Advanced Microeconomics: Lecture Note 5

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1 Introduction of moral hazard

1 So far, we have been considering principle-agent problems, where the agent's private information was affecting the efficient volume of trade.

- We used the name adverse selection for these problems since the agent's private information was about his own type, which affects his performance in accomplishing the delegated task.
- The principal's objective was to offer an optimal contract to the agent in order to achieve a balance between allocative efficiency and information rents, which arise due to their information gap.

This informational asymmetry, however, can also arise between the principal and the agent in other possible forms.

2 The informational asymmetry can be due to possible actions that the agent takes, and not due to possible types he might have.

- The leading candidates for such actions are effort variables, which positively influence the agent's level of production but also create a disutility for the agent.
- In general, the agent's actions affect his performance (therefore, the volume of trade) and they are typically private information.

  As a result, these actions are neither observable by the principal (who offers the contracts), nor by the court of law (who enforces the contracts).

  As such, these hidden actions cannot be contracted upon because no one can verify their value. In such cases we will say that there is moral hazard.

3 Examples:

- The yield of a field depends on the amount of time that the tenant has spent selecting the best crops, or the quality of their harvesting.
- The probability that a driver has a car crash depends on how safely he drives, which also affects his demand for insurance.
- A regulated firm may have to perform a costly and nonobservable investment to reduce its cost of producing a socially valuable good.
- The manager of a large corporation may divert the firm's resources into perks rather than in hiring new engineers for the firm's research lab since he directly benefits from perks.

4 Key elements in moral hazard:

- Moral hazard (as with adverse selection) would not be an issue if the principal and the agent had the same objective function. In other words, because of the conflict between the principal and the agent over which action should be carried out, we may have agency cost arising under moral hazard.
- If the agent's actions were observable or if these actions were perfect determinants of the production levels, then the moral hazard would not be an issue either.
  - However, the agent's actions are in general nonobservable and the production performance is only a noisy signal of the undertaken action.
  - As such, nonobservability of the agent's action prevents an efficient resolution of this conflict of interest because no enforceable contract can dictate which action the agent should take.

5 Asymmetric information plays a crucial role in the design of the optimal incentive contract under moral hazard.

- Instead of being an exogenous uncertainty for the principal, however, uncertainty is now endogenous.
- Indeed, the probabilities of the different states of nature, and thus the expected volume of trade, now depend explicitly on the agent's effort.

  In other words, the realized production level depends on the agent's nonobservable action and this relation is typically non-deterministic.

6 The uncertainty about the agent's actions is key to understanding the contractual problem under moral hazard.

- If the mapping between effort and performance were completely deterministic, the principal and the court of law would have no difficulty in inferring the agent's effort from the observed output.
- In that case, even if the agent's effort was not observable directly, it could be indirectly contracted upon, since output would itself be observable and verifiable.
- In turn, the nonobservability of the effort would not have put any real constraint on the principal's ability to contract with the agent, and their conflict of interests would be costless to solve.

7 In a moral hazard context, however, the principal can only design a contract based on the agent's observable performance.

- This is because the random output aggregates the agent's effort and the realization of pure luck and therefore, it becomes impossible to directly condition the agent's reward on his action.
- As a result, the nonobservability of the agent's effort affects the cost of implementing a given action.
- And therefore, the principal wants to induce, only at a reasonable cost, a high effort from the agent.

2 The basic set-up

8 A principal (employer) hires an agent (employee) for production. The agent can exert a costly effort $e \in \{0, 1\}$. Exerting effort e implies a cost/disutility for the agent that is equal to $\psi(e)$ with the normalizations $\psi(0) = 0$ and $\psi(1) = \psi > 0$. The agent receives a transfer t from the principal.

The agent's utility is assumed to be

$$u(t) - \psi(e),$$

where u is increasing and concave, and $u(0) = 0$. Denote $h = u^{-1}$, which is increasing and convex. We normalize the agent's reservation utility at zero.

9 Profit is stochastic, and effort affects the production level as follows: the stochastic production level q can only take two values $\{q_L, q_H\}$ with $q_H - q_L = \Delta q > 0$, and the stochastic influence of effort on production is characterized by the probabilities

$$\text{Prob}(q = q_H \mid e = 0) = \lambda_0 \text{ and } \text{Prob}(q = q_H \mid e = 1) = \lambda_1,$$

with $\Delta\lambda = \lambda_1 - \lambda_0 > 0$.

	q_H	q_L
$e = 1$	λ_1	$1 - \lambda_1$
$e = 0$	λ_0	$1 - \lambda_0$

10 Effort improves production in the sense of first-order stochastic dominance.

That is, $\text{Prob}(q \leq q^* \mid e)$ is decreasing with e for any given production level q^*.

$$\text{Prob}(q \leq q_L \mid e = 1) = 1 - \lambda_1 < 1 - \lambda_0 = \text{Prob}(q \leq q_L \mid e = 0),$$
$$\text{Prob}(q \leq q_H \mid e = 1) = 1 = \text{Prob}(q \leq q_H \mid e = 0).$$

11 This property implies that any principal who has a utility function v that is increasing in production level prefers the stochastic distribution $(1 - \lambda_1, \lambda_1)$ over $(1 - \lambda_0, \lambda_0)$.

That is, any such principal prefers production induced by the positive effort level $e = 1$ to that induced by the null effort level $e = 0$.

$$\lambda_1 v(q_H) + (1 - \lambda_1)v(q_L) = \lambda_0 v(q_H) + (1 - \lambda_0)v(q_L) + (\lambda_1 - \lambda_0)[v(q_H) - v(q_L)],$$

which is greater than $\lambda_0 v(q_H) + (1 - \lambda_0)v(q_L)$ if v is increasing.

As such, an increase in effort improves production in a strong sense in this model with two possible levels of performance.

12 In a moral hazard environment, the agent's action is not directly observable by the principal.

Thus, the principal can only offer a contract based on the observable production level, i.e., $t(q)$.

Let t_H (resp. t_L) be the payment received by the agent if the production is q_H (resp. q_L).

13 The risk-neutral principal's expected utility is

$$V_1 = \lambda_1 \left[S(q_H) - t_H\right] + (1 - \lambda_1) \left[S(q_L) - t_L\right] \text{ if the agent makes a positive effort } e = 1,$$

and

$$V_0 = \lambda_0 \left[S(q_H) - t_H\right] + (1 - \lambda_0) \left[S(q_L) - t_L\right] \text{ if the agent makes no effort } e = 0.$$

For notational simplicity, we will denote the principal's benefits in each state of nature by

$$S_H = S(q_H) \text{ and } S_L = S(q_L).$$

14 If the agent makes a positive effort $e = 1$, then his expected utility is

$$\lambda_1 u(t_H) + (1 - \lambda_1)u(t_L) - \psi.$$

If the agent chooses $e = 0$, then his expected utility is

$$\lambda_0 u(t_H) + (1 - \lambda_0) u(t_L).$$

15 The problem of the principal is

- to decide whether to induce the agent to exert effort or not and,
- if he chooses to do so, then to decide which contract should be used.

16 The timing is as follows:

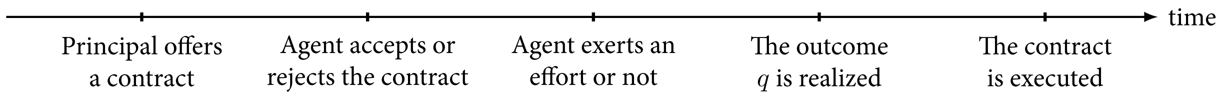


Figure 1: Timing

3 Complete information

17 First assume that the principal and a benevolent court of law can both observe effort.

18 When effort is observable, a contract can be regarded as the form (e, t_H, t_L). That is, the agent is required to exert effort e, and he will receive t_H when the production is high and t_L when the production is low.

Once accepting the contract (e, t_H, t_L), agent needs to exert effort e: If the agent were not exerting effort e, his action could be perfectly detected by the principal, and hence the agent could be heavily punished (for example, $-\infty$).

19 It is convenient to think of this problem in two steps:

- For each $e \in \{0, 1\}$ that might be specified in the contract, what is the best contract (e, t_H, t_L)?
- What is the best choice of e?

20 To induce the agent to exert effort ($e = 1$), the principal's problem is:

$$\begin{aligned} \underset{(t_H, t_L)}{\text{maximize}} \quad & \lambda_1 (S_H - t_H) + (1 - \lambda_1)(S_L - t_L) \\ \text{subject to} \quad & \lambda_1 u(t_H) + (1 - \lambda_1) u(t_L) - \psi \geq 0. \end{aligned}$$

Indeed, only the agent's individual rationality matters for the principal, because the agent can be forced to exert a positive level of effort.

21 Denoting the multiplier of the individual rationality constraint by μ and optimizing with respect to t_H and t_L yields, respectively, the following first-order conditions:

$$\begin{aligned} -\lambda_1 + \mu \lambda_1 u'(t_H^*) &= 0, \\ -(1 - \lambda_1) + \mu (1 - \lambda_1) u'(t_L^*) &= 0, \end{aligned}$$

where t_H^* and t_L^* are the first-best wages.

We immediately derive that $\mu = \frac{1}{u'(t_H^*)} = \frac{1}{u'(t_L^*)} > 0$, and finally that $t^* = t_H^* = t_L^*$.[1]

Because the IR constraint is binding we also obtain the value of this wage, which is just enough to cover the disutility of effort, namely $t^* = u^{-1}(\psi)$.

22 Remark:

- The transfer t^* the agent receives is the same whatever the state of nature—ex post full insurance for agent.
  直觉：已经能观察到行为，因此工资可以仅依赖于行为，非无须再考虑带噪音的结果。
- The transfer $t^* = u^{-1}(\psi)$ is called the first-best cost C^* of implementing the positive effort level.

23 For the principal, inducing effort yields an expected payoff equal to

$$V_1^* = \lambda_1 S_H + (1-\lambda_1) S_L - u^{-1}(\psi).$$

24 Had the principal decided to let the agent exert no effort ($e = 0$), his problem is

$$\begin{aligned} \underset{(t_H, t_L)}{\text{maximize}} \quad & \lambda_0 (S_H - t_H) + (1-\lambda_0)(S_L - t_L) \\ \text{subject to} \quad & \lambda_0 u(t_H) + (1-\lambda_0) u(t_L) \geq 0. \end{aligned}$$

Based on the similar arguments, he would make a zero payment to the agent whatever the realization of profit. In this scenario, the principal would instead obtain a payoff equal to

$$V_0 = \lambda_0 S_H + (1-\lambda_0) S_L.$$

25 Inducing effort is optimal from the principal's point of view when $V_1^* \geq V_0$, i.e.,

$$(\lambda_1 - \lambda_0)(S_H - S_L) \geq u^{-1}(\psi). \tag{1}$$

26 The left-hand side of Equation (1) captures the gain of increasing effort from $e = 0$ to $e = 1$. This gain comes from the fact that the return S_H, which is greater than S_L, arises more often when a positive effort is exerted.

The right-hand side of Equation (1) is instead the first-best cost of inducing the agent's acceptance when he exerts a positive effort.

27 Summary:

- The first-best outcome (effort level) will be achieved:
  - The first-best outcome calls for $e^* = 1$ if and only if $(\lambda_1 - \lambda_0)(S_H - S_L) \geq u^{-1}(\psi)$.
  - When $(\lambda_1 - \lambda_0)(S_H - S_L) \geq u^{-1}(\psi)$, to implement the first-best outcome $e^* = 1$, the principal offers a contract $\left(1, u^{-1}(\psi), u^{-1}(\psi)\right)$ and the agent will accept.
  - When $(\lambda_1 - \lambda_0)(S_H - S_L) < u^{-1}(\psi)$, to implement the first-best outcome $e^* = 0$, the principal offers a contract $(0, 0, 0)$ and the agent will accept.
- The agent gets ex post full insurance.

[1] One can easily derive that $t_H^* = t_L^*$ when u is strictly concave. On the other hand, when u is concave but not strictly concave, one can set $t_H^* = t_L^*$ although there could be multiple optimal solutions.

4 Incomplete information with risk-neutral agent

28 In this situation, a contract is of the form (t_H, t_L). That is, the agent will receive t_H when the production is high and t_L when the production is low, regardless of his effort level.

29 If the agent is risk-neutral, we can assume that (up to an affine transformation) $u(t) = t$ for all t.

30 We consider this problem in two steps:

- If the principal wants the agent to exert positive effort (or zero effort), what is the best contract (t_H, t_L)?
- What is the best choice for the principal, inducing the agent to exert positive effort or zero effort?

31 To induce the agent to exert effort, the principal's problem is

$$\begin{aligned} \underset{(t_H,t_L)}{\text{maximize}} \quad & \lambda_1(S_H - t_H) + (1-\lambda_1)(S_L - t_L) \\ \text{subject to} \quad & \lambda_1 t_H + (1-\lambda_1)t_L - \psi \geq \lambda_0 t_H + (1-\lambda_0)t_L \\ & \lambda_1 t_H + (1-\lambda_1)t_L - \psi \geq 0. \end{aligned}$$

32 The principal's problem is equivalent to

$$\begin{aligned} \underset{(t_H,t_L)}{\text{minimize}} \quad & \lambda_1 t_H + (1-\lambda_1)t_L \\ \text{subject to} \quad & \Delta\lambda t_H \geq \Delta\lambda t_L + \psi \\ & \lambda_1 t_H + (1-\lambda_1)t_L - \psi \geq 0. \end{aligned}$$

33 IR condition should be binding at the optimum; otherwise the principal can decrease t_L without breaking IR condition and IC condition.

34 If the problem has a solution, the expected profit of principal is always

$$V_1^{\text{SB}} = \lambda_1 S_H + (1-\lambda_1)S_L - \psi$$

due to the fact that IR condition is binding.

35 Graphic illustration:

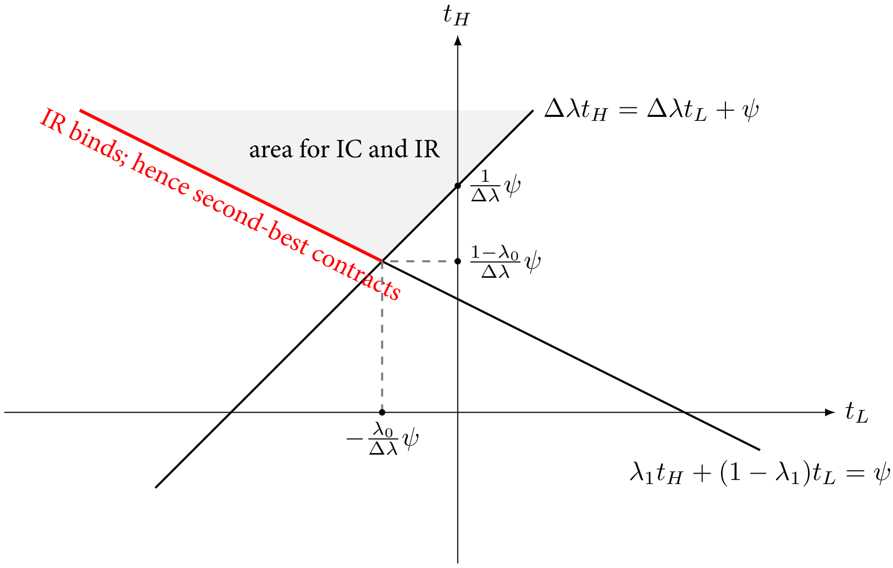


Figure 2: Second-best contracts

36 IC condition is not necessarily binding.

37 To find a solution, we let IC condition be binding. Then we have

$$t_H^{\mathrm{SB}} = \psi + \frac{1-\lambda_1}{\lambda_1-\lambda_0}\psi = \frac{1-\lambda_0}{\Delta\lambda}\psi \text{ and } t_L^{\mathrm{SB}} = \psi - \frac{\lambda_1}{\lambda_1-\lambda_0}\psi = -\frac{\lambda_0}{\Delta\lambda}\psi.$$

- The agent is rewarded if production is high, and his utility is $t_H^{\mathrm{SB}} - \psi = \frac{1-\lambda_1}{\lambda_1-\lambda_0}\psi > 0$.
- The agent is punished if production is low, and his utility is $t_L^{\mathrm{SB}} - \psi = -\frac{\lambda_1}{\lambda_1-\lambda_0}\psi < 0$.

The principal makes an expected payment

$$\lambda_1 t_H^{\mathrm{SB}} + (1-\lambda_1)t_L^{\mathrm{SB}} = \psi,$$

which is equal to the disutility of effort he would incur if he could control the effort level perfectly or if he was carrying the agent's task himself.

38 The transfers $(t_H^{\mathrm{SB}}, t_L^{\mathrm{SB}})$ yield one possible implementation of the first-best outcome, where IC binds.

Let us consider another pair of wages

$$t_H^{\mathrm{SB}'} = \psi + 2\frac{1-\lambda_1}{\lambda_1-\lambda_0}\psi \text{ and } t_L^{\mathrm{SB}'} = \psi - 2\frac{\lambda_1}{\lambda_1-\lambda_0}\psi.$$

Clearly, IR binds and IC is strictly satisfied.

Indeed, there are infinitely many solutions.

39 Graphic illustration:

(1) $t - \psi$ is the agent's utility function when he exerts effort. This curve passes $(\psi, 0)$.

(2) In the complete information case, the agent's utility is zero, and the transfer is always ψ.

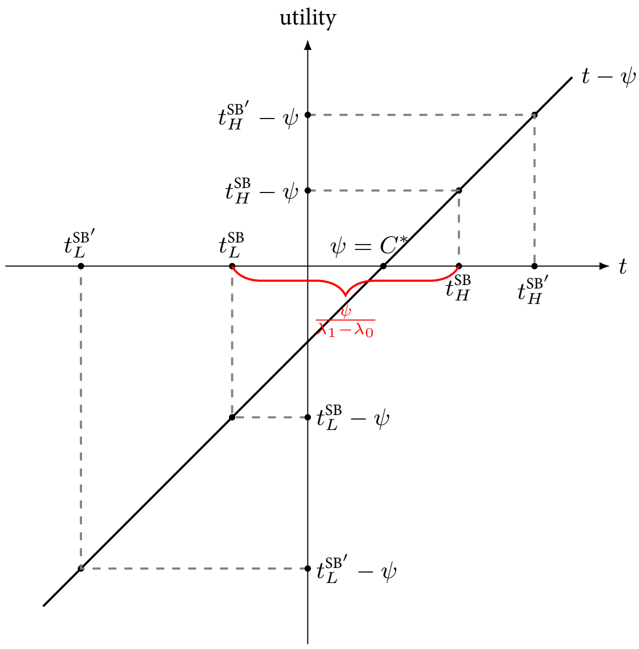


Figure 3: Efficiency vs. Risk

(3) Since IR binds, the contract $(t_H^{\text{SB}}, t_L^{\text{SB}})$ makes the agent's expected utility be zero, shown as in the graph. That is, $\lambda_1 t_H^{\text{SB}} + (1-\lambda_1)t_L^{\text{SB}} - \psi = 0$, or $\frac{|t_H^{\text{SB}}-\psi|}{|t_L^{\text{SB}}-\psi|} = \frac{1-\lambda_1}{\lambda_1}$.
为了能够激励代理人付出努力，需要将 t_H 和 t_L 的差距拉大。从 C^* 开始，沿着曲线 $t-\psi$，向两侧拉开 t_H 和 t_L，同时 u_H 和 u_L 也会同步拉开。拉开过程中保持对应的纵坐标的加权平均值为零（即 $\frac{|t_H-\psi|}{|t_L-\psi|} = \frac{1-\lambda_1}{\lambda_1}$），以保证 IR 条件等号成立。

(4) We obtain $(t_H^{\text{SB}}, t_L^{\text{SB}})$ when IC is binding. The expected transfer should be $\lambda_1 t_H^{\text{SB}} + (1-\lambda_1)t_L^{\text{SB}} = \psi$.
刚好拉开足够的差距时，我们得到了 $(t_H^{\text{SB}}, t_L^{\text{SB}})$；此时 $(t_H^{\text{SB}}, t_H^{\text{SB}}-\psi)$ 和 $(t_L^{\text{SB}}, t_L^{\text{SB}}-\psi)$ 的连线与横轴的交点的横坐标恰好就是 t_H^{SB} 和 t_H^{SB} 的加权平均值（也就是预期支付）。由于 $t-\psi$ 是直线，交点恰好是 $(\psi, 0)$；因此预期支付为 ψ。

(5) To induce the agent to exert effort, the principal needs to set t_H and t_L to satisfy $(\lambda_1-\lambda_0)(t_H-t_L) \geq \psi$. That is, $t_H - t_L$ should be at least $\frac{\psi}{\lambda_1-\lambda_0}$.
为了能够激励代理人付出努力，需要将 t_H 和 t_L 的差距拉大。

(6) IC may not be binding: The principal can increase t_H^{SB} to $t_H^{\text{SB}'}$ and decrease t_L^{SB} to $t_L^{\text{SB}'}$ such that the expected transfer remains the same: $\lambda_1 t_H^{\text{SB}'} + (1-\lambda_1)t_L^{\text{SB}'} = \psi = \lambda_1 t_H^{\text{SB}} + (1-\lambda_1)t_L^{\text{SB}}$.
继续保持比例拉开 t_H 和 t_L，比如得到 $(t_H^{\text{SB}'}, t_L^{\text{SB}'})$；此时 $(t_H^{\text{SB}'}, t_H^{\text{SB}'}-\psi)$ 和 $(t_L^{\text{SB}'}, t_L^{\text{SB}'}-\psi)$ 的连线与横轴的交点依然是 $(\psi, 0)$；因此预期支付仍为 ψ。

40 Had the principal decided to let the agent exert no effort ($e=0$), the principal's problem is

$$\begin{aligned} \underset{(t_H,t_L)}{\text{maximize}} \quad & \lambda_0(S_H-t_H) + (1-\lambda_0)(S_L-t_L) \\ \text{subject to} \quad & \lambda_0 t_H + (1-\lambda_0)t_L \geq \lambda_1 t_H + (1-\lambda_1)t_L - \psi \\ & \lambda_0 t_H + (1-\lambda_0)t_L \geq 0. \end{aligned}$$

Thus, principal would make the following payment:

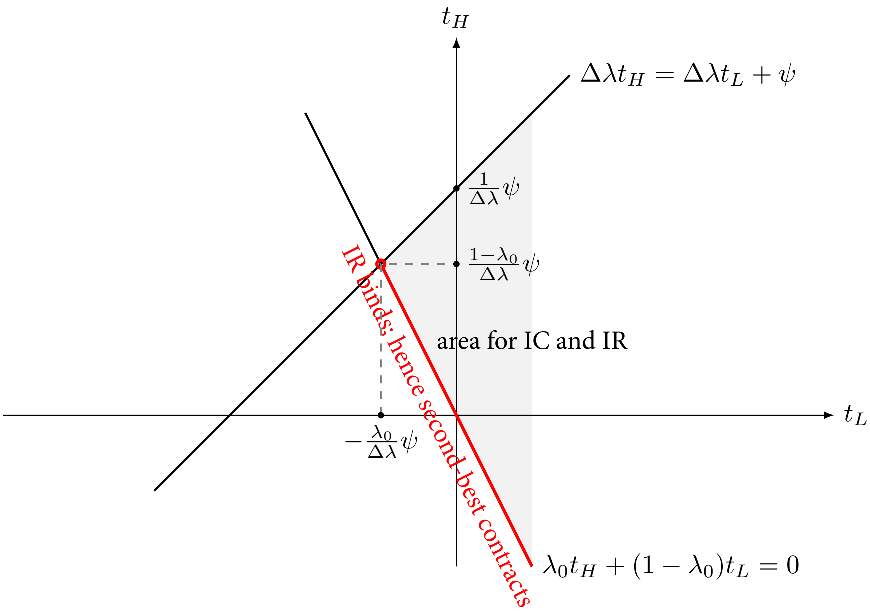
- zero payment to the agent whatever the realization of profit, or

- $t_H^{\text{SB}} = \psi + \frac{1-\lambda_1}{\lambda_1-\lambda_0}\psi - \epsilon_1$ and $t_L^{\text{SB}} = \psi - \frac{\lambda_1}{\lambda_1-\lambda_0}\psi + \epsilon_2$.

The expected profit is

$$V_0 = \lambda_0 S_H + (1-\lambda_0)S_L.$$

41 Graphic illustration:



42 The optimal outcome calls for $e^* = 1$ if and only if $V_1^{\text{SB}} \geq V_0$, i.e.,

$$(\lambda_1 - \lambda_0)(S_H - S_L) \geq \psi = u^{-1}(\psi).$$

Therefore, we have shown: Moral hazard is not an issue with a risk-neutral agent despite the nonobservability of effort. The first-best level of effort is still implemented.

43 The principal can costlessly structure the agent's payment so that the agent has the right incentives to exert effort. Optimal incentives can be provided without incurring any risk-bearing losses.

Indeed, by increasing effort from $e = 0$ to $e = 1$, the agent receives the transfer t_H^{SB} more often than the transfer t_L^{SB}. His expected gain from exerting effort is thus $(\lambda_1 - \lambda_0)(t_H^{\text{SB}} - t_L^{\text{SB}}) = \psi$, i.e., it exactly compensates the agent for the extra disutility of effort that he incurs when increasing his effort from $e = 0$ to $e = 1$.

44 Suppose that $(\lambda_1 - \lambda_0)(S_H - S_L) \geq \psi$. Then the optimal outcome is $e^* = 1$.

(a) Let us consider a pair of transfers

$$t_H^{\text{SB}''} = S_H - T_1 \text{ and } t_L^{\text{SB}''} = S_L - T_1,$$

where T_1 is an up-front payment made by the agent before output realizes.

(b) These transfers satisfy the agent's IC constraint since:

$$(\lambda_1 - \lambda_0)(t_H^{\text{SB}''} - t_L^{\text{SB}''}) = (\lambda_1 - \lambda_0)(S_H - S_L) \geq \psi.$$

(c) The up-front payment T can be adjusted by the principal to have the agent's IR constraint be binding:

$$T_1 = \lambda_1 S_H + (1 - \lambda_1) S_L - \psi.$$

With the transfers $t_H^{\text{SB}''}$ and $t_L^{\text{SB}''}$, the agent becomes residual claimant for the profit of the firm. The up-front payment T_1 is precisely equal to this expected profit. The principal chooses this ex ante payment to reap all gains from delegation.

此处相当于委托人将项目出售给代理人，价格是其预期净利润 T_1。代理人自行管理这个项目，其收入就是项目最终的利润。

45 Suppose that $(\lambda_1 - \lambda_0)(S_H - S_L) < \psi$. Then the optimal outcome is $e^* = 0$.

(a) Let us consider a pair of transfers

$$t_H^{\text{SB}''} = S_H - T_0 \text{ and } t_L^{\text{SB}''} = S_L - T_0,$$

where T_0 is an up-front payment made by the agent before output realizes.

(b) These transfers satisfy the agent's IC constraint since:

$$(\lambda_1 - \lambda_0)(t_H^{\text{SB}''} - t_L^{\text{SB}''}) = (\lambda_1 - \lambda_0)(S_H - S_L) < \psi.$$

(c) The up-front payment T_0 can be adjusted by the principal to have the agent's IR constraint be binding:

$$T_0 = \lambda_0 S_H + (1 - \lambda_0) S_L.$$

With the transfers $t_H^{\text{SB}''}$ and $t_L^{\text{SB}''}$, the agent becomes residual claimant for the profit of the firm. The up-front payment T_0 is precisely equal to this expected profit. The principal chooses this ex ante payment to reap all gains from delegation.

46 One can unify the above arguments as follows:

$$T = \max\{T_1, T_0\},$$

and

$$t_H = S_H - T \text{ and } t_L = S_L - T.$$

Note that $T_1 \geq T_0$ if and only if $\Delta\lambda\Delta S \geq \psi$.

47 Summary:

(a) When the agent is risk neutral, the nonobservability of effort has no effect on the efficiency of trade. Moral hazard does not create any transaction cost.

(b) The principal can achieve the same utility level as if he could directly control the agent's effort.

(c) This first-best outcome is obtained through a contract that is contingent on the level of production.

(d) The agent is "incentivized" by being rewarded for good production levels and penalized otherwise. Since the agent is risk neutral, he is ready to accept penalties and rewards as long as the expected payment he receives satisfies his ex ante participation constraint.

(e) Transfers can be structured to make the agent's participation constraint binding while inducing the desirable effort level. One way of doing so is to make the agent residual claimant for the gains from trade and to grasp all these expected gains by means of an ex ante lump-sum transfer.

5 Incomplete information with limited liability

48 Consider the case $(\lambda_1 - \lambda_0)(S_H - S_L) \geq \psi$, i.e., $e^* = 1$ is the optimal outcome.

49 Clearly, in an optimal contract, t_L has a upper bound: $t_L \leq -\frac{\lambda_0}{\Delta\lambda}\psi$.

In many situation, it also has a lower bound: the responsibility is limited.

50 Let us consider a risk-neutral agent. Let us also assume that the agent's transfer must always be greater than some exogenous level $-l$, with $l \geq 0$.

Limited liability in both states are thus written as

$$t_H \geq -l \text{ and } t_L \geq -l.$$

51 The principal's problem is

$$\begin{aligned} \underset{(t_H, t_L)}{\text{maximize}} \quad & \lambda_1(S_H - t_H) + (1 - \lambda_1)(S_L - t_L) \\ \text{subject to} \quad & \lambda_1 t_H + (1 - \lambda_1)t_L - \psi \geq \lambda_0 t_H + (1 - \lambda_0)t_L \\ & \lambda_1 t_H + (1 - \lambda_1)t_L - \psi \geq 0 \\ & t_H \geq -l \\ & t_L \geq -l \end{aligned}$$

52 For $l > \frac{\lambda_0}{\Delta\lambda}\psi$, the first-best outcome can be implemented, and one optimal wages are

$$t_H^{\text{SB}} = \psi + \frac{1 - \lambda_1}{\lambda_1 - \lambda_0}\psi \text{ and } t_L^{\text{SB}} = \psi - \frac{\lambda_1}{\lambda_1 - \lambda_0}\psi.$$

In this case, the agent has no expected limited liability rent.

53 Graphic illustration:

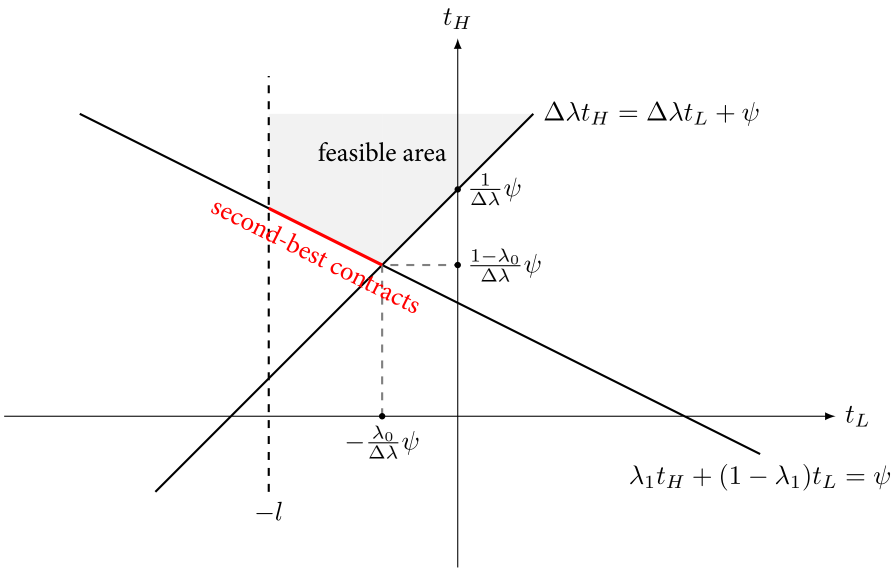


Figure 4: Second-best contracts without limited liability rent

54 For $0 \leq l \leq \frac{\lambda_0}{\Delta\lambda}\psi$, we conjecture that the IC condition and the limited liability condition for low profit are only relevant constraints.

(1) The limited liability condition for high profit is obviously irrelevant (IC implies $t_H \geq \frac{\psi}{\Delta\lambda} + t_L$).

(2) The IR condition is also irrelevant:

$$\lambda_1 t_H + (1-\lambda_1)t_L - \psi \geq \lambda_1\left(-l + \frac{\psi}{\Delta\lambda}\right) + (1-\lambda_1)(-l) - \psi = \frac{\lambda_0}{\Delta\lambda}\psi - l \geq 0.$$

(3) Since the principal is willing to minimize the wages made to the agent, both L-LL and IC constraints must be binding.

(4) Therefore,

$$t_H^{\mathrm{SB}} = -l + \frac{\psi}{\Delta\lambda} \text{ and } t_L^{\mathrm{SB}} = -l.$$

In this case, the agent's expected limited liability rent is non-negative:

$$\lambda_1 t_H^{\mathrm{SB}} + (1-\lambda_1)t_L^{\mathrm{SB}} - \psi = -l + \frac{\lambda_0}{\Delta\lambda}\psi \geq 0.$$

这个租金源于道德风险和有限责任的共同作用所导致的委托人对代理人的额外支付。

55 Graphic illustration:

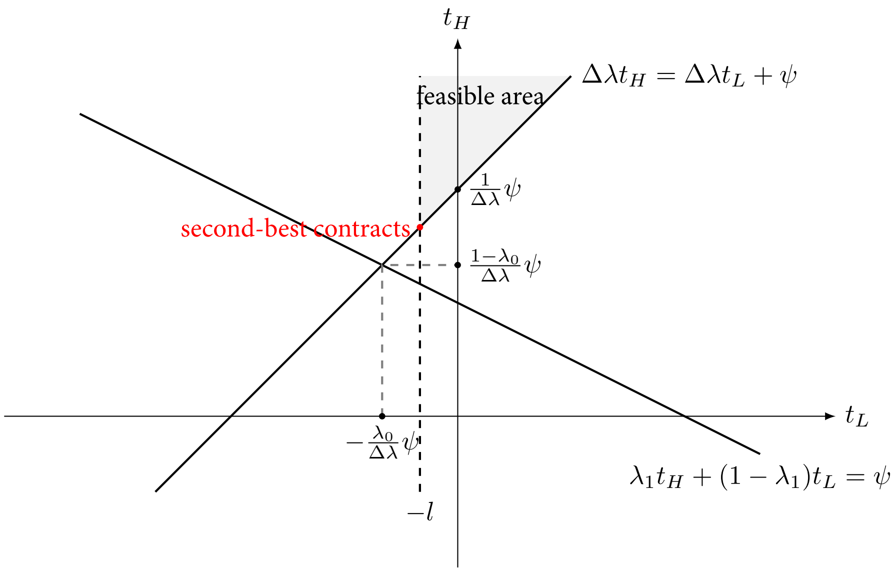


Figure 5: Second-best contracts with limited liability rent

56 Remark:

- Only the limited liability constraint for the bad state may be binding.
- When the limited liability constraint for the bad state is binding, the principal is limited in his punishments to induce effort.

  The principal has to increases awards when high production is realized to induce high effort.

  As a result, the agent receives a non-negative ex ante limited liability rent. Compared with the case without limited liability, this rent is actually the additional payment that the principal must incur because of the conjunction of moral hazard and limited liability.
- As the agent is endowed with more assets, i.e., as l gets larger, the conflict between moral hazard and limited liability diminishes and then disappears whenever l is large enough. In this case, the agent avoids bankruptcy even when he has to pay the optimal penalty to the principal in the bad state of nature.

57 For the sake of simplicity, we assume $l = 0$.

When the principal induces positive effort from the agent, the optimal contract is

$$t_H^{\text{SB}} = \frac{\psi}{\Delta\lambda} \text{ and } t_L^{\text{SB}} = 0,$$

and his expected utility is

$$V_1^{\text{SB}} = \lambda_1 S_H + (1-\lambda_1)S_L - \frac{\lambda_1}{\Delta\lambda}\psi.$$

When the principal gives up the goal of inducing effort from the agent, he can choose $t_H = t_L = 0$ and instead obtain the expected utility level

$$V_0 = \lambda_0 S_H + (1-\lambda_0)S_L.$$

It is worth inducing effort if $V_1^{\text{SB}} \geq V_0$, i.e., when

$$\Delta\lambda\Delta S \geq \frac{\lambda_1}{\Delta\lambda}\psi = \psi + \frac{\lambda_0}{\Delta\lambda}\psi.$$

The left-hand side is the gain of inducing effort, i.e., the gain of increasing the probability of a high production level. The right-hand side is instead the second-best cost C^{SB} of inducing effort, which is the disutility of effort ψ plus the limited liability rent $\frac{\lambda_0}{\Delta\lambda}\psi$. This second-best cost of implementing effort obviously exceeds the first-best cost. It is clear that the limited liability and moral hazard together make it more costly to induce effort.

58 Summary ($l = 0$):

- There is conflict between moral hazard (IC) and limited liability.
- Punishment being now infeasible, the principal is restricted to use only rewards to induce effort. This restriction of the principal's instruments implies that he must give up some ex ante rent to the agent. This limited liability rent is costly for the principal, who then distorts the second-best effort level below its first-best value to reduce the cost of this rent. We have a similar rent extraction-efficiency trade-off leading to a downward distortion in the expected volume of trade.
- IR does not bind. IC binds and limited liability for bad state binds.
- The agent has a positive expected utility $\frac{\lambda_0}{\Delta\lambda}\psi$.
- Efficiency loses since $C^{\text{SB}} = \psi + \frac{\lambda_0}{\Delta\lambda}\psi > \psi = C^*$. The loss part $\frac{\lambda_0}{\Delta\lambda}\psi$ is the limited liability rent for the agent, which is paid by the principal.

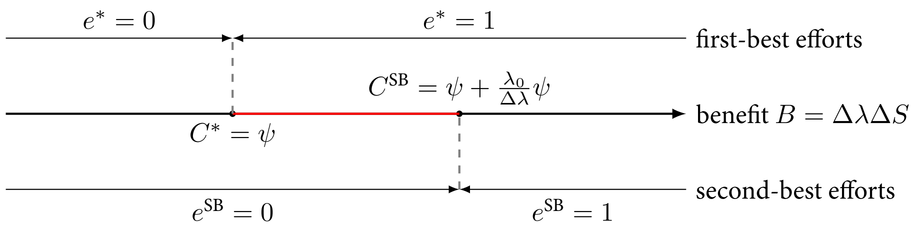


Figure 6: Limited liability rent

6 Incomplete information with risk-averse agent

59 Assume that the agent is risk-averse.

60 We also consider this problem in two steps:

- If the principal wants the agent to exert positive effort (or zero effort), what is the best contract (t_H, t_L)?
- What is the best choice for the principal, inducing the agent to exert positive effort or zero effort?

61 To induce the agent to exert effort, the principal's program is written as:

$$\begin{aligned} \underset{(t_H,t_L)}{\text{maximize}} \quad & \lambda_1(S_H - t_H) + (1-\lambda_1)(S_L - t_L) \\ \text{subject to} \quad & \lambda_1 u(t_H) + (1-\lambda_1)u(t_L) - \psi \geq \lambda_0 u(t_H) + (1-\lambda_0)u(t_L) \\ & \lambda_1 u(t_H) + (1-\lambda_1)u(t_L) - \psi \geq 0. \end{aligned}$$

62 Let $u_H = u(t_H)$ and $u_L = u(t_L)$. Then the principal's program can be written as:

$$\begin{aligned} \underset{(u_H,u_L)}{\text{maximize}} \quad & \lambda_1(S_H - h(u_H)) + (1-\lambda_1)(S_L - h(u_L)) \\ \text{subject to} \quad & \lambda_1 u_H + (1-\lambda_1)u_L - \psi \geq \lambda_0 u_H + (1-\lambda_0)u_L \\ & \lambda_1 u_H + (1-\lambda_1)u_L - \psi \geq 0. \end{aligned}$$

Note that the principal's objective function is now strictly concave in (u_H, u_L) because h is strictly convex. The constraints are now linear and the interior of the constrained set is obviously nonempty, and therefore it is a concave problem, with the Kuhn and Tucker conditions being sufficient and necessary for characterizing optimality.

63 Letting γ and μ be the non-negative multipliers associated respectively with the constraints, the first-order conditions of this program can be expressed as

$$\begin{aligned} 0 &= -\lambda_1 h'(u_H^{\text{SB}}) + \gamma(\lambda_1 - \lambda_0) + \mu\lambda_1 = -\frac{\lambda_1}{u'(t_H^{\text{SB}})} + \gamma(\lambda_1 - \lambda_0) + \mu\lambda_1 \\ 0 &= -(1-\lambda_1)h'(u_L^{\text{SB}}) - \gamma(\lambda_1 - \lambda_0) + \mu(1-\lambda_1) = -\frac{1-\lambda_1}{u'(t_L^{\text{SB}})} - \gamma(\lambda_1 - \lambda_0) + \mu(1-\lambda_1), \end{aligned}$$

where t_H^{SB} and t_L^{SB} are the second-best optimal transfers.

64 Rearranging terms, we get

$$\frac{1}{u'(t_H^{\text{SB}})} = \mu + \gamma\frac{\lambda_1 - \lambda_0}{\lambda_1} \text{ and } \frac{1}{u'(t_L^{\text{SB}})} = \mu - \gamma\frac{\lambda_1 - \lambda_0}{1-\lambda_1}.$$

Multiplying the left equation by λ_1 and the right equation by $1-\lambda_1$, and then adding those two modified equations, we obtain

$$\mu = \frac{\lambda_1}{u'(t_H^{\text{SB}})} + \frac{1-\lambda_1}{u'(t_L^{\text{SB}})} > 0.$$

Hence, the IR condition is binding.

65 The IC condition implies

$$u_H^{\text{SB}} - u_L^{\text{SB}} \geq \frac{\psi}{\lambda_1 - \lambda_0} > 0,$$

and thus $t_H^{\text{SB}} > t_L^{\text{SB}}$.

Therefore,

$$\gamma = \frac{\lambda_1(1-\lambda_1)}{\lambda_1 - \lambda_0}\left(\frac{1}{u'(t_H^{\text{SB}})} - \frac{1}{u'(t_L^{\text{SB}})}\right) > 0,$$

and hence the IC condition is also binding.

66 Since the IR and IC conditions are binding, we have

$$u_H^{\text{SB}} = \psi + \frac{1-\lambda_1}{\lambda_1 - \lambda_0}\psi \text{ and } u_L^{\text{SB}} = \psi - \frac{\lambda_1}{\lambda_1 - \lambda_0}\psi,$$

and hence

$$t_H^{\text{SB}} = h\left(\psi + \frac{1-\lambda_1}{\lambda_1 - \lambda_0}\psi\right) \text{ and } t_L^{\text{SB}} = h\left(\psi - \frac{\lambda_1}{\lambda_1 - \lambda_0}\psi\right).$$

67 The agent receives more than the complete information transfer when a high output is realized, $t_H^{\text{SB}} > h(\psi)$. When a low output is realized, the agent instead receives less than the complete information transfer, $t_L^{\text{SB}} < h(\psi)$.

A risk premium must be paid to the risk-averse agent to induce his participation since he now incurs a risk by the fact that $t_L^{\text{SB}} < t_H^{\text{SB}}$. Indeed, we have

$$\psi = \lambda_1 u(t_H^{\text{SB}}) + (1-\lambda_1) u(t_L^{\text{SB}}) < u\left(\lambda_1 t_H^{\text{SB}} + (1-\lambda_1) t_L^{\text{SB}}\right),$$

where the inequality follows from Jensen's inequality. That is, the expected payment $\lambda_1 t_H^{\text{SB}} + (1-\lambda_1) t_L^{\text{SB}}$ given by the principal is thus larger than the first-best cost $h(\psi)$, which is incurred by the principal when effort is observable.

为了保证代理人获得保留效用（IR 条件），委托人就需要花费更多的奖励——源于风险溢价。

68 The second-best cost of inducing effort under moral hazard is the expected payment made to the agent

$$C^{\text{SB}} = \lambda_1 t_H^{\text{SB}} + (1-\lambda_1) t_L^{\text{SB}} = \lambda_1 h\left(\psi + \frac{1-\lambda_1}{\lambda_1 - \lambda_0}\psi\right) + (1-\lambda_1) h\left(\psi - \frac{\lambda_1}{\lambda_1-\lambda_0}\psi\right) > h(\psi) = C^*,$$

where the inequality follows from Jensen's inequality (h is strictly convex).

基于 IC 条件，委托人提供的合约将导致代理人需要承担风险。对风险厌恶的代理人来说，承担风险会降低他的期望效用。由于合约要满足参与性约束，这个风险成本（风险溢价）最终被转移给委托人。

69 Graphic illustration:

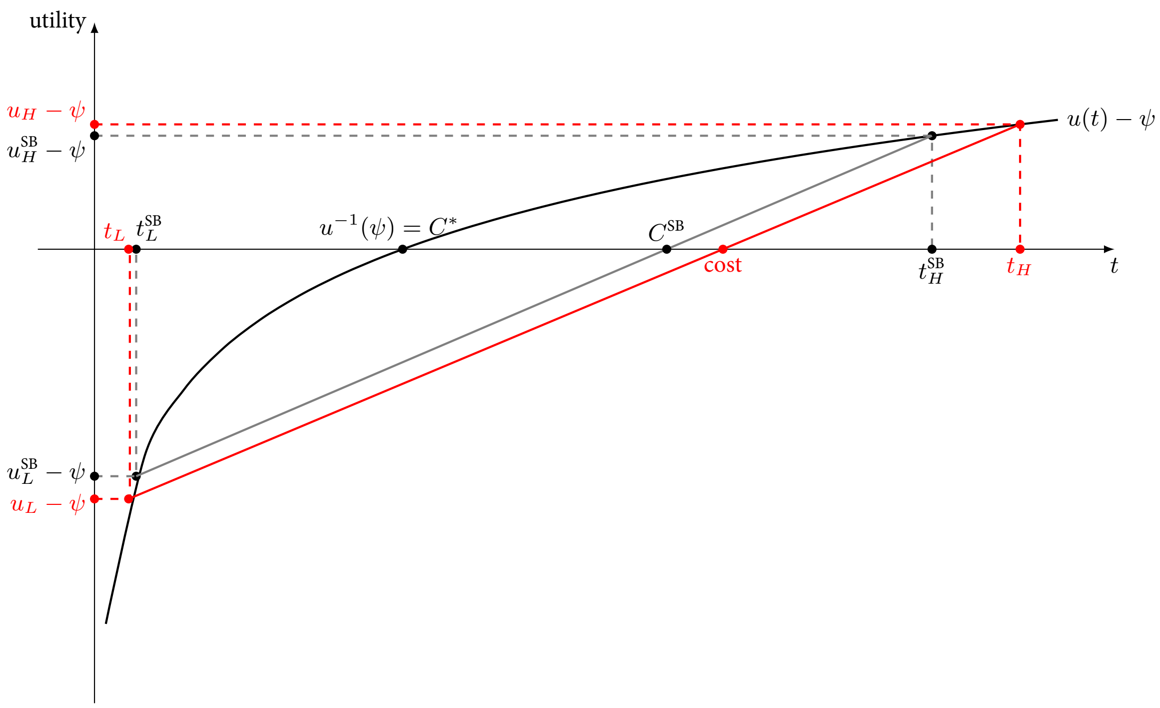


Figure 7: Efficiency vs. Risk premium

(1) $u(t) - \psi$ is the agent's utility function when he exerts effort, which is a convex curve.

(2) In the complete information case, the agent's utility is zero, and the transfer is always $C^* = u^{-1}(\psi)$. So the curve is passing $\left(u^{-1}(\psi), 0\right)$.

(3) Since IR binds, the contract $(t_H^{\text{SB}}, t_L^{\text{SB}})$ makes the agent's expected utility be zero, shown as in the graph. That is, $\lambda_1 u(t_H^{\text{SB}}) + (1-\lambda_1) u(t_L^{\text{SB}}) - \psi = 0$, or $\frac{|u(t_H^{\text{SB}}) - \psi|}{|u(t_L^{\text{SB}}) - \psi|} = \frac{1-\lambda_1}{\lambda_1}$.

为了能够激励代理人付出努力，需要将 t_H 和 t_L 的差距拉大。从 C^* 开始，沿着曲线 $u(t)-\psi$，向两侧拉开 t_L 和 t_H，同时 u_H 和 u_L 也会同步拉开。拉开过程中保持对应的纵坐标的加权平均值为零（即 $\frac{|u(t_H)-\psi|}{|u(t_L)-\psi|}=\frac{1-\lambda_1}{\lambda_1}$），以保证 IR 条件等号成立。

(4) We obtain $(t_H^{\text{SB}}, t_L^{\text{SB}})$ when IC is just binding. The expected transfer should be $\lambda_1 t_H^{\text{SB}} + (1-\lambda_1)t_L^{\text{SB}} = C^{\text{SB}}$.
刚好拉开足够的差距时，我们得到了 $(t_H^{\text{SB}}, t_L^{\text{SB}})$；此时 $(t_H^{\text{SB}}, u_H^{\text{SB}}-\psi)$ 和 $(t_L^{\text{SB}}, u_L^{\text{SB}}-\psi)$ 的连线与横轴的交点的横坐标恰好就是 t_H^{SB} 和 t_L^{SB} 的加权平均值，也就是预期支付；此时为 C^{SB}。

(5) Since u is concave, $C^{\text{SB}} > C^*$.

(6) To induce the agent to exert effort, the principal needs to set t_H and t_L to satisfy $(\lambda_1-\lambda_0)\big(u(t_H)-u(t_L)\big) \geq \psi$. That is, $t_H - t_L$ should be sufficiently large.
为了能够激励代理人付出努力，需要将 t_H 和 t_L 的差距拉大。

(7) IC should be binding; otherwise, the principal can decrease t_H and increase t_L, so that the expected wage $\lambda_1 t_H + (1-\lambda_1)t_L$ decreases.
委托人只会将 t_H 和 t_L 的差距拉大到恰好能够激励代理人付出努力的程度。差距更大的 t_H 和 t_L 虽然可以激励代理人，但会造成额外的成本。例如拉开到图中 t_H 和 t_L，则 $(t_H, u_H-\psi)$ 和 $(t_L, u_L-\psi)$ 的连线与横轴的交点（图中 cost 处）必然位于 C^{SB} 的右侧，即成本高于 C^{SB}。

70 Had the principal decided to let the agent exert no effort, $e=0$, he would (optimally) make a zero payment to the agent whatever the realization of profit. The profit is $\lambda_0 S_H + (1-\lambda_0)S_L$.

71 The benefit of inducing effort is still $(\lambda_1-\lambda_0)(S_H-S_L)$, and a positive effort $e^*=1$ is the optimal choice of the principal whenever

$$(\lambda_1-\lambda_0)(S_H-S_L) \geq C^{\text{SB}} > C^*.$$

72 Summary:

- If the agent is risk averse, a constant wage provides full insurance but induces no effort provision. Inducing effort requires the principal to let the agent bear some risk. To accept such a risky contract, the agent must receive a risk premium. There is now a conflict between the incentive and the participation constraints of the agent. This leads to an insurance-efficiency trade-off. To solve this trade-off the principal must distort the complete information risk-sharing agreement between him and the agent to induce effort provision. A high effort is less often implemented by the principal than under complete information.
- The agent's utility is always zero, although he gets a risk premium.
- The principal sets $t_H^{\text{SB}} > t_L^{\text{SB}}$ to induce the agent to exert effort.
- Efficiency loses since $C^{\text{SB}} > C^*$, which is paid by the principal (“蒸发”掉了).

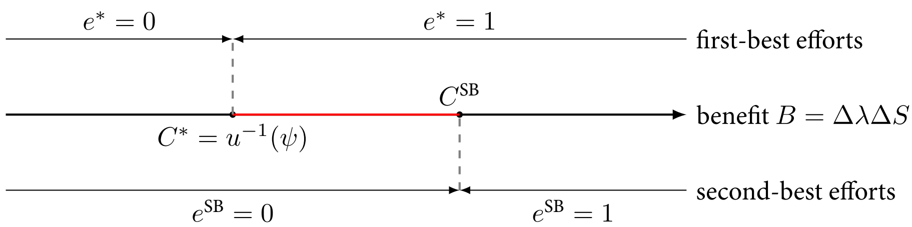


Figure 8: Risk premium

Task

- Reading: 4.1–4.4 in [LM] (required), 5.1 in [S] (required), Appendix 4.2 in [LM] (optional).
- Understanding: 4 summaries.