Advanced Microeconomics: Lecture Note 3

Instructor: Xiang Sun

2022 Fall

1 Overview

1 The complete information optimal contracts are (q_L^*, t_L^*) if $\theta = \theta_L$ and (q_H^*, t_H^*) if $\theta = \theta_H$, where $S'(q_i^*) = \theta_i$ and $t_i^* = \theta_i q_i^*$.

2 Theorem (Optimal contract without shutdown): Under asymmetric information, the optimal menu of contracts entails:

- No output distortion for the efficient type with respect to the first-best, $q_L^{\text{SB}} = q_L^*$. A downward output distortion for the inefficient type, $q_H^{\text{SB}} < q_H^*$ with

$$S'(q_H^{\text{SB}}) = \theta_H + \frac{\lambda}{1-\lambda}\Delta\theta.$$

  Here we assume that the equation above has positive solution. Otherwise q_H^{SB} should be set at zero, and we are in the special case of a contract with shutdown.
  Note that

$$q_L^{\text{SB}} = q_L^* > q_H^* > q_H^{\text{SB}}.$$

- Only the efficient type gets a positive information rent given by

$$U_L^{\text{SB}} = \Delta\theta q_H^{\text{SB}}.$$

- The second-best transfers are respectively given by

$$t_L^{\text{SB}} = \theta_L q_L^* + \Delta\theta q_H^{\text{SB}} > \theta_L q_L^* = t_L^* \text{ and } t_H^{\text{SB}} = \theta_H q_H^{\text{SB}} < \theta_H q_H^* = t_H^*.$$

  Note that

$$t_L^{\text{SB}} = \theta_L q_L^* + \Delta\theta q_H^{\text{SB}} = \theta_L q_L^* + \theta_H q_H^{\text{SB}} - \theta_L q_H^{\text{SB}} = t_H^{\text{SB}} + \theta_L(q_L^* - q_H^{\text{SB}}) > t_H^{\text{SB}}.$$

2 Regulation

3 Literature: Baron and Myerson (1982).

4 Suppose that the principal is a regulator who maximizes a weighted average

- of the consumers' surplus $S(q) - t$ and

- of a regulated monopoly's profit $U = t - \theta q$, with a weight $\alpha < 1$ for the firm's profit.

5 The principal's objective function writes now as

$$V = S(q) - t + \alpha U = S(q) - \theta q - (1-\alpha)U.$$

6 Because $\alpha < 1$, it is socially costly to give up a rent to the firm.

7 As before, with λ probability $\theta = \theta_L$ and with $1-\lambda$ probability $\theta = \theta_H$.

8 The first-best outputs and the complete information optimal contract:

$$\max_{q_i} S(q_i) - \theta_i q_i - (1-\alpha)U_i.$$

We have $S'(q_i^*) = \theta_i$. Then set $U_i^* = 0$ or $t_i^* = \theta_i q_i^*$.

9 The principal's maximization problem of expected social welfare under incentive and participation constraints writes as:

$$\max_{(q_L,U_L),(q_H,U_H)} \lambda \left[S(q_L) - \theta_L q_L\right] + (1-\lambda)\left[S(q_H) - \theta_H q_H\right] - (1-\alpha)\left[\lambda U_L + (1-\lambda)U_H\right],$$

subject to

$$\begin{aligned} U_L &\geq U_H + \Delta\theta q_H, \\ U_H &\geq U_L - \Delta\theta q_L, \\ U_L &\geq 0, \\ U_H &\geq 0. \end{aligned}$$

10 Since we have $U_L = U_H + \Delta\theta q_H$ and $U_H = 0$, while we assume $U_H > U_L - \Delta\theta q_L$ and $U_L > 0$ at the optimal levels $q_L^{\text{SB}} > 0$ and $q_H^{\text{SB}} > 0$, the optimization problem simplifies to

$$\max_{q_L,q_H} \lambda \left[S(q_L) - \theta_L q_L\right] + (1-\lambda)\left[S(q_H) - \theta_H q_H - \frac{\lambda}{1-\lambda}(1-\alpha)\Delta\theta q_H\right].$$

11 Maximizing this objective function leads to $q_L^{\text{SB}} = q_L^*$ for the efficient type, which is given

$$S'(q_L^{\text{SB}}) = \theta_L,$$

and a downward distortion for the inefficient type, $q_H^{\text{SB}} < q_H^*$, which is given by

$$S'(q_H^{\text{SB}}) = \theta_H + \frac{\lambda}{1-\lambda}(1-\alpha)\Delta\theta.$$

12 Note that a higher value of α reduces the output distortion, because the regulator is less concerned by the distribution of rents within society as α increases.

13 If $\alpha = 1$, the firm's rent is no longer costly and the regulator behaves as a pure efficiency maximizer implementing the first-best output in all states of nature.

3 Nonlinear pricing

14 Literature: Maskin and Riley (1984).

15 The principal is the seller of a private good, who faces a continuum of buyers (agents).

16 The principal has production cost cq with $c > 0$. Thus principal's utility function is $V = t - cq$, where q is the quantity consumed and t the payment of the buyer.

17 The tastes of a buyer for the private good are such that his utility function is $U = \theta u(q) - t$, where $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$ for all q.

The characteristics $\theta > 0$ is a number that we can interpret as the buyer's valuation of the good.

18 Suppose that the parameter θ of each buyer is drawn independently from the same distribution on $\{\theta_H, \theta_L\}$ with respective probabilities λ and $1 - \lambda$. Assume $\theta_H > \theta_L$.

19 Remark: We are now in a setting with a continuum of agents.

- It is mathematically equivalent to the framework with a single agent.
- Now the distribution of θ to be considered is the actual distribution of types, i.e., λ is the frequency of type θ_H by the law of large numbers.

It is important to stress this interpretation because it considerably enlarges the relevance of the principal-agent model analyzed before.

20 We assume that the value of θ is known to the buyer, but it is not known to the seller.

- Buyers often know better than sellers how well some particular product meets their preferences.

21 The question of interest here is, what is the best, that is, the profit maximizing, contract (q, t) that the seller will be able to induce the buyer to choose?

Let $\mathcal{A}$ be the set of all feasible contracts, that is,

$$\mathcal{A} = \{(q, t) \mid q \geq 0, t \in \mathbb{R}\}.$$

22 Complete information optimal contract.

By solving the following problem

$$\max_{q_i} \theta_i u(q_i) - cq_i,$$

we have

$$\theta_i u'(q_i^*) = c.$$

To implement them, set $t_i^* = \theta_i u(q_i^*)$.

23 Since $\theta_H > \theta_L$ and u' is decreasing, we have

$$q_H^* > q_L^*.$$

24 Example: $u(q) = \sqrt{q}, c = \frac{2}{3}, \theta_H = 2$ and $\theta_L = 1$. The two curves shown are the indifference curves corresponding to zero utility for the two types of the buyer. The lines tangent to them are isoprofit curves, with equation $t = cq +$ constant. Note that the utility of the buyer increases when going southeast, while the profit of the seller increases when going northwest.

$(q_H^*, t_H^*) = (\frac{9}{4}, 3)$, $(q_L^*, t_L^*) = (\frac{9}{16}, \frac{3}{4})$, $V_H^* = \frac{3}{2}$, $V_L^* = \frac{3}{8}$.

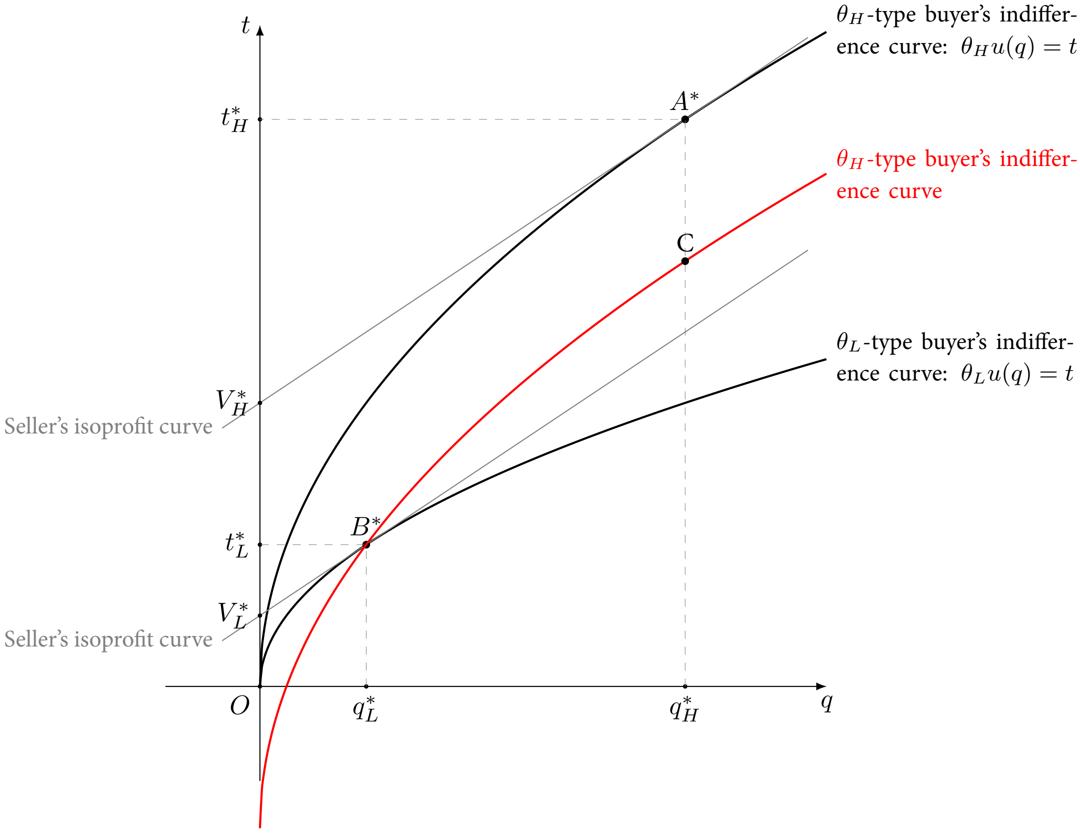

Figure 1: First-best contract

25 We now consider the case where the seller cannot observe directly the buyer's type.

Incentive and participation constraints can as usual be written directly in terms of the information rents $U_H = \theta_H u(q_H) - t_H$ and $U_L = \theta_L u(q_L) - t_L$ as follows:

$$U_H \geq U_L + \Delta\theta u(q_L), \tag{IC$_H$}$$

$$U_L \geq U_H - \Delta\theta u(q_H), \tag{IC$_L$}$$

$$U_H \geq 0, \tag{IR$_H$}$$

$$U_L \geq 0. \tag{IR$_L$}$$

The problem of the seller is therefore to solve

$$\max_{(q_H, U_H), (q_L, U_L)} \lambda \left[\theta_H u(q_H) - c q_H\right] + (1-\lambda)\left[\theta_L u(q_L) - c q_L\right] - \left[\lambda U_H + (1-\lambda) U_L\right],$$

subject to the above four constraints.

26 The analysis is the mirror image of that of our previous discussions, where now the efficient type is the one with the highest valuation for the good θ_H.

Hence, $U_H \geq U_L + \Delta\theta u(q_L)$ and $U_L \geq 0$ are the two binding constraints.

27 As a result, there is no output distortion with respect to the first-best outcome for the high valuation type and $q_H^{SB} = q_H^*$, where $\theta_H u'(q_H^*) = c$.

However, there exists a downward distortion of the low valuation agent's output with respect to the first-best outcome: $q_L^{\mathrm{SB}} < q_L^*$, where

$$\underbrace{\left[\theta_L - \frac{\lambda}{1-\lambda}\Delta\theta\right]}_{<\theta_L} u'(q_L^{\mathrm{SB}}) = c \text{ and } \theta_L u'(q_L^*) = c.$$

28 Example: $u(q) = \sqrt{q}$, $c = \frac{2}{3}$, $\theta_H = 2$, $\theta_L = 1$ and $\lambda = \frac{1}{3}$.

$C = (\frac{9}{4}, \frac{9}{4})$, $V_H^C = \frac{3}{4}$.

$A^{\mathrm{SB}} = (q_H^{\mathrm{SB}}, t_H^{\mathrm{SB}}) = (\frac{9}{4}, 2.625)$, $B^{\mathrm{SB}} = (q_L^{\mathrm{SB}}, t_L^{\mathrm{SB}}) = (0.140625, \frac{3}{8})$, $V_H^{\mathrm{SB}} = 1.125$, $V_L^{\mathrm{SB}} = 0.28125$.

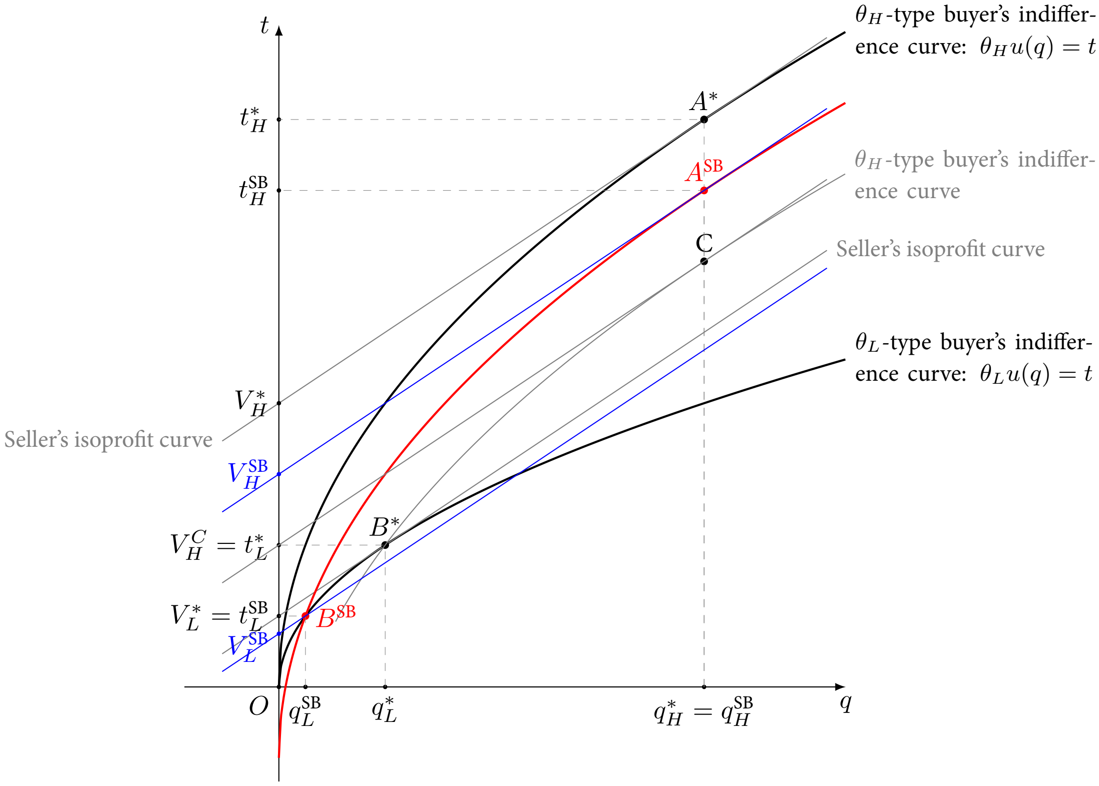

Figure 2: Second-best contract

29 $q_H^{\mathrm{SB}} = q_H^*$ and $q_L^{\mathrm{SB}} < q_L^*$.

The θ_L-type buyer has no surplus, while the other type has a strictly positive information rent:

$$\theta_H v(q_H^{\mathrm{SB}}) - t_H^{\mathrm{SB}} = \theta_H v(q_L^{\mathrm{SB}}) - t_L^{\mathrm{SB}} = (\theta_H - \theta_L) v(q_L^{\mathrm{SB}}).$$

4 Quality and price discrimination

30 Literature: Mussa and Rosen (1978).

31 Consider a similar problem to the one in previous part, where agents buy one unit of a commodity with quality q but are vertically differentiated with respect to their preferences for the good.

32 The marginal cost (and average cost) of producing one unit of quality q is $C(q)$ and the principal has the utility function $V = t - C(q)$, where $C(0) = 0$, $C' > 0$, $C'' > 0$.

33 The utility function of an agent is now $U = \theta q - t$ with θ in $\{\theta_H, \theta_L\}$ with respective probabilities λ and $1 - \lambda$.

34 The first-best outputs and the complete information optimal contracts:

$$\max_{q_i} \theta_i q_i - C(q_i).$$

We have $C'(q_i^*) = \theta_i$. Then set $U_i^* = 0$ or $t_i^* = \theta_i q_i^*$.

35 Incentive and participation constraints can still be written directly in terms of the information rents $U_H = \theta_H q_H - t_H$ and $U_L = \theta_L q_L - t_L$ as follows:

$$U_H \geq U_L + \Delta\theta q_L, \quad (\text{IC}_H)$$
$$U_L \geq U_H - \Delta\theta q_H, \quad (\text{IC}_L)$$
$$U_H \geq 0, \quad (\text{IR}_H)$$
$$U_L \geq 0. \quad (\text{IR}_L)$$

The problem of the seller is therefore to solve

$$\max_{(q_H, U_H), (q_L, U_L)} \lambda \left[\theta_H q_H - C(q_H)\right] + (1-\lambda)\left[\theta_L q_L - C(q_L)\right] - \left[\lambda U_H + (1-\lambda) U_L\right],$$

subject to the above four constraints.

36 Following procedures similar to what we have done so far, only $U_H \geq U_L + \Delta\theta q_L$ and $U_L \geq 0$ are binding constraints.

Finally, we find that the high valuation agent receives the first-best quality $q_H^{\text{SB}} = q_H^*$, where $\theta_H = C'(q_H^*)$.

37 However, quality is now reduced below the first-best for the low valuation agent.

We have $q_L^{\text{SB}} < q_L^*$, where

$$\theta_L = C'(q_L^{\text{SB}}) + \frac{\lambda}{1-\lambda}\Delta\theta \text{ and } \theta_L = C'(q_L^*).$$

38 Example: $C(q) = q^{\frac{3}{2}}$, $\theta_L = \frac{3}{4}$, $\theta_H = \frac{3}{2}$ and $\lambda = \frac{1}{3}$.

First-best contracts: $q_i^* = (\frac{2}{3}\theta_i)^2$, $t_i^* = \theta_i q_i^* = \frac{4}{9}\theta_i^3$. So

$$(q_H^*, t_H^*) = (1, \tfrac{3}{2}),\ (q_L^*, t_L^*) = (\tfrac{1}{4}, \tfrac{3}{16}).$$

Second-best contracts:

$$(q_H^{\text{SB}}, t_H^{\text{SB}}) = (q_H^* = 1, \tfrac{3}{2} - \tfrac{3}{64}),\ (q_L^{\text{SB}}, t_L^{\text{SB}}) = (\tfrac{1}{16}, \tfrac{3}{64}).$$

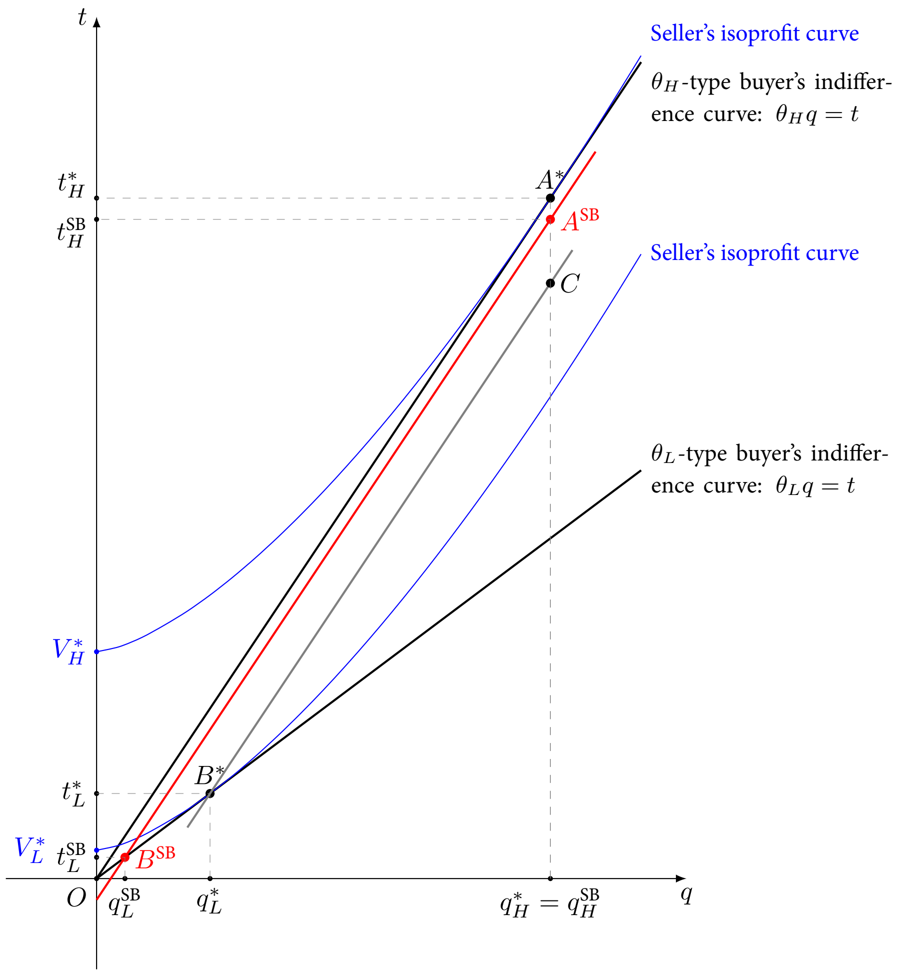

Figure 3: Second-best contract

5 Financial contracts

39 Literature: Freixas and Laffont (1990).

40 Suppose the principal is a lender who provides a loan of size k to a borrower.

41 Capital costs Rk to the lender since it could be invested elsewhere in the economy to earn the risk-free interest rate R.

42 The lender has thus a utility function $V = t - Rk$.

43 The borrower makes a profit $U = \theta f(k) - t$ where $\theta f(k)$ is the production with k units of capital and t is the borrower's repayment to the lender.

44 We assume that $f' > 0$ and $f'' < 0$.

45 The parameter θ is a productivity shock drawn from $\{\theta_H, \theta_L\}$ with respective probabilities λ and $1 - \lambda$.

46 The first-best outputs and the complete information optimal contracts:

$$\max_{k_i} \theta_i f(k_i) - Rk_i.$$

We have $\theta_i f'(k_i^*) = R$. Then set $t_i^* = \theta_i f(k_i^*)$.

47 Incentive and participation constraints can again be written directly in terms of the borrower's information rents $U_H = \theta_H f(k_H) - t_H$ and $U_L = \theta_L f(k_L) - t_L$ as follows:

$$U_H \geq U_L + \Delta\theta f(k_L), \tag{IC$_H$}$$

$$U_L \geq U_H - \Delta\theta f(k_H), \tag{IC$_L$}$$

$$U_H \geq 0, \tag{IR$_H$}$$

$$U_L \geq 0. \tag{IR$_L$}$$

The problem of the seller is therefore to solve

$$\max_{(k_H,U_H),(k_L,U_L)} \lambda \left[\theta_H f(k_H) - Rk_H\right] + (1-\lambda)\left[\theta_L f(k_L) - Rk_L\right] - \left[\lambda U_H + (1-\lambda)U_L\right],$$

subject to the above four constraints.

48 There is no capital distortion with respect to the first-best outcome for the high productivity type and $k_H^{\text{SB}} = k_H^*$ where $\theta_H f'(k_H^*) = R$.

In this case, the return on capital is equal to the risk-free interest rate.

49 However, there also exists a downward distortion in the size of the loan given to a low productivity borrower with respect to the first-best outcome.

We have $k_L^{\text{SB}} < k_L^*$, where

$$\left[\theta_L - \frac{\lambda}{1-\lambda}\Delta\theta\right] f'(k_L^{\text{SB}}) = R \text{ and } \theta_L f'(k_L^*) = R.$$

Task

- Reading: 2.15 in [LM] (required), 3.1 in [S] (required).
- Understanding: