Advanced Microeconomics: Lecture Note 2

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1 Introduction of adverse selection

1 There are many choice situations where a principal delegates the completion of a task to an agent:

- A stockholder delegates the firm's day-to-day decisions to a manager,
- A client delegates his defense to an attorney,
- The landlord delegates the cultivation of his land to a tenant,
- An investor delegates the management of his portfolio to a broker,
- A government procures vaccines from private companies.

2 Delegation can be motivated:

- either by the possibility of benefitting from some increasing returns associated with the division of tasks,
  - e.g., the manager will be the only one to know the business conditions.
- or by the principal's lack of time or lack of any ability to perform the task himself,
  - e.g., the attorney knows better than the client how difficult the case will be.
- or by any other form of the principal's bounded rationality when facing complex problems.
  - e.g., the tenant will be the only one to observe the exact local weather conditions.

3 By the mere fact of this delegation, the agent may get access to information that is not available to the principal.

In other words, the agent may have or gain private information, which is hidden to the principal.

Some examples of pieces of information that may become private knowledge of the agent can be:

- The exact opportunity cost of this task,
- the precise technology used, and how good the matching is between the agent's intrinsic ability and this technology.

In such cases, we will say that there is adverse selection.

4 In order to carry out the delegation of these tasks, the principal and the agent would sign a (bilateral) contract, where the outcomes are verifiable and the consequences are enforceable by a benevolent court of law.

- The key common aspect of all those contracting settings is that the information gap between the principal and the agent has some fundamental implications for the design of the bilateral contract they sign.

- In order to reach an efficient use of economic resources, this contract must elicit the agent's private information.
- This can only be done by giving up some information rent to the privately informed agent, which is costly to the principal.
  - This information cost just adds up to the standard technological cost of performing the task and justifies distortions in the volume of trade achieved under asymmetric information.
- The main objective is to characterize the optimal rent extraction-efficiency trade-off faced by the principal when designing his contractual offer to the agent.
  - The allocative and the informational roles of the contract generally interfere. At the optimal second-best contract, the principal trades off his desire to reach allocative efficiency against the costly information rent given up to the agent to induce information revelation.

5 We proceed in two steps:

- First, we describe the set of allocations (i.e., output to be produced and a distribution of the gains from trade) that the principal can achieve (despite the information gap),
  - incentive compatibility constraints (that are only due to asymmetric information),
  - voluntary participation constraints that ensure that the agent wants to participate in the contract.
- Second, we proceed by optimizing the principal's objective function within the set of incentive feasible allocations.

6 Consequences of hidden information:

- In general, incentive constraints will be binding at the optimum,
  - showing that adverse selection clearly affects the efficiency of trade.
- As such, the optimal second-best contract calls for
  - a distortion in the volume of trade away from the first-best allocation,
  - and for giving up some strictly positive information rents to the most efficient agents.

7 Implicit assumptions:

- We assume that the principal and the agent both adopt an optimizing behavior and maximize their individual utility.
  - In other words, they are both fully rational individualistic agents.
  - Given the contract he receives from the principal, the agent maximizes his utility and chooses output accordingly.
- The principal does not know the agent's private information, but the probability distribution of this information is common knowledge.
  - There exists an objective distribution for the possible types of the agent that is known by both the agent and the principal, and this fact itself is known by the two players.
- The principal is a Bayesian expected utility maximizer.
  - In designing the agent's payoff rule, the principal moves first as a Stackelberg leader under asymmetric information anticipating the agent's subsequent behavior and optimizing accordingly within the set of available contracts.

2 Model

8 Consider a consumer (the principal) who wants to delegate to an agent the production of q units of a good.

9 The value for the principal of these q units is $S(q)$ where $S' > 0$, $S'' < 0$ and $S(0) = 0$.

The marginal value of the good is thus positive and strictly decreasing with the number of units bought by the principal.

10 The production cost of the agent is unobservable to the principal, but it is common knowledge that the marginal cost θ belongs to the set $\Theta = \{\theta_L, \theta_H\}$.

The agent can be either efficient (θ_L) or inefficient (θ_H) with respective probabilities λ and $1 - \lambda$. In other words, he has the cost function

$$c(q, \theta_L) = \theta_L q \text{ with probability } \lambda$$

or

$$c(q, \theta_H) = \theta_H q \text{ with probability } 1 - \lambda.$$

We denote by $\Delta\theta = \theta_H - \theta_L > 0$ the spread of uncertainty on the agent's marginal cost.

Agent has a reservation utility $\bar{u}$, which is assumed to zero. It captures the outside opportunity.

11 The principal's utility, if she purchases q units of the good and pays a monetary transfer t to the agent, is

$$S(q) - t,$$

and at this case the agent's utility is

$$t - c(q, \theta).$$

12 The economic variables are quantity produced q and the transfer t received by the agent.

These variables are both observable and verifiable by a third party such as a benevolent court of law. They can be included in a contract which can be enforced with appropriate penalties if either the principal or the agent deviates from the requested output and transfer.

Let $\mathcal{A}$ be the set of all feasible contract, that is,

$$\mathcal{A} = \{(q, t) \mid q \in \mathbb{R}_+, t \in \mathbb{R}\}.$$

3 Complete information—the first-best outcome

13 First suppose that there is no asymmetry of information between the principal and the agent.

14 The efficient production levels are obtained by maximizing the social value:

$$\max_{q_i \geq 0} S(q_i) - c(q_i, \theta_i) = \max_{q_i \geq 0} S(q_i) - \theta_i q_i.$$

Since $S'' < 0$, the objective function is concave. Then the solution q_i^* must satisfy the first order condition:

$$S'(q_i^*) \begin{cases} \leq \theta_i, & \\ = \theta_i, & \text{if } q_i^* > 0. \end{cases}$$

15 The above equation may not have an interior solution.

- Suppose $S'(q_i) > \theta_i$ for any $q_i \geq 0$. Then there is no solution for the maximization problem.
- Suppose $S'(q_i) < \theta_i$ for any $q_i \geq 0$. Then the only solution is the boundary solution: $q_i^* = 0$.

Hereafter, we assume that an interior solution q_i^* exists (and hence it is unique) for both types.

Interpretation: The efficient production levels q_i^* are obtained by equating the principal's marginal value and the agent's marginal cost:

$$S'(q_i^*) = \theta_i.$$

16 The complete information efficient production levels q_L^* and q_H^* should be both carried out if their social values are non-negative,

$$W_L^* = S(q_L^*) - \theta_L q_L^* \geq 0 \text{ and } W_H^* = S(q_H^*) - \theta_H q_H^* \geq 0.$$

Note that the social value W_L^* is always greater than W_H^*:

$$W_L^* = \underbrace{\overbrace{S(q_L^*) - \theta_L q_L^* \geq S(q_H^*) - \theta_L q_H^*}^{q_L^* \text{ maximizes } S(q_L) - \theta_L q_L} \geq S(q_H^*) - \theta_H q_H^*}_{\theta_L < \theta_H} = W_H^*.$$

For trade to be always carried out, it is thus enough that production be socially valuable for the least efficient type, i.e., the following condition must be satisfied

$$W_H^* = S(q_H^*) - \theta_H q_H^* \geq 0.$$

This condition will be shown to hold automatically later.

3.1 Implementation—payment

17 We have determined the efficient production levels q_i^*.

18 Since the principal cannot force the agent, he must convince the agent to accept the task.

For a successful delegation of the production, the principal must offer the agent a utility level that is at least as high as the utility level that the agent obtains from outside opportunity. We refer to these constraints as the agent's individual rationality constraints or participation constraints.

Here we normalize to zero the agent's outside opportunity utility level (i.e., his status quo utility level), these conditions are written as

$$t_L - \theta_L q_L \geq 0 \text{ and } t_H - \theta_H q_H \geq 0.$$

19 The sequence of play is as follows:

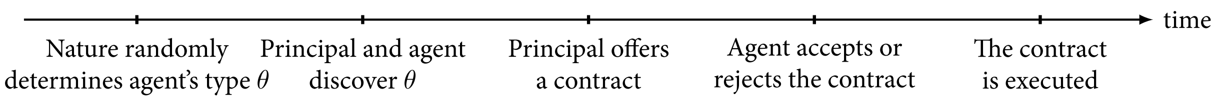

Figure 1: Timing

20 Obviously, the first-best contract menu $\{(q_i^*, t_i^*)\}_{i=H,L}$ satisfies these conditions, if we let $t_i^* = \theta_i q_i^*$.

21 To implement the first-best production levels q_i^*, the principal can make the following take-it-or-leave-it offers to the agent: If $\theta = \theta_i$, the principal offers the transfer t_i^* for the production level q_i^* with $t_i^* = \theta_i q_i^*$.

Whatever his type, agent accepts the offer and makes zero utility. The complete information optimal contracts are thus (q_L^*, t_L^*) if $\theta = \theta_L$ and (q_H^*, t_H^*) if $\theta = \theta_H$.

22 Under complete information, delegation is costless for the principal, who achieves the same utility level that he could get if he was carrying out the task himself (with the same cost function as the agent).

23 Alternative interpretation:

The principal try to maximize her utility subject to inducing the agent to accept the proposed contract. Clearly, the agent obtains 0 if he does not take the principal's contract. So the principal will solve the following problem:

$$\begin{aligned} \underset{(q_i,t_i)\in\mathcal{A}}{\text{maximize}} \quad & S(q_i) - t_i \\ \text{subject to} \quad & t_i - c(q_i, \theta_i) \geq 0. \end{aligned}$$

In any solution, the IR constraint must bind; otherwise, the principal could lower the wage offered and still have the agent accept the contract. Thus, the maximization problem becomes:

$$\max_{q_i \geq 0} S(q_i) - \theta_i q_i.$$

Clearly, $S'' < 0$, and hence the objective function is concave. Then the solution must satisfy the first-order condition:

$$S'(q_i^*) \begin{cases} \leq \theta_i, \\ = \theta_i, \quad \text{if } q_i^* > 0. \end{cases}$$

Assume there is an interior solution q_i^*, i.e., $S'(q_i^*) = \theta_i$. Then the payment is due to the binding IR constraint: $t_i^* = \theta_i q_i^*$.

3.2 The first-best contract

24 The complete information optimal contracts are thus (q_L^*, t_L^*) if $\theta = \theta_L$ and (q_H^*, t_H^*) if $\theta = \theta_H$.

25 Every agent (no matter θ_L or θ_H) obtains exactly 0 from principal, just balancing his reservation utility.

26 We denote by V_H^* (resp. V_L^*) the principal's level of utility when he faces the θ_H-(resp. θ_L-) type:

$$V_i^* = S(q_i^*) - \theta_i q_i^* = W_i^*.$$

Interpretation: Because the principal has all the bargaining power (complete information) in designing the contract, we have $V_i^* = W_i^*$ under complete information.

27 Graphic illustration:

(a) Agent's reservation utility is 0, which is equivalent to the contract $O = (0, 0)$.

(b) Principal seeks to find the most profitable point on the isoutility curve with utility 0, i.e., through the point $O = (0, 0)$.

For the point, the strictly concave indifference curve of the principal is tangent to the zero rent isoutility curve of the corresponding type.

(c) For a θ_i agent, principal pays t_i^* such that $t_i^* - c(q_i^*, \theta_i) = 0$.

(d) For a θ_i agent, principal's profit is $V_i^* = S(q_i^*) - c(q_i^*, \theta_i)$.

This profit is exactly equal to the distance from the origin to the intersection point between the indifference curve through (q_i^*, t_i^*) and the vertical axis:

i. Principal's indifference curve is of the form $S(q_i) - t = \text{constant}$.

ii. The constant should be principal's profit, which is V_i^*.

iii. Letting $q_i = 0$ in the indifference curve $S(q_i) - t_i = V_i^*$, we have $-t_i = V_i^*$. It implies that $V_i^* > 0$.

The complete information optimal contract is finally represented in the following figure by the pair of points (A^*, B^*).

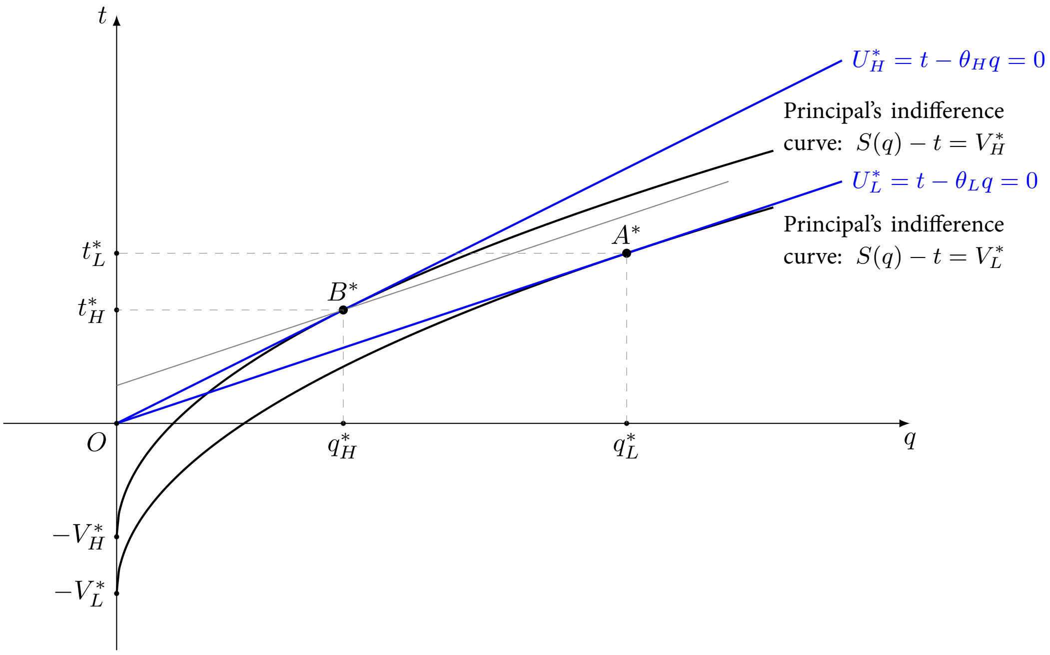

Figure 2: First-best contracts

Suppose instead the reservation utility is $\bar{u} > 0$, which is large enough.

- Then the tangent point and indifference curve will shift up, and hence the profit V_i^* could be negative. In this case, the principal will not provide such a contract—the shutdown occurs.
- Interpretation: If agent's reservation utility is low, principal can attract him to accept some contract; otherwise, agent will not accept any contract that is acceptable for principal.

28 We have $S'(q_i^*) = \theta_i$. Since $S'' < 0$ and $\theta_H > \theta_L$, we have

$$q_L^* > q_H^*,$$

i.e., the optimal production of an efficient agent is greater than that of an inefficient agent.

29 In the figure, the payment t_L^* is greater than t_H^*, but we note that t_L^* can be greater or smaller than t_H^* depending on the curvature of the function S, as it can be easily seen graphically.

Example: $S(q) = -\frac{4}{q+1} + 4, \theta_L = \frac{1}{4}, \theta_H = 1$. Then $(q_L^*, t_L^*) = (3, \frac{3}{4}), (q_H^*, t_H^*) = (1, 1), V_L^* = \frac{24}{7}, V_H^* = 1$.

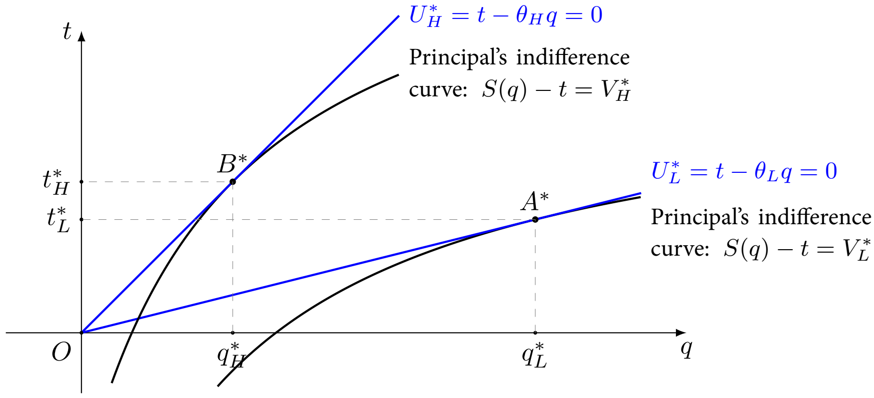

Figure 3: $t_H^* > t_L^*$

30 The principal's utility:

$$V_L^* = W_L^* > W_H^* = V_H^*.$$

From the figure, the indifference curves of the principal correspond to increasing levels of utility when one moves in the southeast direction. Thus, the principal reaches a higher profit when dealing with the efficient type.

4 Incomplete information

31 Suppose that the marginal cost θ is the agent's private information.

We continue to assume that $S'(q_i) = \theta_i$ has a positive solution q_i^*, which implies that $V_i^* = W_i^* > 0$.

32 The sequence of play is as follows:

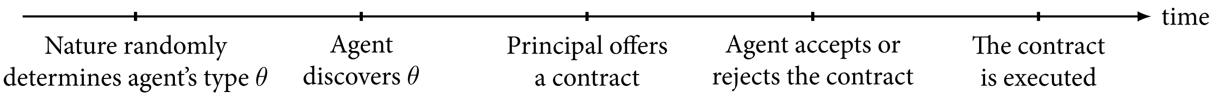

Figure 4: Timing

Note that contracts are offered at the interim stage (事中阶段); there is already asymmetric information between the contracting parties when the principal makes his offer.

33 In the following figure, we draw the indifference curves of a θ_L-agent (heavy curves) and of a θ_H-agent (light curves) in the (q, t) space.

The isoutility curves of both types correspond to increasing levels of utility when one moves in the northwest direction. These indifference curves are straight lines with a slope θ corresponding to the agent's type.

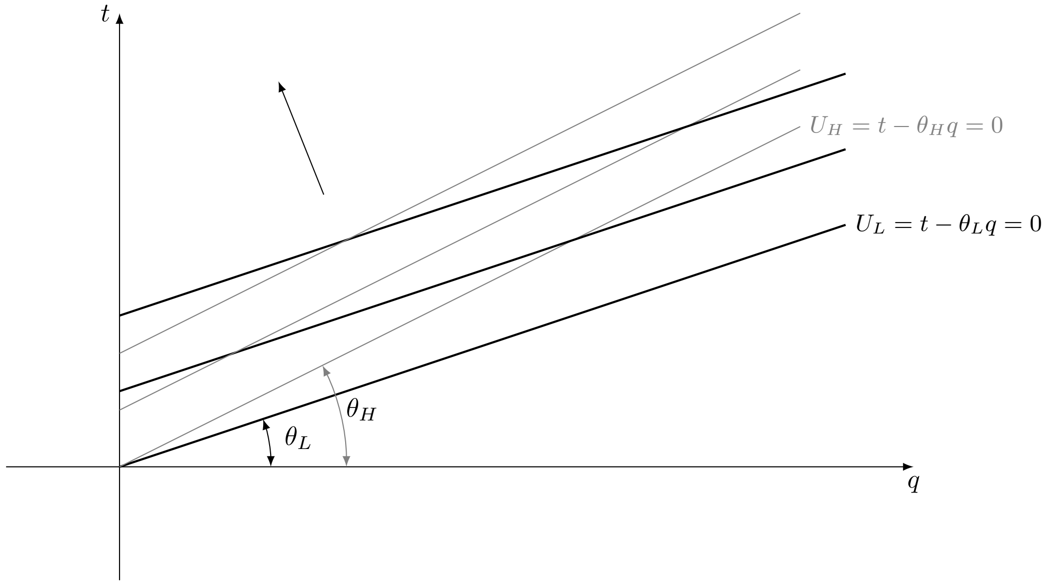

Figure 5: Single-crossing property

Since $\theta_H > \theta_L$, the isoutility curves of the inefficient agent θ_H have a greater slope than those of the efficient agent. Thus, the isoutility curves for different types cross only once. This property is called the single-crossing property (单交叉性质) or Spence-Mirrlees property.

34 Since principal cannot observe agent's type, he cannot offering different contracts for θ_L-agent and θ_H-agent.

In other words, the contract(s) offered to θ_L-agent should coincide with the contract(s) offered to θ_H-agent.

35 Consider the case where the principal offers the menu of contracts $\{(q_L^*, t_L^*), (q_H^*, t_H^*)\}$ hoping that an agent with type θ_L will select (q_L^*, t_L^*) and an agent with type θ_H will select instead (q_H^*, t_H^*).

From Figure 2, we see that B^* is preferred to A^* by both types of agents:

- The θ_L-agent's isoutility curve that passes through B^* corresponds to a positive utility level instead of a zero utility level at A^*.
- The θ_H-agent's isoutility curve that passes through A^* corresponds to a negative utility level, which is less than the zero utility level this type gets by choosing B^*.

Thus, offering the menu (A^*, B^*) fails to have the agents self-selecting properly within this menu. The efficient type mimics the inefficient one and selects also contract B^*. The complete information optimal contracts can no longer be implemented under asymmetric information.

36 A menu of contracts $\{(q_L, t_L), (q_H, t_H)\}$ is incentive compatible when (q_L, t_L) is weakly preferred to (q_H, t_H) by the type-θ_L agent and (q_H, t_H) is weakly preferred to (q_L, t_L) by the type-θ_H agent.

Mathematically,

$$t_L - \theta_L q_L \geq t_H - \theta_L q_H, \tag{IC$_L$}$$

$$t_H - \theta_H q_H \geq t_L - \theta_H q_L. \tag{IC$_H$}$$

37 A menu of contracts $\{(q_L, t_L), (q_H, t_H)\}$ is individually rational if

$$t_L - \theta_L q_L \geq 0, \tag{IR$_L$}$$
$$t_H - \theta_H q_H \geq 0. \tag{IR$_H$}$$

We do not require that (q_L, t_L) is acceptable for θ_H agent and (q_H, t_H) is acceptable for θ_L agent, once we assume IC constraints.

38 Example: Pooling contract.

When the contracts targeted for each type coincide and both types of agent accept this contract, we have a pooling contract.

$$q_L = q_H = q^P \text{ and } t_L = t_H = t^P.$$

- Incentive compatibility is trivially satisfied, but at the cost of an obvious loss of flexibility in allocations that are no longer dependent on the state of nature.
- Only the participation constraints matter now; the hardest participation constraint to satisfy is that of the inefficient agent. This is because Equation (IR$_H$) directly implies Equation (IR$_L$) for a pooling contract, which is efficient agent's participation constraint.

39 Example: Shutdown contract.

When one of the contracts is the null contract $(0, 0)$ and the nonzero contract (q^s, t^s) is only accepted by the efficient type.

- Then, Equation (IC$_L$) and Equation (IR$_L$) both reduce to $t^s - \theta q^s \geq 0$.
- The Equation (IC$_H$) reduces to $0 \geq t^s - \theta_H q^s$. If this inequality is strict, only the efficient type accepts the contract.
- With such a contract, the principal gives up production if the agent is a θ_H-type. We will say that it is a contract with shutdown of the least efficient type.

$\{(q_L^*, t_L^*), (0, 0)\}$ is a shutdown contract, where the contract (q_L^*, t_L^*) is only accepted by the efficient type.

40 If a menu of contracts $\{(q_L, t_L), (q_H, t_H)\}$ is incentive compatible, then

$$\overbrace{\theta_L(q_H - q_L) \geq t_H - t_L}^{\text{By Equation (IC}_L)} \geq \theta_H(q_H - q_L)$$
$$\theta_L(q_H - q_L) \geq \underbrace{t_H - t_L \geq \theta_H(q_H - q_L)}_{\text{By Equation (IC}_H)},$$

and hence

$$q_H - q_L \leq 0. \tag{M}$$

It is called the monotonicity constraint.

Incentive compatibility alone (regardless of the principal's preferences) implies that the production level requested from a θ_H-agent cannot be higher than the one requested from a θ_L-agent.

41 A pair of outputs (q_L, q_H) is said to be implementable if it can be reached by an incentive compatible contract.

Implementability is equivalent to monotonicity constraint. Suppose $q_H - q_L \leq 0$. For IC constraints to satisfy, we should have

$$t_L - \theta_L q_L \geq t_H - \theta_L q_H \text{ and } t_H - \theta_H q_H \geq t_L - \theta_H q_L.$$

Then we have

$$\theta_L(q_H - q_L) \leq t_H - t_L \leq \theta_H(q_H - q_L).$$

It is enough to take transfers (t_L, t_H) such that the above equation holds.

4.1 Principal's problem

42 Recall that under complete information, the principal is able to maintain all types of agents at their zero status quo utility level. Their respective utility levels U_L^* and U_H^* at the first-best contracts satisfy

$$U_L^* = t_L^* - \theta_L q_L^* = 0 \text{ and } U_H^* = t_H^* - \theta_H q_H^* = 0.$$

Generally this will not be possible anymore under incomplete information, at least when the principal wants both types of agents to be active.

43 Take any IC and IR menu of contracts $\{(q_L, t_L), (q_H, t_H)\}$. Let

$$U_L = t_L - \theta_L q_L \geq 0 \text{ and } U_H = t_H - \theta_H q_H \geq 0$$

denote the respective information rent (the utility in excess of the reservation utility) of each type.

(a) Consider the utility level that a θ_L-agent would get by mimicking a θ_H-agent. By doing so, he would get

$$t_H - \theta_L q_H = t_H - \theta_H q_H + \theta_H q_H - \theta_L q_H = U_H + \Delta\theta q_H.$$

(b) As such, even if the θ_H-agent utility level is reduced to its lowest utility level fixed at zero; that is, $U_H = t_H - \theta_H q_H = 0$, the θ_L-agent benefits from an information rent $\Delta\theta q_H$ coming from his ability to possibly mimic the less efficient type.

(c) So, as long as the principal insists on a positive output for the inefficient type, $q_H > 0$, the principal must give up a positive rent to a θ_L-agent. This information rent is generated by the informational advantage of the agent over the principal.

(How about θ_H-agent mimicking θ_L-agent?)

The principal's problem is to determine the smartest way to give up the rent provided by any given IC and IR menu of contracts.

44 According to our timing of the contractual game, the principal must offer a menu of contracts before knowing which type of agent he is facing.

Therefore, he will compute the benefit of any menu of contracts $\{(q_L, t_L), (q_H, t_H)\}$ in expected terms.

The principal's problem is to solve

$$\begin{aligned} \underset{(q_L,t_L),(q_H,t_H)}{\text{maximize}} \quad & \lambda\big(S(q_L) - t_L\big) + (1-\lambda)\big(S(q_H) - t_H\big) \\ \text{subject to} \quad & \text{Constraints } (\text{IC}_L)\text{–}(\text{IR}_H). \end{aligned}$$

45 Since $U_L = t_L - \theta_L q_L$ and $U_H = t_H - \theta_H q_H$, we can replace transfers in the principal's objective function as functions of information rents and outputs so that the new optimization variables are now $\{(q_L, U_L), (q_H, U_H)\}$. The focus on outputs allows us to analyze its impact on allocative efficiency and the overall gains from trade.

46 With this change of variables, the principal's objective function can then be rewritten as

$$\underbrace{\lambda\big(S(q_L)-\theta_L q_L\big)+(1-\lambda)\big(S(q_H)-\theta_H q_H\big)}_{\text{Expected social value/allocative efficiency}}-\underbrace{\big(\lambda U_L+(1-\lambda)U_H\big)}_{\text{Expected information rent}}.$$

This new expression clearly shows that the principal wishes to maximize the expected social value of trade minus the expected rent of the agent.

There is a tradeoff between distortions away from efficiency in order to decrease the agent's information rent.

47 The incentive constraints and individual rationality constraints are rewritten as

$$U_L \geq U_H + \Delta\theta q_H, \tag{IC$'_L$}$$
$$U_H \geq U_L - \Delta\theta q_L, \tag{IC$'_H$}$$
$$U_L \geq 0, \tag{IR$'_L$}$$
$$U_H \geq 0. \tag{IR$'_H$}$$

4.2 Solving the principal's problem

48 The major technical difficulty of principal's problem, and more generally of incentive theory, is to determine which of the many constraints imposed by incentive compatibility and participation are the relevant ones, i.e., the binding ones at the optimum of the principal's problem.

49 Let us first consider contracts without shutdown, i.e., such that $q_H > 0$. The condition will be determined later.

50 Step 1: The constraint (IR$'_L$) is always strictly satisfied due to constraints (IC$'_L$) and (IR$'_H$).

The ability of the θ_L-agent to mimic the θ_H-agent implies that the θ_L-agent's participation constraint is always strictly satisfied.

If a menu of contracts enables an inefficient agent to reach his status quo utility level, it will also be the case for an efficient agent who can produce at a lower cost.

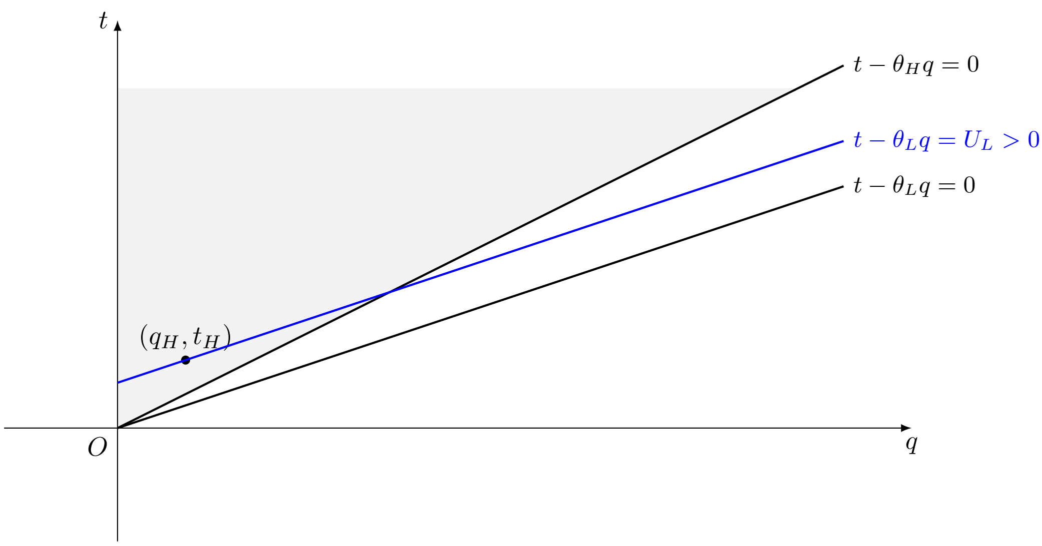

Figure 6: IR for θ_L

Graphic illustration:

(a) By Equation (IR'_H), (q_H, t_H) must lie in the shaded region.

(b) By Equation (IC'_L), (q_L, t_L) must lie on or above the θ_L-indifference curve through (q_H, t_H).

(c) This implies that θ_L-agent's utility is at least 0.

51 Step 2: The constraint (IR'_H) is binding at the optimum, i.e., $U_H = 0$.

Suppose that $U_H = \varepsilon > 0$ at the optimum. Then the principal can decease U_H by ε and consequently also U_L by ε and gain ε. Contradiction.

52 Step 3: The constraint (IC'_L) is binding at the optimum, i.e., $U_L = \Delta\theta q_H$.

Suppose that $U_L - \Delta\theta q_H = \varepsilon > 0$ at the optimum. Then the principal can decrease U_L by ε and gain $\lambda\varepsilon$. Contradiction.

53 IC for θ_H-agent seems irrelevant because the difficulty comes from a θ_L-agent willing to claim that he is inefficient rather than the reverse.

We ignore this condition for now and then get a solution. We can verify whether the solution satisfies this condition.

Remark: When (IC'_L) is binding, (IC'_H) is equivalent to (M):

$$t_H^{\mathrm{SB}} - \theta_H q_H^{\mathrm{SB}} - t_L^{\mathrm{SB}} + \theta_H q_L^{\mathrm{SB}} = \Delta\theta(q_L^{\mathrm{SB}} - q_H^{\mathrm{SB}}).$$

54 Step 4: By Steps 2 and 3, we obtain a reduced program

$$\underset{q_L, q_H}{\text{maximize}} \quad \lambda\big(S(q_L) - \theta_L q_L\big) + (1-\lambda)\big(S(q_H) - \theta_H q_H\big) - \lambda\Delta\theta q_H.$$

Compared with the full information setting, asymmetric information alters the principal's optimization simply by the subtraction of the expected rent that has to be given up to the efficient type.

The inefficient type gets no rent, but the efficient type θ_L gets the information rent that he could obtain by mimicking the inefficient type θ_H. This rent depends only on the level of production requested from this inefficient type.

55 Step 5: The first order condition on q_L implies

$$S'(q_L^{\mathrm{SB}}) = \theta_L, \text{ that is, } q_L^{\mathrm{SB}} = q_L^*.$$

Hence, there is no distortion away from the first-best for the efficient type's output. Here, the superscript SB means the second-best.

The first order condition on q_H implies

$$(1-\lambda)\big(S'(q_H^{\mathrm{SB}}) - \theta_H\big) \begin{cases} \le \lambda\Delta\theta, & \\ = \lambda\Delta\theta, & \text{if } q_H^{\mathrm{SB}} > 0. \end{cases}$$

Since we have assumed $S'(q_H^*) - \theta_H = 0$, the equation above does have a solution. We first assume there is an interior solution q_H^{SB}. This equation expresses the important trade-off between efficiency and rent extraction which arises under asymmetric information. The expected marginal efficiency gain (resp. cost) and the expected marginal cost (resp. gain) of the rent brought about by an infinitesimal increase (resp. decrease) of the inefficient type's output are equated.

At the second-best optimum, the principal is neither willing to increase nor to decrease the inefficient agent's output.

56 Step 6: We have the following inequality

$$q_L^{\text{SB}} = q_L^* > \underbrace{q_H^* > q_H^{\text{SB}}}_{S''<0},$$

and hence

$$U_H^{\text{SB}} = 0 > \Delta\theta q_H^{\text{SB}} - \Delta\theta q_L^{\text{SB}} = U_L^{\text{SB}} - \Delta\theta q_L^{\text{SB}}.$$

That is, the constraint (IC'_H) is strictly satisfied.

向上的激励相容条件（upward incentive compatibility，低能力模仿高能力）不是问题。另一方面，向下的激励相容条件（downward incentive compatibility，高能力模仿低能力）更为关键，需要谨慎处理。

57 We have assumed $q_H^{\text{SB}} > 0$. That is, the equation $(1-\lambda)\big(S'(q_H) - \theta_H\big) = \lambda\Delta\theta$ has to admit an interior solution (which is unique).

58 Theorem (Optimal contract without shutdown): Under asymmetric information, the optimal menu of contracts entails:

- No output distortion for the efficient type with respect to the first-best, $q_L^{\text{SB}} = q_L^*$.
- A downward output distortion for the inefficient type, $q_H^{\text{SB}} < q_H^*$ with

$$S'(q_H^{\text{SB}}) = \theta_H + \frac{\lambda}{1-\lambda}\Delta\theta.$$

  Here we have assumed that the equation above has positive solution. Otherwise q_H^{SB} should be set at zero, and we are in the special case of a contract with shutdown, which will be discussed later.

  Note that

$$q_L^{\text{SB}} = q_L^* > q_H^* > q_H^{\text{SB}}.$$

- Only the efficient type gets a positive information rent given by

$$U_L^{\text{SB}} = \Delta\theta q_H^{\text{SB}}.$$

- The second-best transfers are respectively given by

$$t_L^{\text{SB}} = \theta_L q_L^* + \Delta\theta q_H^{\text{SB}} > \theta_L q_L^* = t_L^* \text{ and } t_H^{\text{SB}} = \theta_H q_H^{\text{SB}} < \theta_H q_H^* = t_H^*.$$

  Note that

$$t_L^{\text{SB}} = \theta_L q_L^* + \Delta\theta q_H^{\text{SB}} = \theta_L q_L^* + \theta_H q_H^{\text{SB}} - \theta_L q_H^{\text{SB}} = t_H^{\text{SB}} + \theta_L(q_L^* - q_H^{\text{SB}}) > t_H^{\text{SB}}.$$

59 “顶部无扭曲”与“单向扭曲/向下扭曲”是两条最基本的规律。

- 对于高能力，不存在产出水平的扭曲（其产出水平与完全信息最优时的产出水平一致），但代价是需要给其支付信息租金。
- 对于低能力，其付出的产出水平低于完全信息最优时的产出水平，但没有信息租金。

4.3 Graphic illustration

60 $q_H^{\text{SB}} \leq q_H^*$.

(a) Suppose $q_H^{\text{SB}} > q_H^*$.

(b) Since θ_H-IR binds, $(q_H^{\text{SB}}, t_H^{\text{SB}})$ lies on the indifference curve through $(0, 0)$.

(c) To make θ_H-IC and θ_L-IC hold, $(q_L^{\text{SB}}, t_L^{\text{SB}})$ lies in the shaded region.

(d) Principal can raise her profit by moving $(q_H^{\text{SB}}, t_H^{\text{SB}})$ to (q_H^*, t_H^*): θ_H-IC and θ_L-IC still hold.

(e) Thus, $q_H^{\text{SB}} > q_H^*$ cannot be optimal.

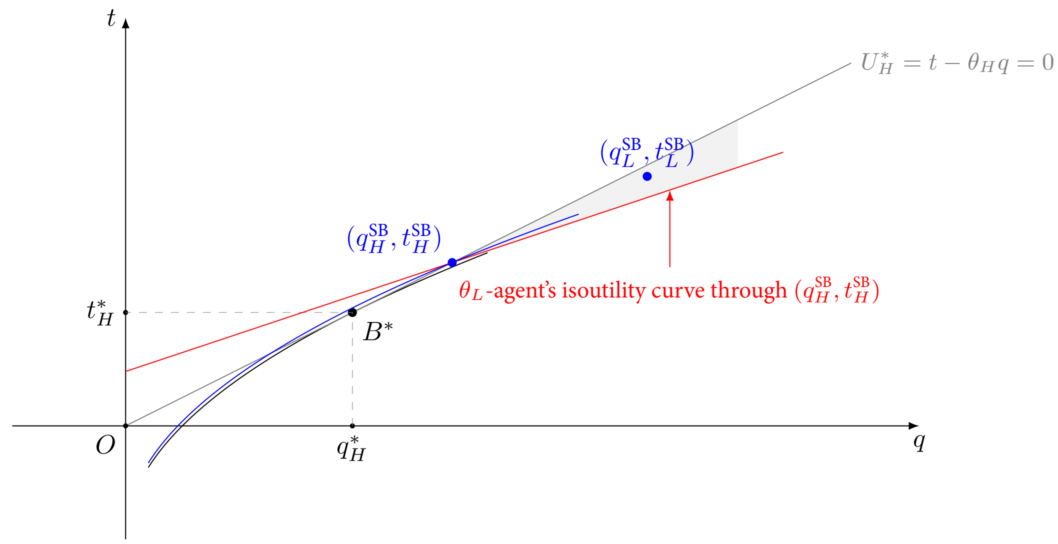

Figure 7: $q_H^{\text{SB}} \leq q_H^*$

61 $q_L^{\text{SB}} = q_L^*$.

(a) Suppose that $q_H^{\text{SB}} \leq q_H^*$.

(b) To make θ_H-IC and θ_L-IC hold, $(q_L^{\text{SB}}, t_L^{\text{SB}})$ lies in the shade region.

(c) Principal's problem is to find the allocation of $(q_L^{\text{SB}}, t_L^{\text{SB}})$ that maximizes her profit.

(d) The optimal solution occurs at a point of tangency between the indifference curve of θ_H-agent through $(q_L^{\text{SB}}, t_L^{\text{SB}})$ and an isoprofit curve for principal.

(e) All points of tangency between indifference curves of θ_H-agent and isoprofit curves of principal occur at q_L^*.

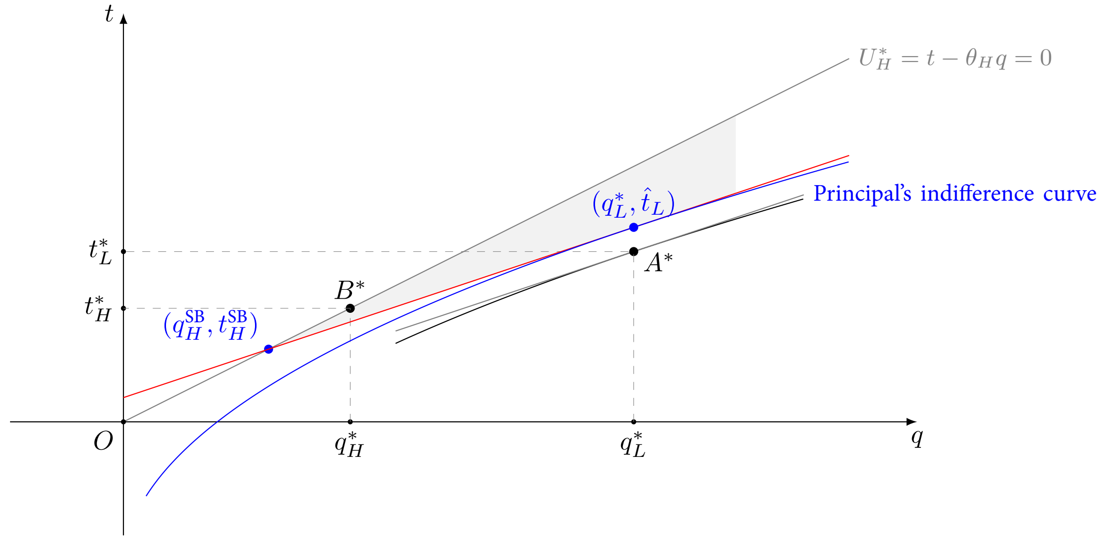

Figure 8: $q_L^{\rm SB} = q_L^*$

62 Starting from the complete information optimal contract (A^*, B^*) that is not incentive compatible, we can construct an incentive compatible contract (C, B^*) with the same production levels by giving a higher transfer to the agent producing q_L^* (Figure 9).

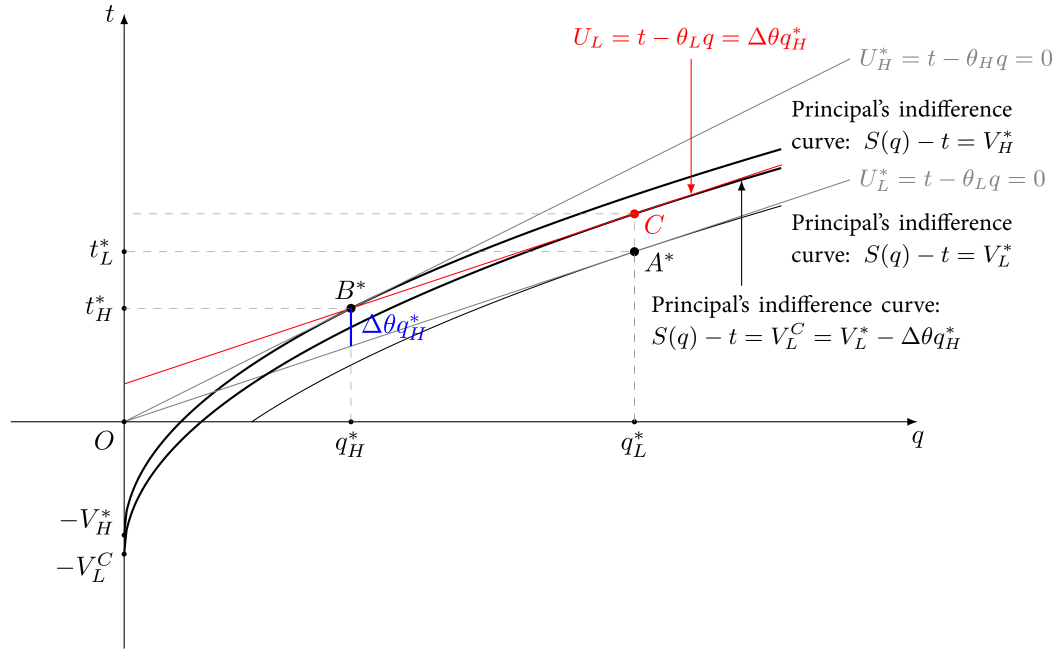

Figure 9: Rent needed to implement the first-best outputs

(a) The contract C is on the θ_L-agent's indifference curve passing through B^*.

(b) Hence, the θ_L-agent is now indifferent between B^* and C. (B^*, C) becomes an incentive-compatible menu of contracts.

(c) The rent that is given up to the θ_L-agent is now $\Delta\theta q_H^*$.

63 Rather than insisting on the first-best production level q_H^* for an inefficient type, the principal can slightly decrease

q_H by a small amount.

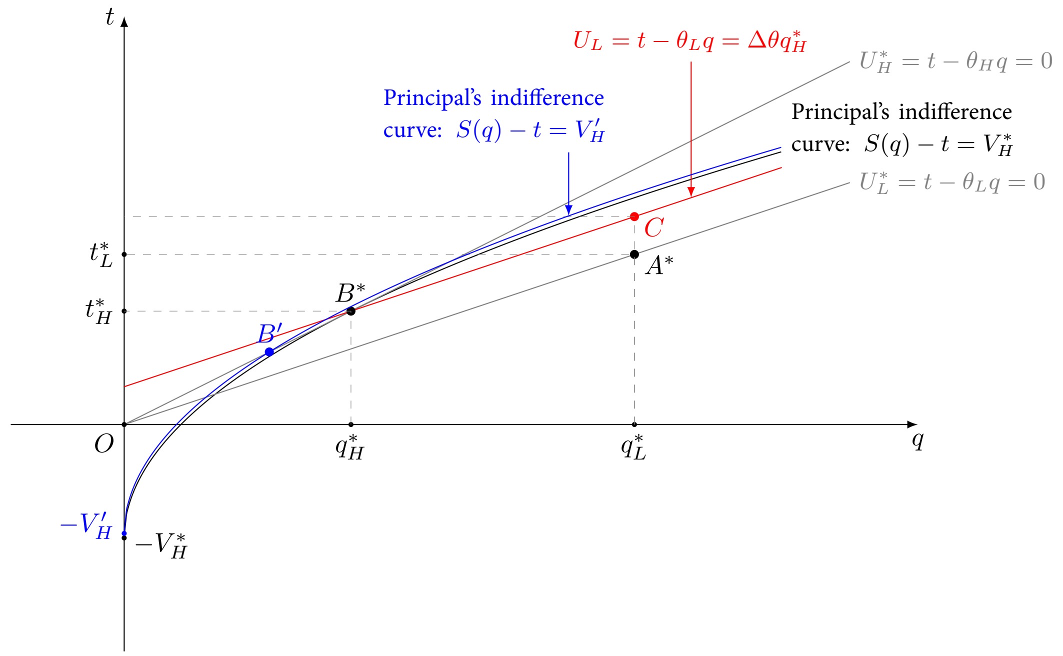

Figure 10: Profit loss in θ_H

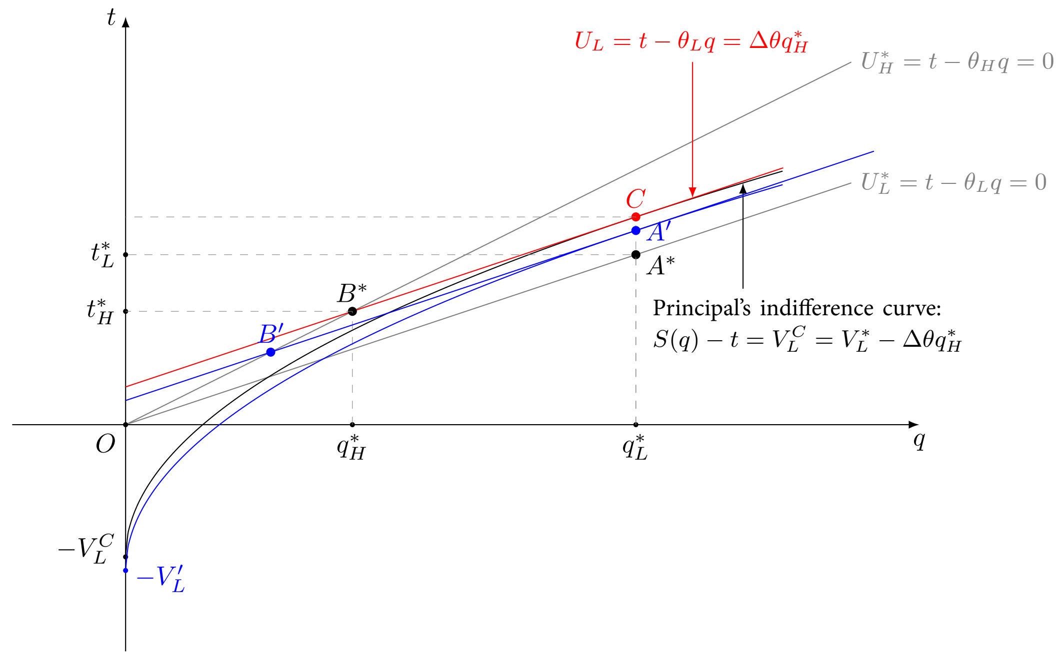

Figure 11: Profit gain in θ_L

(a) Principal firstly moves $B^* = (q_H^*, t_H^*)$ downwards along θ_H-agent's indifference curve through $(0, 0)$, for example, to B'.

(b) This change lowers the profit that principal earns from θ_H agents: from V_H^* to $V_H' < V_H^*$. (Figure 10)

(c) On the other hand, it relaxes θ_L-agent's IC constraint.

(d) Principal then moves C to A'.

(e) This change increases the profit that principal earns from θ_L agents: from V_L^C to $V_L' > V_L^C$. (Figure 11)

(f) Comparison: By slightly decreasing q_H by an amount dq:

- By doing so, expected efficiency is just diminished by a second-order term $\frac{1}{2}|S''(q_H^*)|(\mathrm{d}q)^2$ since q_H^* is the first-best output that maximizes efficiency when the agent is inefficient:

$$\left[S(q_H^* - \mathrm{d}q) - \theta_H(q_H^* - \mathrm{d}q)\right] - \left[S(q_H^*) - \theta_H q_H^*\right] = \frac{1}{2}S''(q_H^*)(\mathrm{d}q)^2 + o\big((\mathrm{d}q)^3\big).$$

- Instead, the information rent left to the efficient type diminishes to the first-order term $\Delta\theta\,\mathrm{d}q$:

$$\left[\Delta\theta(q_H^* - \mathrm{d}q)\right] - \Delta\theta q_H^* = -\Delta\theta\,\mathrm{d}q.$$

(g) Of course, the principal stops reducing the inefficient type's output when a further decrease would have a greater efficiency cost than the gain in reducing the information rent it would bring about. The optimal trade-off finally occurs at $(A^{\mathrm{SB}}, B^{\mathrm{SB}})$ as shown in Figure 12.

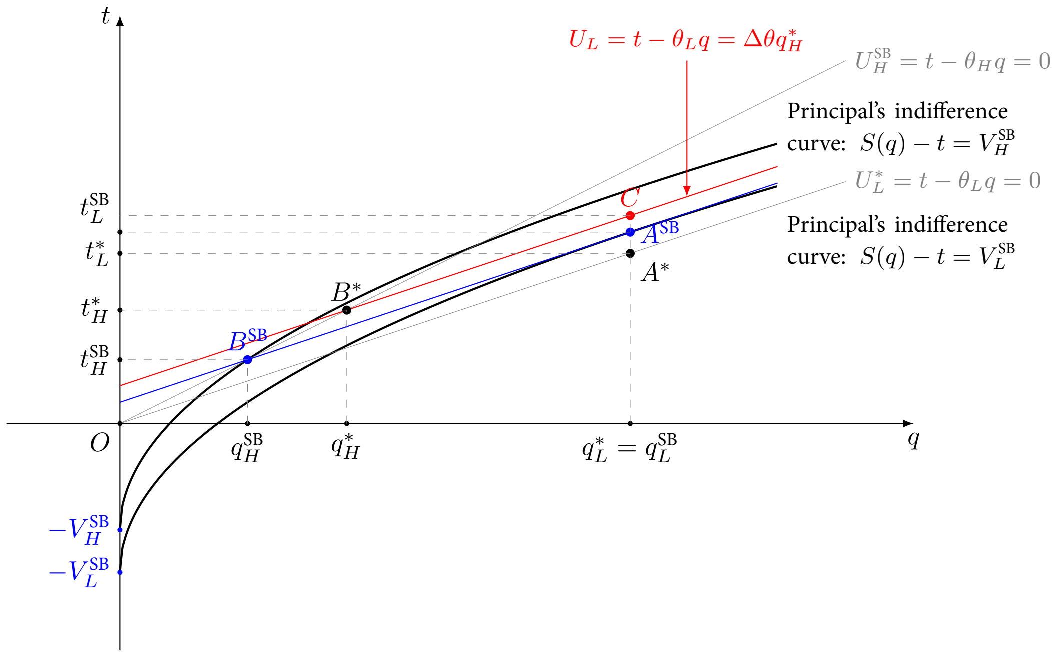

Figure 12: Second-best contracts

64 配置效率与信息租金之间的权衡：

- 为了让 θ_L 选择为其设计的产出水平，需要给他一定好处的信息租金；该信息租金取决于 θ_H 的产出水平，以及 θ_L 和 θ_H。
- 之所以降低 θ_H 的产出水平，是为了尽可能减少支付给 θ_L 的信息租金。
- Principal 扭曲的 θ_H 的产出水平，依赖于两种 agent 之间的差异。
  - 当 $\theta_H - \theta_L \to 0$ 时，θ_L 的信息租金趋于零，此时 θ_H 会趋于有效的产出水平 q_H^*。
  - 而当 $\theta_H - \theta_L \to \infty$ 时，θ_L 的信息租金趋于无穷大，此时 principal 会采取将 θ_H 停工的排斥性合约，以避免支付高额的信息租金。

4.4 Optimal contract with shutdown

65 Consider the first order condition of θ_H-agent:

$$(1-\lambda)\big(S'(q_H^{\text{SB}})-\theta_H\big)\begin{cases}\le \lambda\Delta\theta,\\ = \lambda\Delta\theta, \quad \text{if } q_H^{\text{SB}}>0.\end{cases}$$

We assumed $S'(q_H^{\text{SB}})=\theta_H+\frac{\lambda}{1-\lambda}\Delta\theta$ has a positive solution.

66 Theorem (Optimal contract with shutdown).

(a) If the equation $S'(q_H^{\text{SB}})=\theta_H+\frac{\lambda}{1-\lambda}\Delta\theta$ has no positive solution, q_H^{SB} should be set at zero.

(b) Then B^{SB} coincides with O and A^{SB} with A^* in Figure 12.

(c) No rent is given up to the θ_L-agent by the unique non-null contract (q_L^*, t_L^*) offered and selected only by agent θ_L.

(d) The shutdown of the agent occurs when $\theta=\theta_L$.

With such a contract, a significant inefficiency emerges because the inefficient type does not produce. The benefit of such a contract is that no rent is given up to the efficient type.

67 直觉：

- 如果 θ_L 的比例很大（λ 接近于 1），导致一阶条件没有正数解：若给 θ_H 提供非零合约，或者说提高 θ_H 的配置效率，则甄别中需要支付给 θ_L 过多的信息租金，对于 principal 并不划算。
- 如果两种 agent 的差异较大（$\theta_H-\theta_L$ 很大），导致一阶条件没有正数解：若给 θ_H 提供非零合约，则甄别中需要支付给 θ_L 过多的信息租金，principal 也会选择不给 θ_H 提供合约。

68 Numerical example: $S(q)=\log(q+1), \theta_H=\frac{1}{2}, \theta_L=\frac{1}{3}, \lambda=\frac{6}{7}$.

69 More generally, such a shutdown contract is optimal when

$$\lambda\big(S(q_L^*)-\theta_L q_L^*\big)\ge\lambda\big(S(q_L^{\text{SB}})-\theta_L q_L^{\text{SB}}-\Delta\theta q_H^{\text{SB}}\big)+(1-\lambda)\big(S(q_H^{\text{SB}})-\theta_H q_H^{\text{SB}}\big)$$

or, noting that $q_L^*=q_L^{\text{SB}}$, when

$$\lambda\Delta\theta q_H^{\text{SB}}\ge(1-\lambda)\big(S(q_H^{\text{SB}})-\theta_H q_H^{\text{SB}}\big).$$

- The left-hand side represents the expected cost of the efficient type's rent due to the presence of the inefficient one when the latter produces a positive amount q_H^{SB}.
- The right-hand side represents the expected benefit from transacting with the inefficient type at the second-best level of output.
- Thus, shutdown for the inefficient type is optimal when this expected benefit is lower than the expected cost.

70 When Inada condition $S'(0)=+\infty$ is satisfied and $\lim_{q\to 0}S'(q)q=0$, the shutdown is never desirable.

(1) q_H^{SB} defined by $S'(q_H^{\text{SB}})=\theta_H+\frac{\lambda}{1-\lambda}\Delta\theta$ is necessarily strictly positive since $S'(0)=+\infty$.

(2)

$$S(q_H^{\text{SB}})-\big(\theta_H+\frac{\lambda}{1-\lambda}\Delta\theta\big)q_H^{\text{SB}}=S(q_H^{\text{SB}})-S'(q_H^{\text{SB}})q_H^{\text{SB}}$$

is strictly positive since $S(q) - S'(q)q$ is strictly increasing with q and is equal to zero for $q = 0$. Hence,

$$\lambda \Delta\theta q_H^{\mathrm{SB}} < (1-\lambda)\big(S(q_H^{\mathrm{SB}}) - \theta_H q_H^{\mathrm{SB}}\big)$$

and the shutdown of the least efficient type does not occur.

5 General utility function for the agent

71 Consider a general cost function $C(q,\theta)$ with the assumption

$$C(0,\theta) = 0,\ C_q > 0,\ C_\theta > 0,\ C_{qq} > 0,\ C_{qq\theta} > 0.$$

72 The generalization of the Spence-Mirrlees property used so far is now

$$C_{q\theta} > 0.$$

This condition still ensures that the different types of the agent have indifference curves which cross each other at most once.

(a) A typical indifference curve of θ-agent is $t - C(q,\theta) =$ constant, i.e., $t = C(q,\theta) +$ constant. Then, at any (q,t), the marginal rate of substitution between transfers and outputs is

$$\frac{\mathrm{d}t}{\mathrm{d}q} = C_q(q,\theta),$$

which describes the slope of the indifference curve.

(b) The slope $C_q(q,\theta)$ is increasing in θ since $C_{q\theta}(q,\theta) > 0$. Thus, at a given point $(\hat{q},\hat{t})$, for two indifference curves passing it,

$$\begin{aligned}\text{Slope of } \theta_L\text{-indifference curve} = \frac{\mathrm{d}t(q,\theta_L)}{\mathrm{d}q}\Big|_{(\hat{q},\hat{t})} &= C_q(\hat{q},\theta_L) \\ < C_q(\hat{q},\theta_H) = \frac{\mathrm{d}t(q,\theta_H)}{\mathrm{d}q}\Big|_{(\hat{q},\hat{t})} &= \text{Slope of } \theta_H\text{-indifference curve.}\end{aligned}$$

(c) The increasing rate of slope $C_{qq}(q,\theta)$ is increasing in θ since $C_{qq\theta} > 0$.

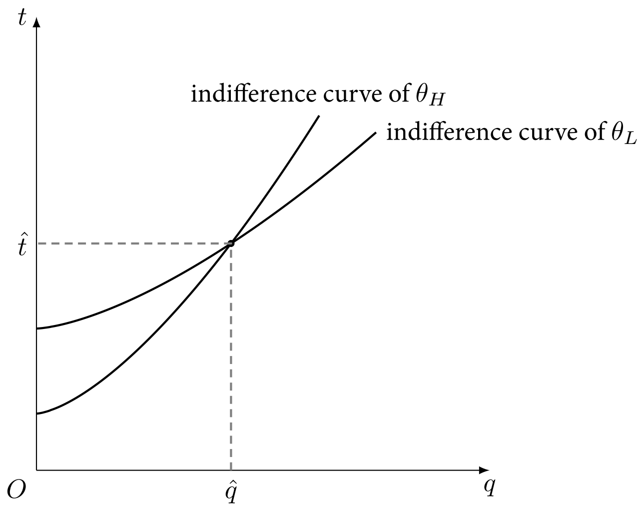

Figure 13: Spence-Mirrlees property

73 It is obviously satisfied in the linear case $C(q,\theta) = \theta q$ that was analyzed before.

Economically, this Spence-Mirrlees property is quite clear; it simply says that a more efficient type is also more efficient at the margin.

74 Incentive compatibility constraints are

$$\begin{aligned} t_L - C(q_L, \theta_L) &\geq t_H - C(q_H, \theta_L), \\ t_H - C(q_H, \theta_H) &\geq t_L - C(q_L, \theta_H). \end{aligned}$$

Individual rationality constraints are

$$t_L - C(q_L, \theta_L) \geq 0 \text{ and } t_H - C(q_H, \theta_H) \geq 0.$$

75 IC constraints imply monotonicity constraint:

$$\int_{q_H}^{q_L} C_q(q, \theta_H)\,\mathrm{d}q = \overbrace{C(q_L, \theta_H) - C(q_H, \theta_H) \geq t_L - t_H}^{\text{By } \theta_H\text{-IC}} \hspace{-5.5em} \underbrace{ \geq C(q_L, \theta_L) - C(q_H, \theta_L)}_{\text{By } \theta_L\text{-IC}} = \int_{q_H}^{q_L} C_q(q, \theta_L)\,\mathrm{d}e,$$

and hence $q_L \geq q_H$.

76 Let $U_L = t_L - C(q_L, \theta_L)$ and $U_H = t_H - C(q_H, \theta_H)$ denote information rents. Then we can rewrite the constraints as:

$$\begin{aligned} U_L &\geq U_H + \Phi(q_H), \\ U_H &\geq U_L - \Phi(q_L), \\ U_L &\geq 0, \\ U_H &\geq 0, \end{aligned}$$

where $\Phi(q) = C(q, \theta_H) - C(q, \theta_L)$. Then $\Phi'(q) = C_q(q, \theta_H) - C_q(q, \theta_L) > 0$ and $\Phi''(q) = C_{qq}(q, \theta_H) - C_{qq}(q, \theta_L) > 0$.

77 Following the same steps as before, the incentive constraint of an efficient type and the participation constraint for the inefficient type in are the two relevant constraints for optimization.

78 These constraints are both binding at the second-best optimum, and so we have

$$U_L = U_H + \Phi(q_H) \text{ and } U_H = 0.$$

It leads to the following expression of the efficient type's rent

$$U_L = \Phi(q_H).$$

Since $\Phi' > 0$, reducing the inefficient agent's output also reduces, as before, the efficient agent's information rent.

79 Also, using the information rents and binding constraints, we can transform the principal's objective function from

$$\lambda\left[S(q_L) - C(q_L, \theta_L)\right] + (1-\lambda)\left[S(q_H) - C(q_H, \theta_H)\right] - \left[\lambda U_L + (1-\lambda) U_H\right]$$

to

$$\lambda\left[S(q_L) - C(q_L, \theta_L)\right] + (1-\lambda)\left[S(q_H) - C(q_H, \theta_H) - \tfrac{\lambda}{1-\lambda}\Phi(q_H)\right].$$

80 With the assumptions made on C, one can also check that the principal's objective function is strictly concave with respect to outputs.

81 By ignoring θ_H-IC and the first order approach, optimal contract entails:

- No output distortion with respect to the first-best outcome for the efficient type, $q_L^{\text{SB}} = q_L^*$ with

$$S'(q_L^*) = C_q(q_L^*, \theta_L).$$

- A downward output distortion for the inefficient type, $q_H^{\text{SB}} < q_H^*$ with

$$S'(q_H^*) = C_q(q_H^*, \theta_H)$$

  and

$$S'(q_H^{\text{SB}}) = C_q(q_H^{\text{SB}}, \theta_H) + \frac{\lambda}{1-\lambda}\Phi'(q_H^{\text{SB}}).$$

- Only the efficient type gets a positive information rent given by $U_L^{\text{SB}} = \Phi(q_H^{\text{SB}})$.
- The second-best transfers are respectively given by $t_L^{\text{SB}} = C(q_L^*, \theta_L) + \Phi(q_H^{\text{SB}})$ and $t_H^{\text{SB}} = C(q_H^{\text{SB}}, \theta_H)$.

82 The first order conditions characterize the optimal solution if the neglected θ_H-IC is satisfied.

(a) For this to be true, we need to have

$$t_H^{\text{SB}} - C(q_H^{\text{SB}}, \theta_H) \geq t_L^{\text{SB}} - C(q_L^{\text{SB}}, \theta_H) = t_H^{\text{SB}} - C(q_H^{\text{SB}}, \theta_L) + C(q_L^{\text{SB}}, \theta_L) - C(q_L^{\text{SB}}, \theta_H),$$

which amounts to

$$0 \geq \Phi(q_H^{\text{SB}}) - \Phi(q_L^{\text{SB}}).$$

(b) Since $\Phi' > 0$, it is equivalent to $q_H^{\text{SB}} \leq q_L^{\text{SB}}$.

(c) We still have

$$q_L^{\mathrm{SB}} = q_L^* > q_H^* > q_H^{\mathrm{SB}}.$$

- $S'(q_L^*) = C_q(q_L^*, \theta_L) < C_q(q_L^*, \theta_H)$ because $C_{q\theta} > 0$. Hence, using the fact that $S(q) - C(q, \theta_H)$ is concave in q and maximum for q_H^*, we have $q_L^* > q_H^*$.
- $\Phi' > 0$ implies that $S'(q_H^{\mathrm{SB}}) > C_q(q_H^{\mathrm{SB}}, \theta_H)$. Thus, $q_H^{\mathrm{SB}} < q_H^*$.

(d) So the Spence-Mirrlees property guarantees that only the efficient type's incentive constraint has to be taken into account.

Task

- Reading: 2.1–2.6 and 2.10 in [LM] (required), 14.C in [MWG] (required), 2.2–2.3 in [S] (optimal).
- Understanding:
  - 在逆向选择模型中，由于委托人与代理人之间的信息差异，可能对经济体的效率产生影响。
  - 这时，委托人需要设计合约，诱导代理人真实反映其类型，自发选择为其定制的合约。
  - 激励代理人这样做，是有成本的（即信息租金，与产出水平相关），从而无法实现完全信息时的最优结果，只能得到次优结果。
  - 其中的基本问题在于信息租金的抽取和配置的效率之间进行权衡和取舍。