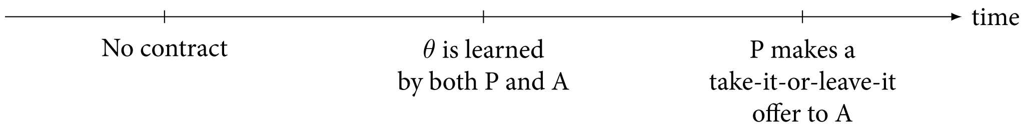
1 No ex ante contract

6 We consider the case where the principal and the agent do not write any contract ex ante. Bargaining over the gains from trade takes place ex post, i.e., once the state of nature is commonly known.

1.1 Principal has full bargaining power

7 We assume that the principal has all the bargaining power at the ex post stage.

8 The sequence of play is as follows:



After being informed about θ, the principal can make a take-it-or-leave-it offer to the agent under complete information.

9 The offer can implement the first-best outcome:

- If agent is of high ability, then he will make effort e_H^* such that $\pi'(e_H^*) = g_e(e_H^*, \theta_H)$, receive wage $w_H^* = g(e_H^*, \theta_H)$.
- If agent is of low ability, then he will make effort e_L^* such that $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$, receive wage $w_L^* = g(e_L^*, \theta_L)$.

1.2 Bargaining

10 We assume that the principal and the agent have equal weights in the negotiation at the ex post stage.

11 The sequence of play is as follows:

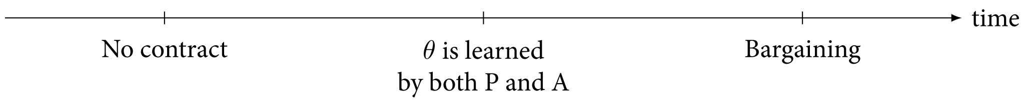


Figure 1

12 We use Nash bargaining solution.

13 A two-person bargaining problem, denoted by $\langle U, d \rangle$, consists of

- U is the set of possible agreements in terms of utilities that they yield to 1 and 2. An element of U is a pair $u = (u_1, u_2)$.
- d is a pair (d_1, d_2), called the disagreement point or threat point.

If agreement $u = (u_1, u_2) \in U$ is reached, then 1 gets utility u_1 and 2 gets utility u_2. If no agreement is reached then 1 gets utility d_1 and 2 gets utility d_2.

The set of two-person bargaining games is denoted by W.

14 Convention: Assume that

- U is compact and convex.
- U contains a point y for which $y_i > d_i$ for $i = 1, 2$, that is, bargaining is worthwhile for both the players.

15 The Nash bargaining solution is a mapping $f\colon W \to \mathbb{R}^2$ that associates a unique element $f(U, d)$ with the game $\langle U, d\rangle$, satisfying the following axioms:

N1. Feasibility: $f(U, d) \in U$.

N2. Individual rationality: $f(U, d) \geq d$ for all $\langle U, d\rangle \in W$.

N3. Pareto optimality: $f(U, d)$ is Pareto optimal. That is, there does not exist a point $(u_1, u_2) \in U$ such that

$$u_1 \geq f_1(U, d), u_2 \geq f_2(U, d), (u_1, u_2) \neq f(U, d).$$

N4. Symmetry: If $\langle U, d\rangle \in W$ satisfies $d_1 = d_2$ and $(x_1, x_2) \in U$ implies $(x_2, x_1) \in U$, then $f_1(U, d) = f_2(U, d)$.

N5. Invariance under linear transformations: Let $a_1, a_2 > 0$, $b_1, b_2 \in \mathbb{R}$, and $\langle U, d\rangle, \langle U', d'\rangle \in W$ where $d_i' = a_i d_i + b_i$, $i = 1, 2$, and $U' = \{x \in \mathbb{R}^2 \mid x_i = a_i y_i + b_i, i = 1, 2, y \in U\}$. Then $f_i(U_i', d_i') = a_i f_i(U, d) + b_i$, $i = 1, 2$.

N6. Independence of irrelevant alternatives: If $\langle U, d\rangle, \langle U', d'\rangle \in W$, $d = d'$, $U \subseteq U'$, and $f(U', d') \in U$, then $f(U, d) = f(U', d')$.

The interpretation is that, given any bargaining problem $\langle U, d\rangle$, the solution function tells us that the agreement $u = f(U, d)$ will be reached.

16 Theorem: A game $\langle U, d\rangle \in W$ has a unique Nash solution $u^* = f(U, d)$ satisfying Conditions N1 to N6. Furthermore, the solution u^* satisfies Conditions N1 to N6 if and only if

$$(u_1^* - d_1)(u_2^* - d_2) > (u_1 - d_1)(u_2 - d_2)$$

for all $u \in U$, $u \geq d$, and $u \neq u^*$.

17 Remark:

- Existence of an optimal solution: Since the set U is compact and the objective function is continuous, there exists an optimal solution.
- Uniqueness of the optimal solution: The objective function is strictly quasi-concave. Therefore, maximization problem has a unique optimal solution.

18 When the agent is of high ability, they shall agree on effort e and wage w, which are solutions to the problem

$$\max_{(e,w)} \big(\pi(e) - w\big)\big(w - g(e, \theta_H)\big).$$

19 Solution:

- Wage $w_H^{\mathrm{NB}} = \frac{1}{2}[\pi(e_H^*) + g(e_H^*, \theta_H)]$.
- Effort is first-best effort e_H^* such that $\pi'(e_H^*) = g_e(e_H^*, \theta_H)$;

Both principal and agent receive an equal share of the first-best gains.

20 Similarly for the low ability case.

- Wage $w_L^{\text{NB}} = \frac{1}{2}[\pi(e_L^*) + g(e_L^*, \theta_L)]$.
- Effort is first-best effort e_L^* such that $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$;

Both principal and agent receive an equal share of the first-best gains.

21 Summary:

- Bargaining over the gains from trade takes place ex post, i.e., once the state of nature is commonly known.
- If the principal has all the bargaining power ex post, the first-best outcome is implemented, with the agent being maintained at his status quo utility level.
- If we had considered a more even distribution of the bargaining power ex post, outcome efficiency would still be preserved, but the distribution of the gains from trade would be more egalitarian: the principal (resp. the agent) would obtain a lower (resp. higher) utility level.
- If the principal does not expect to have all the bargaining power at the ex post stage, he strictly prefers to design a mechanism at the ex ante stage when he still has all the bargaining power.

2 Ex ante contract

22 Instead of waiting for the realization of the state of nature, the principal can offer to the agent, at the ex ante stage, a menu of contracts.

23 The contract can only be written in terms of the verifiable variables. θ is not verifiable and cannot be written into a contract.

A nonlinear price $w(e)$ or a menu $\{(e_H, w_H), (e_L, w_L)\}$ is a feasible instrument.

- When facing the menu $\{(e_H, w_H), (e_L, w_L)\}$, agent accepts the menu itself or not.
- In contrast, in the screening model, agent chooses (e_H, w_H), (e_L, w_L), or neither when he faces a menu $\{(e_H, w_H), (e_L, w_L)\}$.

When he accepts such a contract $\{(e_H, w_H), (e_L, w_L)\}$, the agent anticipates that

- his choice of outputs e_H in state θ_H will satisfy the following interim constraint

$$w_H - g(e_H, \theta_H) \geq w_L - g(e_L, \theta_H).$$

- his choice of outputs e_L in state θ_L will satisfy the following interim constraint

$$w_L - g(e_L, \theta_L) \geq w_H - g(e_H, \theta_L).$$

These constraints are the same as the standard incentive compatibility constraints as in adverse selection.

24 The sequence of play is as follows:

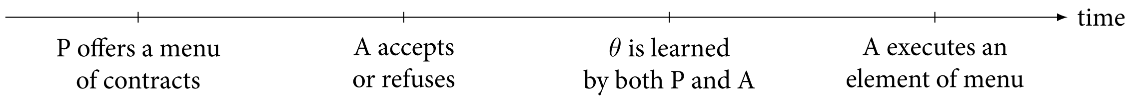


Figure 2

25 Agent's problem:

- IR: $\lambda\left(w_H - g(e_H, \theta_H)\right) + (1-\lambda)\left(w_L - g(e_L, \theta_L)\right) \geq 0$.
- IC: $w_H - g(e_H, \theta_H) \geq w_L - g(e_L, \theta_H)$ and $w_L - g(e_L, \theta_L) \geq w_H - g(e_H, \theta_L)$.

26 Principal's problem:

$$\begin{aligned} \underset{(e_L,w_L),(e_H,w_H)}{\text{maximize}} \quad & \lambda\big(\pi(e_H) - w_H\big) + (1-\lambda)\big(\pi(e_L) - w_L\big) \\ \text{subject to} \quad & \text{IR and IC.} \end{aligned}$$

27 IR should be binding at the optimum. Otherwise, principal can lower w_H and w_L simultaneously.

28 Ignoring IC, principal's problem is

$$\max_{e_H, e_L} \lambda\left(\pi(e_H) - g(e_H, \theta_H)\right) + (1-\lambda)\left(\pi(e_L) - g(e_L, \theta_L)\right).$$

SOC and FOC imply that $\pi'(e_H^*) = g_e(e_H^*, \theta_H)$ and $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$.

29 IC conditions can be satisfied by setting

$$w_H^* = \pi(e_H^*) - T^* \text{ and } w_L^* = \pi(e_L^*) - T^*,$$

where T^* is a lump-sum payment:

$$\begin{aligned} w_H^* - w_L^* = \pi(e_H^*) - \pi(e_L^*) = \int_{e_L^*}^{e_H^*} \pi'(e)\,\mathrm{d}e \geq \int_{e_L^*}^{e_H^*} g_e(e, \theta_H)\,\mathrm{d}e = g(e_H^*, \theta_H) - g(e_L^*, \theta_H), \\ w_H^* - w_L^* = \pi(e_H^*) - \pi(e_L^*) = \int_{e_L^*}^{e_H^*} \pi'(e)\,\mathrm{d}e \leq \int_{e_L^*}^{e_H^*} g_e(e, \theta_L)\,\mathrm{d}e = g(e_H^*, \theta_L) - g(e_L^*, \theta_L). \end{aligned}$$

To make IR bind, we can choose $T^* = \lambda\left(\pi(e_H^*) - g(e_H^*, \theta_H)\right) + (1-\lambda)\left(\pi(e_L^*) - g(e_L^*, \theta_L)\right)$.

30 This implementation of the first-best outcome amounts to having the principal selling the benefit of the relationship to the risk-neutral agent for a fixed up-front payment T^*. The agent benefits from the full value of the good and trades off the value of any production against its cost just as if he was an efficiency maximizer.

31 Summary: Efficiency is always achieved when the single crossing property is satisfied for the agent's objective function:

- The first-best outcome can be implemented: $\pi'(e_H^*) = g_e(e_H^*, \theta_H)$ and $\pi'(e_L^*) = g_e(e_L^*, \theta_L)$,

  $$w_H^* = \pi(e_H^*) - T^* \text{ and } w_L^* = \pi(e_L^*) - T^*,$$

  where $T^* = \lambda\left(\pi(e_H^*) - g(e_H^*, \theta_H)\right) + (1-\lambda)\left(\pi(e_L^*) - g(e_L^*, \theta_L)\right)$.

32 If agent is risk-averse, ex ante contracting fails to achieve efficiency. (Bonus question)

3 Nash implementation

33 The principal and agent can achieve ex post efficiency through an ex ante contract when they are both risk neutral. This contract uses only agent's message but fails to achieve efficiency when the agent is risk-averse.

34 Consider the following mechanism:

- If both principal and agent report that θ_H has realized, the contract (e_H^*, w_H^*) is enforced, where

$$\pi'(e_H^*) = g_e(e_H^*, \theta_H) \text{ and } w_H^* = g(e_H^*, \theta_H).$$

- If both principal and agent report that θ_L has realized, the contract (e_L^*, w_L^*) is enforced, where

$$\pi'(e_L^*) = g_e(e_L^*, \theta_L) \text{ and } w_L^* = g(e_L^*, \theta_L).$$

- If they disagree, then nothing is enforced.

		Principal	
		θ_H	θ_L
Agent	θ_H	(e_H^*, w_H^*)	$(0,0)$
	θ_L	$(0,0)$	(e_L^*, w_L^*)

Note that the same game form must be played by the agent and the principal, whatever the true θ.

The goal of this mechanism is to ensure that there exists a truthful Nash equilibrium in each θ that implements the first-best outcome.

35 Proposition: The first-best outcome can be implementable in Nash equilibrium.

Proof. First consider θ_H.

- Given that agent reports θ_H, principal gets $\pi(e_H^*) - g(e_H^*, \theta_H)$ by reporting the truth and zero otherwise.
- By assumption, the delegation is valuable: $\pi(e_H^*) - g(e_H^*, \theta_H) = \pi(e_H^*) - w_H^* \geq 0$.
- Telling the truth is a best response for principal.
- Agent is indifferent telling the truth or not when principal reports θ_H.

Next consider θ_L.

- Given that agent reports θ_L, principal gets $\pi(e_L^*) - g(e_L^*, \theta_L)$ by reporting the truth and zero otherwise.
- By assumption, the delegation is valuable: $\pi(e_L^*) - g(e_L^*, \theta_L) = \pi(e_L^*) - w_L^* \geq 0$.
- Telling the truth is a best response for principal.
- Agent is indifferent telling the truth or not when principal reports θ_L.

□

36 When θ_H realizes, (θ_H, θ_H) is not the unique Nash equilibrium.

37 Consider the following mechanism:

		Principal	
		θ_H	θ_L
Agent	θ_H	(e_H^*, w_H^*)	$(\hat{e}_2, \hat{w}_2)$
	θ_L	$(\hat{e}_1, \hat{w}_1)$	(e_L^*, w_L^*)

38 The outcomes $(\hat{e}_1, \hat{w}_1)$ and $(\hat{e}_2, \hat{w}_2)$ may be different from the no-trade option used above, in order to give more flexibility to the court in designing off-the equilibrium punishments, ensuring both the truthful revelation and the uniqueness of the equilibrium. Let us now see how it is possible to do so.

39 The conditions for having a truthful Nash equilibrium in θ_H are:

- For principal, reporting θ_H is better than reporting θ_L: $\pi(e_H^*) - w_H^* > \pi(\hat{e}_2) - \hat{w}_2$. (红线左上方)
- For agent, reporting θ_H is better than reporting θ_L: $0 = w_H^* - g(e_H^*, \theta_H) > \hat{w}_1 - g(\hat{e}_1, \theta_H)$. (2 号线右下方)

Similarly, the conditions for having a truthful Nash equilibrium in θ_L are:

- For principal, reporting θ_L is better than reporting θ_H: $\pi(e_L^*) - w_L^* > \pi(\hat{e}_1) - \hat{w}_1$. (蓝线左上方)
- For agent, reporting θ_L is better than reporting θ_H: $0 = w_L^* - g(e_L^*, \theta_L) > \hat{w}_2 - g(\hat{e}_2, \theta_L)$. (4 号线右下方)

40 Since $\pi(e_L^*) - w_L^* > \pi(\hat{e}_1) - \hat{w}_1$, principal still prefers to report θ_L when θ_H.

Thus, when θ_H, to ensure (θ_L, θ_L) not to be a Nash equilibrium, we must have: agent prefers to report θ_H:

$$\hat{w}_2 - g(\hat{e}_2, \theta_H) > w_L^* - g(e_L^*, \theta_H).$$

(5 号线左上方)

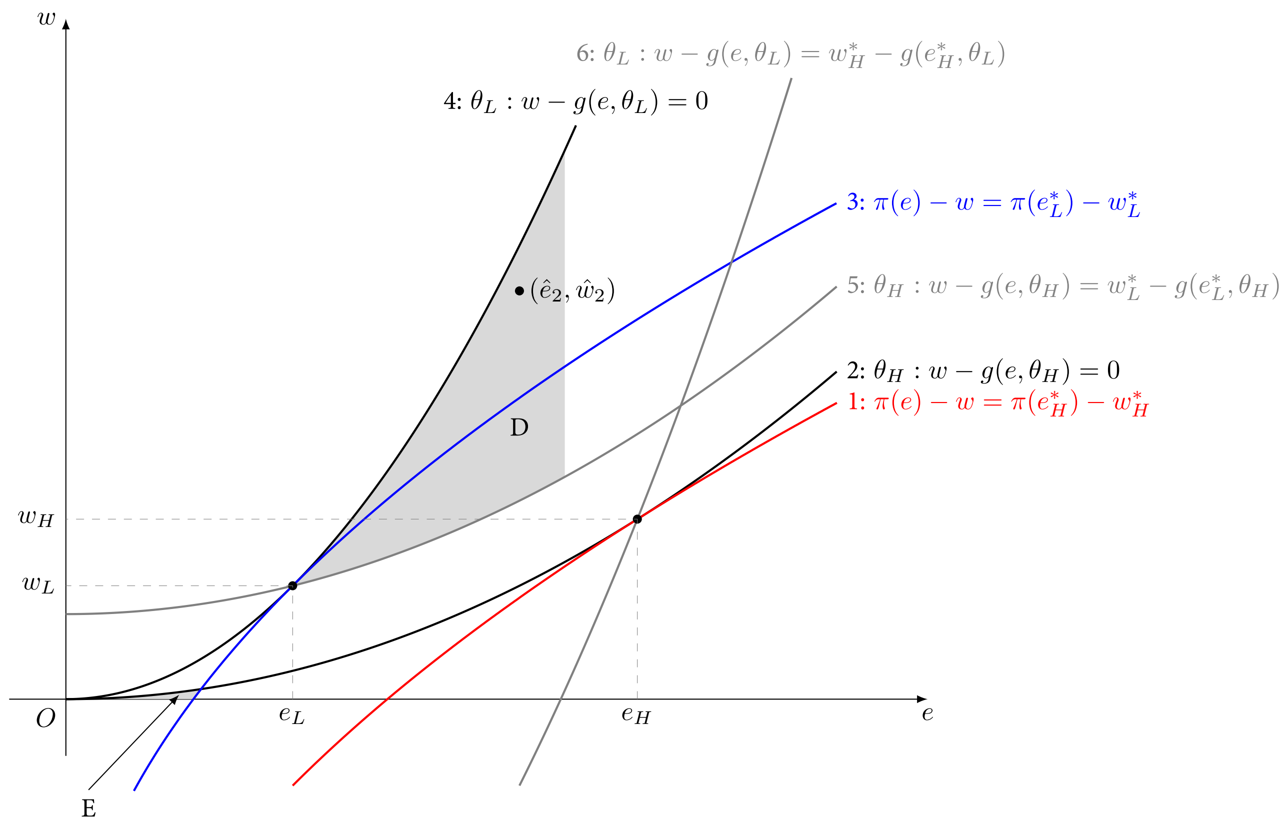
Similarly, when θ_L, to ensure (θ_H, θ_H) not to be a Nash equilibrium, we must have: agent prefers to report θ_L:

$$\hat{w}_1 - g(\hat{e}_1, \theta_L) > w_H^* - g(e_H^*, \theta_L).$$

(6 号线左上方)

41 Proposition: The first-best outcome can be uniquely implementable in Nash equilibrium.

Proof. Consider the following graph.



Pick $(\hat{e}_1, \hat{w}_1)$ in region E and $(\hat{e}_2, \hat{w}_2)$ in region D. □

42 Summary:

- The principal offers a mechanism that is designed to ensure that the noncooperative play of the game by both the principal and the agent yields the desired first-best allocation.
- The standard principal-agent models are such that the first-best is implementable in Nash equilibrium with rather simple mechanisms.

4 Homework

- Reading: Chapter 6 in *The Theory of Incentives*, Chapter 6 in 信息与激励经济学