# Advanced Microeconomics: Lecture Note 15

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2022 Fall

## 1 Herding

1. Herding (从众) is the act of changing individual behavior to follow a trend.

   Examples:

   - Bubble
   - New fashion trend

2. Question: Why do people follow suit and cluster on the same (occupational, cultural, consumption) choice?

3. There are many possible answers: similarity in taste, social conformity, positive network externality.

4. Bikhchandani, Hirshleifer, and Welch (JPE, 1992) look at the particular angle of observational learning and they offer a rational justification of people's incentive to abandon private opinion and follow the crowd, even if the crowd's choice is not necessarily correct.

5. The theory of informational cascades is a theory that explains fads, fashion, custom and cultural changes from the perspective of observational learning.

   It rationalizes herding behavior as an outcome of observational learning.

## 2 Setup

6. There are two new restaurants in town, $A$ and $B$.

   Which restaurant is better is uncertain with equal prior probability, i.e., true state is either $A$ or $B$ with

   $$\text{Prob}(A) = \text{Prob}(B) = \tfrac{1}{2}.$$

7. There are an infinite sequence of consumers who arrive one by one.

   Each consumer $i$ receives an i.i.d. private signal $s_i \in \{s_A, s_B\}$.

   - $s_A$ means "I personally think restaurant A is better";
   - $s_B$ means "I personally think restaurant B is better".

8. The accuracy of the private signal is $p > \frac{1}{2}$:

$$\begin{aligned} \text{Prob}(s_A \mid A) = \text{Prob}(s_B \mid B) = p > \tfrac{1}{2}, \\ \text{Prob}(s_A \mid B) = \text{Prob}(s_B \mid A) = 1 - p < \tfrac{1}{2}. \end{aligned}$$

9. Consumers have identical payoff functions: Each consumer gets payoff 1 if he chooses the better restaurant and 0 otherwise.

   Tie-breaking rule: When a consumer is indifferent, he flips a coin. That is, he chooses restaurant $A$ with probability $\frac{1}{2}$.

10. Timeline of the game:

    - The first consumer arrives and picks a restaurant based on his private signal $s_1$. Denote his choice as $C_1$.
    - The second consumer observes the choice of the first consumer as well as his private signal $s_2$, and picks a restaurant $C_2$.
    - For any $k \geq 2$, the $k$-th consumer observes the choices of all previous consumers, $C_1, C_2, \ldots, C_{k-1}$, as well as his private signal $s_k$. Based on these information, he picks a restaurant $C_k$.

## 3 Decision rule

11. Each consumer has two types: The private signal can be either $s_A$ or $s_B$.

12. Each consumer's strategy is a function from the "set of histories $\times$ the set of types" to $\{A, B\}$.

    Given the history of the game, each consumer $i$'s strategy is a pair $(C_i^A, C_i^B)$, where $C_i^A$ denotes his choice if his signal is $s_A$ and $C_i^B$ denotes his choice if the signal is $s_B$.

13. Perfect Bayesian equilibrium requires each consumer $i$ always chooses the best option according to his belief about the true state.

    (a) Suppose that, conditional on all available information, consumer $i$ believes that restaurant $A$ is better with probability $x$.

    (b) Then the expected utilities are

    $$\begin{aligned} \mathbf{E}[u(A)] &= u(\text{choose A when A is better}) \cdot x + u(\text{choose A when B is better}) \cdot (1 - x) = x, \\ \mathbf{E}[u(B)] &= u(\text{choose B when A is better}) \cdot x + u(\text{choose B when B is better}) \cdot (1 - x) = 1 - x. \end{aligned}$$

    (c) Thus, $\mathbf{E}[u(A)] > \mathbf{E}[u(B)]$ when $x > \frac{1}{2}$, and $\mathbf{E}[u(A)] < \mathbf{E}[u(B)]$ when $x < \frac{1}{2}$.

14. The decision rule for each consumer is:

$$\begin{cases} \text{choose } A, & \text{if Prob}(A \mid \text{current information}) > \frac{1}{2}, \\ \text{choose } B, & \text{if Prob}(A \mid \text{current information}) < \frac{1}{2}, \\ \text{choose } A \text{ with probability } \frac{1}{2}, & \text{if Prob}(A \mid \text{current information}) = \frac{1}{2}. \end{cases}$$

# 4 Belief updating

15. Perfect Bayesian equilibrium also require that each consumer correctly update his belief about the true state using Bayes' rule.

