Game Theory

Introduction

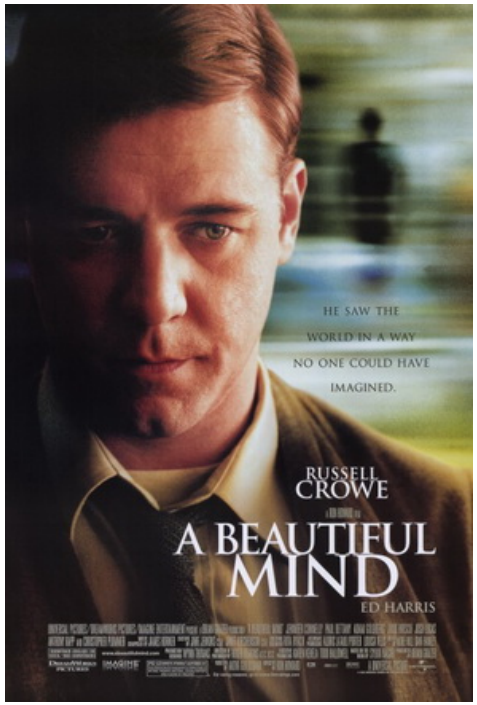
Xiang Sun

2021 Summer

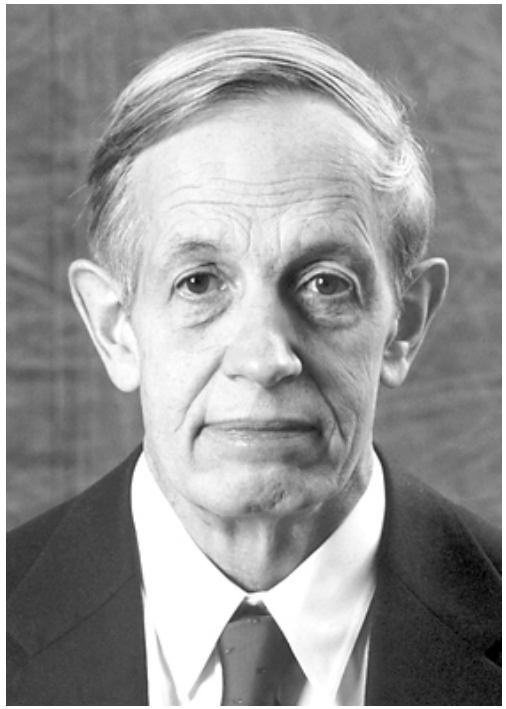
A Motivating Example

- In the movie, "A Beautiful Mind," John Nash hangouts with three buddies in a club. Five women walk in, four brunettes and a stunning blonde. Each of the three buddies starts forward to introduce himself to the blonde.
- Nash stops them, though, saying, "if we all go for the blonde, we will all be rejected and none of the brunettes will talk to us afterwards because they will be offended. So let's go for the brunettes".
- The next thing we see is the four buddies dancing with the four brunettes and the blonde standing alone, looking unhappy.
- Question: Is John Nash right? What should the four buddies do instead?

A Motivating Example



(a) A Beautiful Mind



(b) John Nash

A Motivating Example

- The example above involves a situation in which different individuals need to make some decisions: each of the four buddies wants to choose a woman to dance with.
- There is strategic interaction (策略性的互动) among individuals: when each buddy decides which woman to approach, he needs to think about the possible choices that his friends will make.
- This example can be considered as a game in which strategic interactions take place.
- We need some tools to understand or predict how people behave in such strategic settings $\Rightarrow$ Game theory

What is Game Theory?

- It sounds obvious that life is all a game!
- "Man is a gaming animal. He must always be trying to get the better in something or other."

  —C. Lamb, *Essays of Elia* (伊利亚随笔集), 1823
- Game theory is the study of mathematical models of strategic interaction among rational decision-makers.

  It provides a mathematical tool for multi-person decision making.
- 实际场景 → 博弈模型 → 博弈的解（参与者在博弈中应该如何行动）

Why do We Need to Study Game Theory?

"Nowadays one cannot find a field of economics (or of disciplines related to economics, such as finance, accounting, marketing, political science) in which understanding the concept of a Nash equilibrium is not nearly essential to the consumption of the recent literature $\cdots$ the basic notions of non-cooperative game theory have become a staple in the diet of students of economics."

—D. M. Kreps (*Game Theory and Economic Modelling*, 1990)

Why do We Need to Study Game Theory?

- Game theory plays a significant role in much of the contemporary research in economics.
- Applications of game theory have arisen in many fields of economics, such as industrial organization, international economics, labor economics, finance, and political science.
- Examples found:
  - Oligopoly competition in industrial organization
  - Bargaining and auctions in microeconomics
  - Firms' input choice problems in labor markets or in finance
  - Trade policies in international trade
  - Government policy choice in macroeconomics or political economics
  - etc.

Game theory vs. decision theory

- Decision theory studies the formal theory of decision-making of a single individual.
- Many theories in microeconomics belong to the region of decision theory (e.g., consumer theory, producer theory).
- In particular, decision theory can be viewed as the theory of games where there is only one individual.
- Decision theory cannot deal with strategic settings with many individuals.

Rationality

- One key assumption in game theory is the rationality (理性) of individuals.
- In economics, the standard form of rationality means that a decision maker chooses an action that yields maximum (expected) utility among all possible actions, given the decision maker's information.
- In game theory, a player is rational if the player chooses an action that maximizes his (expected) payoff, given the player's beliefs about opponents' strategy choices.
- In this sense, game theory studies the formal theory of decision-making of rational individuals.

Philosophy

- The philosophy of game theory helps to understand and hopefully to resolve human conflicts.
- The normative and positive work of game theory has been motivated by an interest in the rigorous understanding of human behavior in strategic contexts.
- Game theory is not descriptive but rather conditionally prescriptive. The standard game theory does not state how players do behave, but how they should behave in a certain sense.
- Most game-theoretic solution concepts tell not how people behave in practice, but how rational and intelligent agents should behave.

Methodology of Game Theory

- We first focus on the non-cooperative game theory framework, which treats all players' actions as individual actions. An individual action is determined by a player himself, independent of others in the strategic environment.
- We study different solution concepts in different game environments.
- The standard solution is equilibrium (均衡).
- Mathematics is a language, what's important is the ability to reason.

Classification

- We separate different types of non-cooperative games according to how the game is played (static vs. dynamic) and players' information about others (complete information vs. incomplete information).
- Static: one-shot, simultaneous-move
- Complete information: each player's payoff function is common knowledge among all players.

Classification

- Four types of non-cooperative games we consider
  1. Static games with complete information
  2. Dynamic games with complete information
  3. Static games with incomplete information
  4. Dynamic games with incomplete information
- Four corresponding solution concepts
  1. Nash equilibrium 纳什均衡
  2. Subgame perfect Nash equilibrium 子博弈精炼均衡
  3. Bayesian Nash equilibrium 贝叶斯纳什均衡
  4. Perfect Bayesian equilibrium 精炼贝叶斯均衡

Knowledge

- Example:
  - Each of 100 individuals is wearing a hat that is either white or black. Two of these individuals are wearing a white hat.
  - Each individual can see the hats of the other individuals, but not his own.
  - An observer publicly announces: "Each of you is wearing a hat that is either white or black. At least one of the hats is white. I will start to count slowly. After each number you will have the opportunity to raise a hand. You may do so only when you know the color of your hat."
- Question: When, for the first time, will any individual raise his hand?

Knowledge

- Answer: The two wearing a white hat will raise their hands when the observer counts the number "2".
- Reason:
  1. When the observer counts the number "1", no one knows the color of his/her hat. The two wearing a white hat see (1 white and 98 black) and the others see (2 white and 97 black).
  2. When the observer counts the number "2", the two wearing a white hat now know the color of their hats. Each of the two who are wearing a white hat can reason as follows: Since the one wearing a white hat (I saw last time) did not raise his/her hand, there should be another white hat which must be on my head.

Knowledge

- An event E is mutual knowledge (相互知识) among the players if all players know E.
- Mutual knowledge by itself does not imply anything about knowledge that anyone attributes to anyone else.
- An event E is said to be common knowledge (共同知识) among the players if all players know E, all players know that they all know E, all players know that they all know that they all know E, and so on ad infinitum.
- An easier way to interpret this is that all players gather around a table where E is displayed, so that each player can verify that the others observe E and also can verify the same about everyone else.

Knowledge

莊子與惠子游於濠梁之上。

- 莊子曰：儵魚出游從容，是魚之樂也。
- 惠子曰：子非魚，安知魚之樂？
- 莊子曰：子非我，安知我不知魚之樂？
- 惠子曰：我非子，固不知子矣；子固非魚也，子之不知魚之樂，全矣！

莊子・外篇・秋水

Knowledge

There are four kinds of men:

1. He who knows not and knows not he knows not: he is a fool - shun him;
2. He who knows not and knows he knows not: he is simple - teach him;
3. He who knows and knows not he knows: he is asleep - wake him;
4. He who knows and knows he knows: he is wise - follow him.

—Arabian Proverb

Knowledge

- Suppose we extend the example to a general case where there are N individuals and $k \geq 1$ of them are wearing a white hat.
- Will all individuals who wear a white hat know the color of their hats? If so, when for the first time, will anyone raise his hand?
- It is common knowledge that "at least one hat is white", if all k individuals who wear a white hat will know the color of their hats eventually, for any k, N and $1 \leq k \leq N$.
- We can show by induction that all individuals who are wearing a white hat will raise their hands when the observer announces "k". (Exercise)

Augustin Cournot

In 1838, the book *Researches into the Mathematical Principles of the Theory of Wealth* by Antoine Augustin Cournot (安托万・奥古斯丁・库尔诺).

In Chapter 7 of the book, "On the competition of producers", Cournot discussed the special case of duopoly and utilises a solution concept that is a restricted version of the Nash equilibrium.

Ernst Zermelo

In 1913, Zermelo's theorem by Ernst Zermelo (恩斯特・策梅洛).

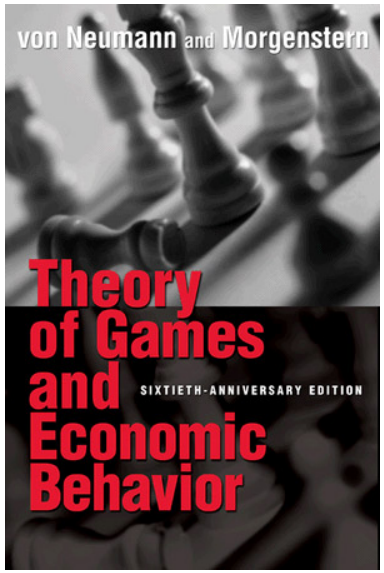
More formally, every finite extensive-form game exhibiting full information has a Nash equilibrium that is discoverable by backward induction. If every payoff is unique, for every player, this backward induction solution is unique.
When applied to chess, Zermelo's theorem states "either white can force a win, or black can force a win, or both sides can force at least a draw."

John von Neumann

In 1928, Zur Theorie der Gesellschaftsspiele (团队游戏之理论) by John von Neumann (约翰·冯·诺伊曼).

John von Neumann proved the minimax theorem in this paper. It states that every two-person zero-sum game with finitely many pure strategies for each player is determined, i.e., when mixed strategies are admitted, this variety of game has precisely one individually rational payoff vector. This paper also introduced the extensive form of a game.

Theory of Games and Economic Behavior



This book is considered the groundbreaking text that created the interdisciplinary research field of game theory. As well as expounding two-person zero sum theory this book is the seminal work in areas of game theory such as the notion of a cooperative game, with transferable utility, its coalitional form and its von Neumann-Morgenstern stable sets. It was also the account of axiomatic utility theory given here that led to its wide spread adoption within economics.