## 4.1 Consumer 1

16. The first consumer observes only his private signal $s_1$, so he updates his belief according to the realization of his signal:

$$\begin{aligned}
\text{Prob}(A \mid s_A) &= \frac{\text{Prob}(s_A \mid A)\,\text{Prob}(A)}{\text{Prob}(s_A)} = \frac{\frac{1}{2}p}{\text{Prob}(s_A)}, \\
\text{Prob}(A \mid s_B) &= \frac{\text{Prob}(s_B \mid A)\,\text{Prob}(A)}{\text{Prob}(s_B)} = \frac{\frac{1}{2}(1-p)}{\text{Prob}(s_B)}.
\end{aligned}$$

Since

$$\begin{aligned}
\text{Prob}(s_A) &= \text{Prob}(s_A \mid A) \times \text{Prob}(A) + \text{Prob}(s_A \mid B) \times \text{Prob}(B) = \tfrac{1}{2}p + \tfrac{1}{2}(1-p) = \tfrac{1}{2}, \\
\text{Prob}(s_B) &= \text{Prob}(s_B \mid A) \times \text{Prob}(A) + \text{Prob}(s_B \mid B) \times \text{Prob}(B) = \tfrac{1}{2}(1-p) + \tfrac{1}{2}p = \tfrac{1}{2},
\end{aligned}$$

we have

$$\text{Prob}(A \mid s_A) = p > \tfrac{1}{2} \text{ and } \text{Prob}(A \mid s_B) = 1 - p < \tfrac{1}{2}.$$

17. Based on the decision rule, the first consumer always chooses according to his private signal, i.e., his strategy is

$$(C_1^A, C_1^B) = (A, B).$$

## 4.2 Consumer 2

18. When the second consumer arrives, he observes not only his private signal $s_2$ but also the choice of the first consumer, $C_1$.

    Moreover, the second consumer also knows that the choices of the first consumer perfectly reveals his private signal according to his strategy $(C_1^A, C_1^B) = (A, B)$.

    Based on these information, he updates his belief in each of the four possible scenarios.

19. Case 1: When $C_1 = A$ and $s_2 = s_A$,

$$\begin{aligned}
\text{Prob}(A \mid C_1 = A, s_2 = s_A) &= \text{Prob}(A \mid s_1 = s_A, s_2 = s_A) \\
&= \frac{\text{Prob}(s_1 = s_A, s_2 = s_A \mid A)\,\text{Prob}(A)}{\text{Prob}(s_1 = s_A, s_2 = s_A)} \\
&= \frac{\text{Prob}(s_1 = s_A, s_2 = s_A \mid A)\,\text{Prob}(A)}{\text{Prob}(s_1 = s_A, s_2 = s_A \mid A)\,\text{Prob}(A) + \text{Prob}(s_1 = s_A, s_2 = s_A \mid B)\,\text{Prob}(B)} \\
&= \frac{p^2 \frac{1}{2}}{p^2 \frac{1}{2} + (1-p)^2 \frac{1}{2}} \\
&= \frac{p^2}{p^2 + (1-p)^2} > \frac{p^2}{p^2 + p^2} = \tfrac{1}{2}.
\end{aligned}$$

Therefore, the second consumer strictly prefers to choose $A$.

20. Case 2: When $C_1 = A$ and $s_2 = s_B$, we can use the Bayes' rule to get

$$\begin{aligned}
\text{Prob}(A \mid C_1 = A, s_2 = s_B) &= \text{Prob}(A \mid s_1 = s_A, s_2 = s_B) \\
&= \frac{\text{Prob}(s_1 = s_A, s_2 = s_B \mid A)\,\text{Prob}(A)}{\text{Prob}(s_1 = s_A, s_2 = s_B)} \\
&= \frac{\text{Prob}(s_1 = s_A, s_2 = s_B \mid A)\,\text{Prob}(A)}{\text{Prob}(s_1 = s_A, s_2 = s_B \mid A)\,\text{Prob}(A) + \text{Prob}(s_1 = s_A, s_2 = s_B \mid B)\,\text{Prob}(B)} \\
&= \frac{p(1-p)\frac{1}{2}}{p(1-p)\frac{1}{2} + (1-p)p\frac{1}{2}} = \tfrac{1}{2}.
\end{aligned}$$

This means that the second consumer is indifferent between $A$ and $B$ and will flip a coin.

21. Case 3: When $C_1 = B$ and $s_2 = s_A$,

$$\text{Prob}(A \mid C_1 = B, s_2 = s_A) = \tfrac{1}{2}.$$

The second consumer is indifferent between $A$ and $B$ and will flip a coin.

22. Case 4: When $C_1 = B$ and $s_2 = s_B$,

$$\text{Prob}(A \mid C_1 = B, s_2 = s_B) = \frac{(1-p)^2}{(1-p)^2 + p^2} < \tfrac{1}{2},$$

and the second consumer strictly prefers to choose $B$.

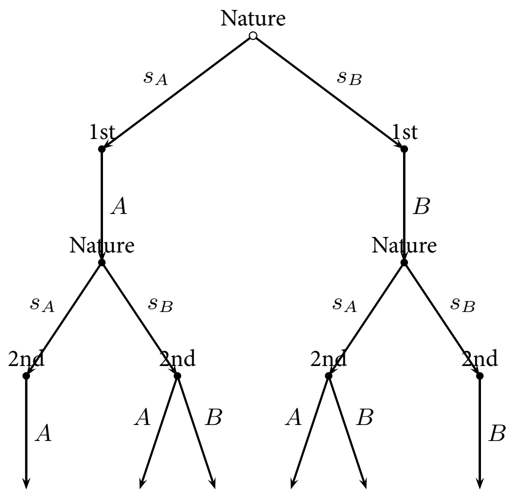
23. Based on these four scenarios, the strategy of the second consumer is conditional on the action of the first consumer:

- If the first consumer chose $A$, then

$$(C_2^A, C_2^B) = \big(A, \tfrac{1}{2}A + \tfrac{1}{2}B\big).$$

- If the first consumer chose $B$, then

$$(C_2^A, C_2^B) = \big(\tfrac{1}{2}A + \tfrac{1}{2}B, B\big).$$



## 4.3 Consumer 3

24. When the third consumer arrives, he observes both choices from consumer 1 and 2 as well as his private signal $s_3$. There are four possible realizations of choices from the first two consumers:

$$(C_1, C_2) \in \{(A, A), (A, B), (B, A), (B, B)\}.$$

To simplify things, let's classify them into two categories.

25. Case 1: $C_1 \neq C_2$.

    (a) If the choice of the second consumer differs from the choice of the first, then it must be the case that the second consumer's private signal is opposite of the first one's.

    (b) In this case, the choices of the first consumers perfectly reveals their (opposite) private signals. That is,

$$(C_1, C_2) = (A, B) \Longrightarrow (s_1, s_2) = (s_A, s_B),$$
$$(C_1, C_2) = (B, A) \Longrightarrow (s_1, s_2) = (s_B, s_A).$$

    (c) Since the consumers' signals are equally accurate, the effect of a pair of opposite signals cancel out:

$$\text{Prob}(A \mid s_A, s_B) = \text{Prob}(A \mid s_B, s_A) = \tfrac{1}{2}.$$

    (d) Therefore, when $C_1 \neq C_2$, the third consumer cannot learn anything from the choices of the first two consumers and he has to rely his choice solely on his own private signal.

    (e) In other words, when the first pair of consumers differ in their choices, we can simply delete them from the sequence. The third consumer acts as if he is the first consumer in the sequence; he chooses according to his private signal only:

$$(C_3^A, C_3^B) = (A, B).$$

26. Case 2: $C_1 = C_2$.

27. Subcase 1: Suppose $C_1 = C_2 = A$ and $s_3 = s_A$.

$$\begin{aligned}
&\text{Prob}(A \mid C_1 = C_2 = A, s_3 = s_A) \\
=\ &\text{Prob}(A \mid s_1 = s_3 = s_A, s_2 = s_A \text{ or } s_2 = s_B \text{ but the coin flip suggests } A) \\
=\ &\frac{\text{Prob}(s_1 = s_3 = s_A, s_2 = s_A \text{ or } s_2 = s_B \text{ but the coin flip suggests } A \mid A)\,\text{Prob}(A)}{\text{Prob}(s_1 = s_3 = s_A, s_2 = s_A \text{ or } s_2 = s_B \text{ but the coin flip suggests } A)} \\
=\ &\frac{p^2[p + (1-p)\frac{1}{2}]\frac{1}{2}}{p^2[p + (1-p)\frac{1}{2}]\frac{1}{2} + (1-p)^2[(1-p) + p\frac{1}{2}]\frac{1}{2}} \\
=\ &\frac{p^2[p + \frac{1}{2}(1-p)]}{p^2[p + \frac{1}{2}(1-p)] + (1-p)^2[(1-p) + \frac{1}{2}p]}.
\end{aligned}$$

    (a) Since $p > \frac{1}{2}$, we have $p^2 > (1-p)^2$ and $p + \frac{1}{2}(1-p) = \frac{1}{2} + \frac{1}{2}p > 1 - \frac{1}{2}p = (1-p) + \frac{1}{2}p$.

    (b) Then $p^2[p + \frac{1}{2}(1-p)]\frac{1}{2} > (1-p)^2[(1-p) + \frac{1}{2}p]$, and hence $\text{Prob}(A \mid C_1 = C_2 = A, s_3 = s_A) > \frac{1}{2}$.

    (c) Therefore, the third consumer strictly prefers to choose $A$.

In summary, the third consumer continues to receive a private signal that suggests the same action, he will again choose the same action $C_3 = C_1 = C_2$.

28. Subcase 2: Suppose $C_1 = C_2 = A$ and $s_3 = s_B$.

The third consumer updates his belief:

$$\text{Prob}(A \mid C_1 = C_2 = A, s_3 = s_B) = \frac{\text{Prob}(C_1 = C_2 = A, s_3 = s_B \mid A)\,\text{Prob}(A)}{\text{Prob}(C_1 = C_2 = A, s_3 = s_B)},$$

where

$$\begin{aligned}
\text{Prob}(C_1 = C_2 = A, s_3 = s_B) =\; & \text{Prob}(C_1 = C_2 = A, s_3 = s_B \mid A)\,\text{Prob}(A) \\
& + \text{Prob}(C_1 = C_2 = A, s_3 = s_B \mid B)\,\text{Prob}(B), \\
\text{Prob}(C_1 = C_2 = A, s_3 = s_B \mid A) =\; & \text{Prob}(s_1 = s_2 = s_A, s_3 = s_B \mid A) + \tfrac{1}{2}\,\text{Prob}(s_1 = s_A, s_2 = s_3 = s_B \mid A) \\
=\; & p^2(1-p) + \tfrac{1}{2}p(1-p)^2, \\
\text{Prob}(C_1 = C_2 = A, s_3 = s_B \mid B) =\; & \text{Prob}(s_1 = s_2 = s_A, s_3 = s_B \mid B) + \tfrac{1}{2}\,\text{Prob}(s_1 = s_A, s_2 = s_3 = s_B \mid B) \\
=\; & (1-p)^2 p + \tfrac{1}{2}(1-p)p^2.
\end{aligned}$$

Therefore,

$$\text{Prob}(A \mid C_1 = C_2 = A, s_3 = s_B) = \frac{p^2(1-p) + \frac{1}{2}p(1-p)^2}{p^2(1-p) + \frac{1}{2}p(1-p)^2 + (1-p)^2 p + \frac{1}{2}(1-p)p^2} = \frac{1+p}{3} > \frac{1}{2},$$

the last inequality is due to $p > \frac{1}{2}$.

Based on the decision rule, this means that, if the first two consumers both chose restaurant $A$, the third consumer will choose restaurant $A$ even if his private signal is $s_B$.

29. The third consumer will follow suit and choose $A$ regardless of his private signal. This is called an informational cascade of $A$.

30. Similar result holds for the case when $C_1 = C_2 = B$.

# 5 Informational cascade

31. A player is said to be in an informational cascade of action $X$ if he chooses $X$ regardless of his private signal.

32. Question: If the third consumer is choosing $A$ regardless of his private signal, what about the other consumers?

33. Answer: All of them choose $A$ regardless of their private signals.

Intuition:

- If the third consumer's choice is irrelevant of his private signal, his choice becomes completely uninformative to the fourth consumer because $C_3$ reveals nothing about $s_3$.
- Therefore, the fourth consumer faces the exact same decision problem as the third consumer; he chooses $A$ regardless of his private signal, too.
- The choice of the fourth consumer is uninformative to the fifth.
- The fifth faces the same problem as the third; again, he chooses $A$ regardless of his signal.
- …
- The same result applies for every consumer later in the (infinite) sequence.

34. Theorem: If player $k$ is in an informational cascade of action $X$, every player $i > k$ is in the information cascade of action $X$.

35. Summary: In the perfect Bayesian equilibrium of this game

- A pair of choices $(A, A) \Longrightarrow$ an informational cascade of $A$;
- A pair of choices $(B, B) \Longrightarrow$ an informational cascade of $B$;
- A pair of choices $(A, B)$ or $(B, A) \Longrightarrow$ delete this uninformative pair and restart the game.

36. When a cascade occurs, the choice of infinitely many consumers is, in fact, determined by the private signals of only two consumers.

37. When there are infinite consumers, eventually we will encounter a pair of $(A, A)$ or $(B, B)$ in the sequence. Thus, in the limit, a cascade occurs with probability 1.

 (a) Suppose the true state be $A$.

 (b) The probability that a correct cascade starts from the third consumer is equal to $\text{Prob}(C_1 = C_2 = A \mid A) = p^2 + \frac{1}{2}p(1-p)$.

 The probability that a wrong cascade starts from the third consumer is equal to $\text{Prob}(C_1 = C_2 = B \mid A) = (1-p)^2 + \frac{1}{2}(1-p)p$.

 (c) The probability that a correct cascade starts exactly from consumer $2k+1$ is equal to

$$\text{Prob}(k-1 \text{ pairs of opposite actions followed by a pair of } A \mid A) = [p(1-p)]^{k-1} \times [p^2 + \tfrac{1}{2}p(1-p)].$$

 The probability that a wrong cascade starts exactly from consumer $2k+1$ is equal to

$$\text{Prob}(k-1 \text{ pairs of opposite actions followed by a pair of } B \mid A) = [p(1-p)]^{k-1} \times [(1-p)^2 + \tfrac{1}{2}(1-p)p].$$

 (d) A correct cascade exists with probability

$$\sum_{k=1}^{\infty}[p(1-p)]^{k-1} \times [p^2 + \tfrac{1}{2}p(1-p)] = \frac{p^2 + \frac{1}{2}p(1-p)}{1-p(1-p)}.$$

 A wrong cascade exists with probability

$$\sum_{k=1}^{\infty}[p(1-p)]^{k-1} \times [(1-p)^2 + \tfrac{1}{2}(1-p)p] = \frac{(1-p)^2 + \frac{1}{2}(1-p)p}{1-p(1-p)}.$$

 (e) A cascade exists with probability

$$\frac{p^2 + \frac{1}{2}p(1-p)}{1-p(1-p)} + \frac{(1-p)^2 + \frac{1}{2}(1-p)p}{1-p(1-p)} = 1.$$

38. Because a cascade is essentially triggered by only one pair of identical signals, a small change in the information environment can easily change the formation of a cascade.

39. For example: If there is an expert (whose signal is more accurate than the others) in the sequence:

- If this expert shows up after a cascade already occurred, he may be able to overturn the current trend.

- If this expert is the first player in the sequence, then a cascade starts immediately.

40. Because cascades stop social learning, delaying cascades can expose more private signals and improve social welfare.

    For example, if there is a tiny percentage of anonymous laypeople (whose signals are less accurate than normal people) in the population:

    (a) Overall signal accuracy of the population decreases slightly.

    (b) But normal people do not jump into a cascade so quickly $\Longrightarrow$ enhanced social learning.

    (c) In general, a tiny percentage of laypeople can help decrease Prob$(incorrect)$; see Wu (JEBO 2015).

## Task

- Reading:
- Understanding: